ELECTRIC AND MAGNETIC MEASUREMENTS

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New York
THE MACMILLAN COMPANY
1919

Set up and electrotyped. Published February, 1917.

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

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INTRODUCTION

PART I. LABORATORY METHODS

1. Treatment of Errors in Laboratory Work. In everyday life it is not uncommon to hear positive statements as to the accuracy of frequently measured magnitudes, such as mass or length, although every one knows that instruments of increased precision would show more or less important deviations from the stated values. In accurate scientific work, the *ideal* is to make measurements which shall be without error. This ideal is not attainable, however, and the methods chosen, as well as the values observed, must be studied carefully in order to determine the most probable value of the quantity which is being measured. In every measurement attention must be paid to all the conditions which may affect the correctness of the value sought, and an estimate must be made of the reliability, or probable precision, or trustworthiness, of the final result.

Let us suppose that a galvanometer deflection of 8.55 cm. is observed, and let us assume that a careful estimate of the probable precision of the measurements leads to the conclusion that the observer can be sure of a single reading to within 0.02 cm. The deflection will be recorded in the form

$$d = 8.55 \pm 0.02 \text{ cm.},$$

which means that the recorded result will not be in error by more than 0.23 %.

Errors of various kinds arise in making measurements, the most important of which are the following:

Errors of observation, due to lack of ability on the part of the observer to make accurate readings.

Simple mistakes, which grow less troublesome as the observer gains in training and experience.

Instrumental errors, due to incorrect calibration of scales or to faulty adjustment of comparison standards.

Systematic errors, due to defects in the method used.

It is readily seen that the effect of errors of observation is reduced by taking the average of a great many measurements, in which the chance that some readings are too large is offset by the chance that others are too small. Apparatus of a higher grade, more carefully made and calibrated, is important for the elimination of instrumental errors, while repeating the experiment by independent methods reduces the systematic errors.

2. Laboratory Methods. In order to accomplish results in the laboratory it is of fundamental importance first to understand thoroughly what is to be done, and then to proceed in a systematic manner to do it.

In general, formulas and circuit diagrams should not be memorized, but instead they should be reproduced as the result of careful study and logical thought. The student should be able, before performing the experiment, to derive the formulas and to draw the circuits. This should be the outcome of a thorough understanding of the argument of the problem, however, rather than definite acts of memory.

The following points should be carefully observed:

(1) Make sure that you know fully the purpose and the theory of the experiment, including the meaning of all the leading terms.

(2) Be sure that you know precisely what data are to be secured.

(3) Prepare in advance a schedule or program according to which the observations will be taken, and make a ruled table with headings for the record. This makes it unlikely that any step will be omitted, and furnishes a check that the necessary data are in hand.

(4) Secure a laboratory notebook or observation journal of a convenient size, preferably permanently bound, and keep it exclusively for this course, dating each set of entries.

(5) Frequently in engineering practice, one person takes the observations and another person reduces and discusses them, perhaps in a distant city, and after a considerable interval of time. This renders it imperative to make a set of observational data complete in itself, perfectly clear and readable and with explanatory notes sufficiently copious so that the influence of the experimental environment may be fully grasped. The student should keep this in mind, no matter how simple may be the problem assigned him.

(6) Scrutinize the conditions under which the observations are taken and state as a part of the original data the probable precision of the single readings or measurements.

(7) Proceed systematically in making the required adjustments. The student will be greatly aided in this work by arranging the circuits in a neat and orderly manner, always avoiding a tangled set of connecting wires.

Always sketch out in the observation journal the precise circuit as it was used, before it is disconnected.

(8) **The report** should contain the following subdivisions:

(*a*) A brief statement of the purpose of the experiment.

(*b*) A list of the apparatus used. Each piece used should be listed, together with the maker's name and the number, for purposes of identification, and such pieces as are of special design, or of exceptional interest in any way, should be described.

(*c*) A brief analysis of the entire procedure, which will include a concise review of the whole problem, the physical principles involved, and the fundamental definitions.

(*d*) A brief statement of the actual steps of the manipulation.

(*e*) A table of data.

(*f*) Any formulas used, together with the meaning of each symbol, and a sample calculation. The derivation of the formula is usually necessary in order to make a clear statement of the problem as required under (*c*) above.

(*g*) A statement of the degree of precision attained in the observations and in the final result.

(*h*) Do not omit the units in which the result, and any other important quantities, are expressed.

(*i*) Give any necessary discussion of the results and answer any questions asked in the text, or by the instructor.

(9) **Precautions.** Bear in mind that the apparatus used in the electrical laboratory is expensive and essentially delicate. Handle each piece with the utmost care, and report promptly any accidents which may occur. In using electrical measuring instruments *proceed with caution.* Use the lowest shunt first, or the highest series resistance, or the scale of greatest range, in order to avoid currents too large for the apparatus. In general the circuit should be looked over and approved by an instructor before final connections are made. The connection to the source of power should always be the last one made, and this should be done cautiously. The importance of good contacts must not be overlooked. The ends of connecting wires must be scraped clean and joined under pressure with double connectors or in binding posts, never by loosely twisting them together with the fingers.

Ordinarily tap keys and small switches are only intended for use with feeble currents and with small differences of potential. Sparking and consequent oxidation of the contact points will result from the use of keys or switches with currents which are beyond their intended capacity.

Part II. Fundamental Definitions and Units

3. Fundamental Concepts and Definitions. The three quantities upon which most of the fundamental relations of electric measurement are based are magnetic pole strength, magnetic field strength, and current strength.

The unit of magnetic pole strength is that strength of pole which will repel a similar pole of equal strength at a distance of one centimeter, in air, with a force of one dyne.

The unit of magnetic field strength is that field in which the unit pole is acted on by a force of one dyne.

A magnetic field is always specified in terms of its effect on the unit pole. The direction of the field is the direction in which the free north-seeking pole will move, and its intensity is measured by the force which acts upon a unit pole placed in the field.

When a magnetic pole of strength m units is placed in a field of strength H units, the force F, in dynes, acting on the pole is given by the formula

$$F = mH \text{ dynes.}$$

The unit of current strength, in the electromagnetic system, is that current which, flowing through a conductor placed normal to the field, will experience a side thrust of one dyne for each centimeter of its length.

Direction of the magnetic field. The direction of the magnetic field about a straight wire carrying a current is clockwise as one looks along the wire in the direction of the current. In a circular loop or solenoid the current is clockwise to an observer looking along the axis in the direction of the field.

Reactions between magnetic fields. It is often convenient to regard magnetic fields as composed of discrete lines of force. According to this conception the following rules for aiding the memory are useful.

Two magnetic fields when superposed can develop force

actions between themselves only when they have components which are parallel. If such parallel components are in the same direction, the fields will repel one another, and if in opposite directions, they will attract one another. In representing these fields on paper, the plane of the paper must contain these parallel components.

Current uniform over any cross-section. A steady current which is maintained through a circuit of constant resistance by means of an electromotive force has the same value throughout the entire circuit. There cannot be any increase or decrease of current strength in one portion of the circuit as compared with any other portion.

Ohm's law expresses the relation which exists between the three fundamental quantities, current strength I, electromotive force or potential difference E, and resistance R. It may be written in any one of three ways:

$$I = \frac{E}{R}. \tag{1}$$

$$E = IR. \tag{2}$$

$$R = \frac{E}{I}. \tag{3}$$

Two cases of the application of this law will be considered: first, when the entire circuit is taken into account, and second, when only a portion of the circuit is concerned.

(1) If the entire circuit is considered, the value of E in the above equations must be the maximum potential difference, or the E. M. F. of the generator. If more than one generator is in the same circuit, E will be the algebraic sum of all the separate electromotive forces. Similarly, R must be the sum of all the resistances in the circuit, including the internal resistance of the generator, or of all the generators, if there is more than one. For this case, the law will be written in the form

$$I = \frac{\Sigma \pm E}{\Sigma R}. \tag{4}$$

(2) If a limited portion of a circuit is considered, in which there is no generator, the value of E in the above equations is the potential difference impressed across the terminals of the constant resistance R (Fig. 1).

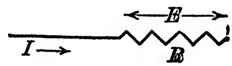

FIG. 1.

Any variation in the potential difference is accompanied by a corresponding variation in the current strength, and the ratio E/I is constant, and always equal to R.

Potential drop. The product of the resistance of any part of a circuit, by the current flowing in that part, equation (2) above, is called the ***potential drop***, the ***fall of potential***, or the ***IR drop***. The algebraic sum of all such products taken around the entire circuit is then equal to the effective or resultant E. M. F. in the circuit.

Series and parallel combinations of resistance. When several resistances of values r_1, r_2, r_3 ··· are connected in series, the equivalent resistance of the group is given by the formula

$$R = r_1 + r_2 + r_3 \cdots.$$

When several resistances of values r_1, r_2, r_3 ··· are connected in parallel, or in multiple, the equivalent resistance of the group is given by the formula

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \cdots.$$

4. Establishment of Electric Units. For measurements of length, mass, and time, the absolute units of the C. G. S. system are of convenient magnitudes. However, the electromagnetic units derived from these are much too large or too small for practical uses. During the early development of electric theory the units were in a confused state, frequently with different values for units of the same name.

Several international conferences, with the leading physicists and engineers of the world as delegates, were called from time to time. At the International Electrical Congress, held in

Chicago in 1893, formal definitions for the principal electric units[1] were adopted, and the numerical magnitudes of the ohm, volt, and ampere were fixed. All of these units were designated as the ***International Units.*** These units were made legal in the United States in January, 1894. Since that time slight changes have been recommended by the International Conference at London in 1908, and by its authorized committee.[2] The units as adopted by this Conference are defined below.

5. Fundamental Electric Units. The ***ohm*** is the unit of resistance, which has the value of 10^9 in terms of the centimeter and the second.

The ***ampere*** is the unit of current strength, which has the value of 10^{-1} in terms of the centimeter, gram, and second.

The ***volt*** is the unit of electromotive force, which has the value of 10^8 in terms of the centimeter, gram, and second.

The ***watt*** is the unit of power, which has the value of 10^7 in terms of the centimeter, gram, and second.

6. The International Electric Units. In order to represent these fundamental units practically for purposes of actual measurement, and as a basis for legislation, the Conference recommended the adoption of the International Units, defined as follows:[3]

The ***international ohm*** is the resistance offered to an unvarying electric current by a column of mercury[4] at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area, and of a length of 103.300 centimeters.

[1] The volt, ampere, ohm, coulomb, farad, henry, joule, and watt.

[2] For details of this Conference see: LONDON ELECTRICIAN, Vol. 62, 1908–09, p. 104. U. S. BUREAU OF STANDARDS: (*a*) *Circular No.* 29, (*b*) Miscellaneous publications. *Report of the International Committee on Electrical Units and Standards.*

[3] U. S. BUREAU OF STANDARDS, *Circular No.* 29.

[4] For exact specifications see *Report of the London Conference*, LONDON ELECTRICIAN, Vol. 62, 1908–09.

The ***international ampere*** is the unvarying electric current which, when passed through a solution of silver nitrate in water,[1] deposits silver at the rate of 0.00111800 gram per second.

The ***international volt*** is the electric pressure which, when steadily applied to a conductor the resistance of which is one international ohm, will produce a current of one international ampere.

The ***international watt*** is the energy expended per second by an unvarying electric current of one international ampere, under an electric pressure of one international volt.

Based upon the foregoing, the following international units are readily defined.

The ***joule***, equivalent to 10^7 C. G. S. units, is the work done when one ampere flows for one second, under an electric pressure of one volt.

The ***coulomb***, equivalent to 10^{-1} C. G. S. units, is the quantity of electricity transferred by a current of one ampere in one second.

The ***farad***, equivalent to 10^{-9} C. G. S. units, is the capacity of a condenser which is charged to a potential of one volt by one coulomb.

The ***henry***, equivalent to 10^9 C. G. S. units, is the inductance of a circuit in which an E. M. F. of one volt is established by a current varying at the rate of one ampere per second.

In addition to the above units, there were adopted by the International Convention of Electrical Engineers at Paris in 1900 the following:

The ***maxwell***, equivalent to one C. G. S. line of force, is the unit of magnetic flux.

The ***gauss***, equivalent to one maxwell per square centimeter, is the unit of magnetic flux density.

[1] For exact specifications see *Report of the London Conference*, LONDON ELECTRICIAN, Vol. 62, 1908–09.

PART III. DIMENSIONS AND DIMENSIONAL FORMULAS

7. Units and Dimensions. In order to define any physical quantity, there must be given (a) a *unit* and (b) a *numerical coefficient* which shows how many times the unit is repeated. This is clearly seen in the expression of such quantities as 10 pounds, or 30 miles per hour.

It has been found that nearly all of the units used in physics can be referred back to, and expressed in terms of, three fundamental units, length, mass, and time. The *powers* to which these fundamental units are severally raised in the formula for any derived unit are called the ***dimensions*** of that unit. For example, an area is the square of a length, hence the *dimension* of area is 2 as regards length. Similarly, a volume has the dimension 3 in length, while a velocity has the dimension 1 in length and -1 in time.

In writing the dimensions of units attention must be paid to the defining equation of the quantities concerned. In the table below, the dimensions of a few units are given, together with the defining equations of the quantities.

QUANTITY	DEFINING EQUATION	DIMENSIONS
Area	$a = l^2$	$[L^2]$
Volume	$v = l^3$	$[L^3]$
Velocity	$v = \frac{l}{t}$	$[LT^{-1}]$
Acceleration	$a = \frac{v}{t} = \frac{l}{t^2}$	$[LT^{-2}]$
Force	$f = ma$	$[MLT^{-2}]$
Work	$w = fl$	$[ML^2T^{-2}]$
Power	$p = \frac{w}{t}$	$[ML^2T^{-3}]$

The expressions in the third column are called ***dimensional formulas***, and it is customary to represent the three fundamental units by capital letters, and to enclose the entire group in square brackets, using negative coefficients where units enter into the denominator of the defining equations. These formulas indicate how the units of the various quantities involve the three fundamental units in terms of which they are defined, without any reference whatever to the numerical coefficients which may be associated with them. These numerical coefficients may be either the numbers which represent the multiples of the units used, or they may be

quantities like π, or the trigonometric functions. In any case such numerical coefficients have no dimensions, and do not enter into dimensional formulas.

8. Dimensional Equations. An equation signifies that the quantities connected by the sign of equality must be alike in kind, as well as identical in magnitude. The actual numbers substituted in formulas in making computations are the numerical coefficients, but all equations must be carefully examined in order to ascertain whether the dimensions of the units on both sides of the equation are identical. Such equations with their various qualities expressed in terms of dimensions, as shown above, are called ***dimensional equations.***

9. Uses of the Theory of Dimensions. The study of the dimensions of physical quantities is useful in three ways:

(1) *To check formulas.* The dimensional formulas may be used to determine whether an equation is homogeneous with respect to the fundamental units involved in it.

Consider the familiar equation

$$x = s_0t + \tfrac{1}{2}\,at^2,$$

where x is the distance passed over in t seconds, by a particle with initial speed s_0, and a constant acceleration a. Written in dimensions this becomes

$$[L] = [LT^{-1}T] + [LT^{-2}T^2].$$

It is seen that x is the sum of two lengths. Indeed, quantities of different kinds would have no meaning if added in this way.

Again, the equation for the period of a simple pendulum is

$$T = 2\,\pi\sqrt{\frac{l}{g}}.$$

Written with regard to dimensions only, this becomes

$$[T] = [L^{\frac{1}{2}}L^{-\frac{1}{2}}T]$$

or

$$[T] = [T].$$

In these examples it appears that every term in any one of the equations has the same dimensions. If this were not so, it would be proof that there was some error in the formula.

Dimensional equations do not check the correctness of the numerical work in computing; they only show whether the reasoning has been correct with regard to the fundamental units involved. This may be further illustrated by applying dimensions to the distinction between mass and weight, which are frequently used incorrectly by the student.

Weight is a force and hence has the dimensions $[MLT^{-2}]$, while mass is itself a fundamental unit with dimension $[M]$.

(2) *To designate unnamed units.* It is frequently convenient to use the fundamental units as they appear in the dimensional formulas when no name has otherwise been given to a unit. Thus, the unit of speed is given as foot/second or centimeter/second; the unit of acceleration is foot/second^2 or centimeter/second^2; the unit of moment of inertia as gram-centimeter^2 or pound-foot^2.

(3) *To find the new value of the numerical coefficient when the system of units is changed.* If n_1 is the numerical coefficient of a given quantity whose unit is q_1, and if n_2 is the numerical coefficient of the same quantity when expressed in terms of another unit q_2, then

$$n_1q_1 = n_2q_2$$

or

$$n_2 = n_1\frac{q_1}{q_2}.$$

For example, if the quantity 10 pounds is to be expressed in terms of the gram as a unit, then n_2 equals the product of n_1 by the ratio of the pound to the gram, and

$$n_2 = 10 \times 453.6.$$

As another example of the usefulness of the method, we may find what number will represent 33,000 foot-pounds per minute when C. G. S. units are used. The number 33,000 must first be reduced to the equivalent number of absolute units in the F. P. S. system or

$$\tfrac{33000}{60} \times 32.1 = 550 \times 32.1 \text{ foot-poundals per second.}$$

Then

$$n_1[M_1L_1^2T_1^{-3}] = n_2[M_2L_2^2T_2^{-3}],$$

$$n_1[(\text{gram})(\text{centimeter})^2(\text{second})^{-3}] = n_2[(\text{pound})(\text{foot})^2(\text{second})^{-3}],$$

$$n_1 = 550 \times 32.1\left[\left(\frac{\text{pound}}{\text{gram}}\right)\left(\frac{\text{foot}}{\text{centimeter}}\right)^2\left(\frac{\text{second}}{\text{second}}\right)^{-3}\right].$$

$$n_1 = 550 \times 32.1[453.6 \times (30.5)^2] = 745^{+} \times 10^7 \text{ ergs per second.}$$

10. Dimensions of Electric Units. There are two systems of electric units, the ***electrostatic*** and the ***electromagnetic***, based, respectively, on the unit charge and the unit magnetic pole.

Electric charge. The fundamental equation in the electrostatic system which expresses the force action between two electric charges q_1 and q_2, at a distance apart r, is given by

$$F = \frac{q_1q_2}{kr^2}.$$

In this equation k is a constant which depends upon the medium. The dimensional formula for charge is derived in the following way. From the above equation

$$
\begin{aligned}
q_1q_2 &= Fkr^2,\\
q &= [(Fk)^{\frac{1}{2}}r],\\
q &= [(MLT^{-2})^{\frac{1}{2}}k^{\frac{1}{2}}L],\\
q &= [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}k^{\frac{1}{2}}].
\end{aligned}
$$

Magnetic pole strength. The fundamental equation in the electromagnetic system which expresses the force action between two magnetic poles, m_1 and m_2, at a distance apart r, is given by

$$F = \frac{m_1m_2}{\mu r}.$$

In this equation μ is a constant which depends upon the medium. The dimensional formula for magnetic pole strength is derived in a manner similar to that for charge:

$$
\begin{aligned}
m_1m_2 &= F\mu r^2,\\
m &= [(F\mu)^{\frac{1}{2}}r],\\
m &= [(MLT^{-2})^{\frac{1}{2}}\mu^{\frac{1}{2}}L],\\
m &= [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}\mu^{\frac{1}{2}}].
\end{aligned}
$$

In the above equations for charge and magnetic pole strength it is difficult to understand the significance of the fractional exponents. Under the usual assumption that k and μ are both unity it appears that the dimensions of m and q are the same, which could only mean that they are dimensionally identical. This seems peculiar, and the explanation of the apparent absurdity lies in the fact that both k and μ may themselves possess dimensions. But since we do not know the precise mechanics of the electrostatic and magnetic phenomena of the ether, we cannot write the dimensional formulas of these quantities. If we could do so, the fractional exponents might be rationalized. Valuable theoretical results follow from the use of the equations containing k and μ, but the usefulness of the method, so far as the immediate applications of dimensions are concerned, is not impaired by omitting them. The remaining equations will be derived on the assumption that the phenomena take place in air, in which case k and μ are taken as equal to unity. The dimensions of electric quantities based on the electrostatic system will not be considered further.

11. The Electromagnetic System. In deriving the dimensional formulas based on the electromagnetic system of units, the starting point is always the force action associated with the unit magnetic pole.

Magnetic field strength. When a magnetic pole of strength m is placed in a magnetic field of strength H, the force action is given by

$$F = mH$$

and

$$H = \frac{F}{m}.$$

The dimensional formula for this is

$$H = \frac{MLT^{-2}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}} = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}].$$

Magnetic flux. Magnetic flux is the product of field strength by the area of cross-section, taken at right angles to the field, or

$$\phi = Ha.$$

Hence its dimensions are

$$\phi = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}L^2] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}].$$

Electric current. In deriving the dimensional formula for any quantity, any defining equation may be used, so long as the dimensions of all of its involved units are known, except the one to be derived. In the case of current, we may start with the force on a conductor of length l, placed at right angles to a magnetic field of strength H, and carrying a current of strength i.

$$F = iHl$$

and

$$i = \frac{F}{Hl}.$$

The dimensions of current are then

$$i = [MLT^{-2}M^{-\frac{1}{2}}L^{\frac{1}{2}}TL^{-1}] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}].$$

Quantity of electricity. The quantity of electricity which passes through a circuit with a constant current i, in time t, is given by

$$Q = it.$$

Its dimensions are

$$Q = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}T] = [M^{\frac{1}{2}}L^{\frac{1}{2}}].$$

Potential difference. The difference of potential between two points is measured by the ratio of the work done to the charge carried, or

$$V = \frac{W}{Q}.$$

Its dimensions are

$$V = [ML^2T^{-2}M^{\frac{1}{2}}L^{-\frac{1}{2}}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}].$$

Resistance. Resistance may be defined from Ohm's law, in terms of current and potential difference:

$$R = \frac{V}{i}.$$

Its dimensions are

$$R = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}M^{-\frac{1}{2}}L^{-\frac{1}{2}}T^{1}] = [LT^{-1}].$$

Inductance. Inductance may be defined in terms of the number of linkings per unit current, where N is given by the product of the magnetic flux by the number of wire turns:

$$L = \frac{N}{i} = \frac{n\phi}{i} = \frac{nHa}{i}$$

where a is the area of the cross-section.

Since n is a pure number it need not be further regarded, and the dimensional formula of inductance is

$$L = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} \cdot L^{2} \cdot M^{-\frac{1}{2}}L^{-\frac{1}{2}}T] = [L].$$

Capacity. The charge in a condenser is given by the product of the capacity by the charging potential difference, whence

$$Q = CV$$

and

$$C = \frac{Q}{V}.$$

Its dimensions are

$$C = [M^{\frac{1}{2}}L^{\frac{1}{2}}M^{-\frac{1}{2}}L^{-\frac{3}{2}}T^{2}] = [L^{-1}T^{2}].$$

The student is not advised to attempt to memorize dimensional formulas. The defining equations will be familiar, however, and from these the dimensional formulas for all the important quantities can readily be derived.

EXERCISES

1. Find the number of joules equivalent to 100 foot-pounds.

2. A mass of 100 lb. moves with a speed of 500 ft. per second. Find its kinetic energy in foot-pounds, also in kilogram-meters.

3. Derive the dimensional formula for inductance from the intrinsic energy equation.

4. Check the correctness of the Helmholtz equation by means of dimensions.

5. Derive the dimensional formula for resistance from Joule's law.

6. Derive the dimensional formula for potential difference from the Faraday equation.

Part IV. Miscellaneous Information

12. Prefixes. Certain prefixes are so frequently used in scientific work that they should be quite familiar. The more common ones with their meanings are

meg- or mega-	one million
kilo-	one thousand
hekto-	one hundred
deka-	ten
deci-	one tenth
centi-	one hundredth
milli-	one thousandth
micro-	one millionth

13. Keys and Switches. It is frequently desirable in electric work to open or close a circuit quickly, or to change the connections between circuits rapidly, without the delay or inconvenience of adjusting wires in binding posts. Various forms of keys and switches are used for this purpose. The simple tap key is shown in Fig. 2. A leaf spring is held rigidly at *e* on a base *b*, and is provided with

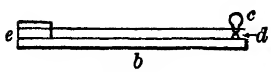

Fig. 2.

a platinum pin at *d*, which may be brought into contact with a platinum surface beneath it when the finger is pressed on the knob *c*. If the lower contact and the spring at *e* are connected to a pair of binding posts which are in turn connected to any circuit, the current may be interrupted or established as desired. Platinum contacts are necessary in order to prevent oxidation. Such keys are intended for use with feeble currents and low voltage only.

A switch is any device for making, breaking, or changing the connections in an electric circuit. A single-pole, single-throw switch is shown in Fig. 3, and in any given case one will be

chosen of sufficient size to carry safely the current required. An arrangement of two such switches on the same base, with their movable blades rigidly connected by an insulating bar, is called a double-pole, single-throw switch.

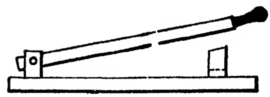

FIG. 3.

If the blades are so arranged that they may be thrown clear over, and made to engage jaws on the opposite side of the rocking point, it is called a double-pole, double-throw switch. A plan view of such a device is shown in Fig. 4. By throwing over the switch blades pivoted at *cd*, the circuit attached to the terminals *cd* may be connected to either of two other circuits connected respectively at *ab* or *ef*.

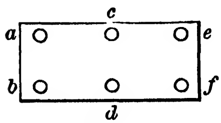

FIG. 4.

By putting cross connections between the terminal posts as shown in Fig. 5, the current in a circuit connected at *ef* may be reversed through the circuit *cd* by throwing the switch blades from one extreme position to the other.

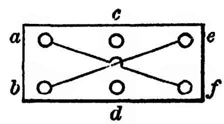

FIG. 5.

CHAPTER I

GALVANOMETERS

PART I. GENERAL TYPES AND CHARACTERISTICS

14. Classification of Instruments. Nearly all methods of electric or magnetic testing require the measurement of one or more of three quantities, current strength, potential difference, or charge. Frequently, however, the problem in hand requires only that the presence or absence of current, potential difference, or charge, be shown. In general, essentially the same instrument, with certain modifications, may be used either to detect the existence of one of these quantities or to measure its magnitude.

An instrument for simply indicating the absence of a current in a circuit is called a ***detector*** or an ***indicating galvanometer.***

When used to measure current strength, it is called a ***current galvanometer***, or simply a ***galvanometer.*** When provided with a scale graduated to read amperes, it becomes an ***ammeter.***

When arranged to measure a potential difference, it is called a ***potential galvanometer***, and if provided with a scale graduated to read volts, a ***voltmeter.***

If arranged to measure quantity, it is called a ***ballistic galvanometer.***

In general, the term *galvanometer* is reserved for a class of instruments used for either measuring or indicating relatively feeble currents by means of their magnetic effect.

15. Galvanometer Types. Many instruments of different kinds have been designed for the purpose just mentioned. Most of them depend upon force actions between magnetic fields. Whatever the scale of the instrument reads, or whatever the quantity to be measured, the observed effect is really due to a feeble current flowing through fixed or movable coils, in the neighborhood of movable or fixed magnets or other coils. The observed motion results from the attraction and repulsion of the magnetic fields. One member of the system must be free to move about its supports. Two types are common, the ***suspended-needle type*** and the ***suspended-coil type.***

In the *suspended-needle type*, a magnetic needle is freely suspended by a light fiber of silk or quartz at the center of a coil of wire through which the current flows. The field about this coil tends to set the needle parallel to itself, and any motion of the suspended system may be observed directly, or more accurately by means of a beam of light reflected from a small mirror fastened to the moving part. It is possible to so design the parts that the deflections are very nearly proportional to the currents passing through the coils.

In the *suspended-coil type*, a permanent magnet is fixed in position, and between its poles a coil of wire through which the current is to pass is hung by a thin ribbon of phosphor bronze or steel. A helix of similar material attached to the bottom of the coil serves as a flexible connection with one of the coil terminals, the other being attached to the upper suspension strip.

16. Torque. Any rotation of the movable parts of the instrument is due to a torque or turning moment applied to it. Figures 6 and 8, pages 20 and 22, show respectively how the reaction of the magnetic fields causes the torque in each of the two types.

For the suspended-needle type (Fig. 6) imagine a single circular loop of wire, whose plane lies in the magnetic meridian *NS*, and which is perpendicular to the plane of the paper intersecting this plane in *A* and *B*. If a current is flowing clockwise in this loop as seen from the right-hand side, the

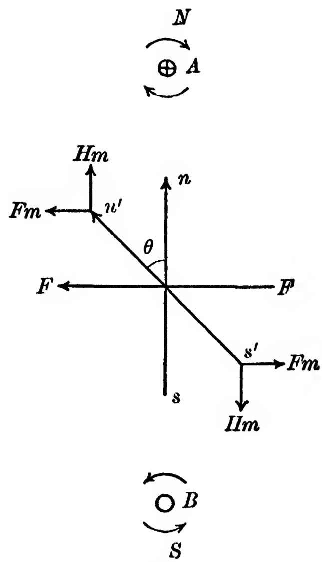

Fig. 6.

lines of force will be shown by the arrowheads about *A* and *B*, giving a resultant field *FF* at the center of the coil. A magnetic needle originally in the position *ns* will be deflected through an angle θ to $n's'$, at which position the moment of the deflecting couple is just compensated by the moment of the restoring couple. Assume the strength of pole of the needle to be *m* units. It will then be acted upon by the field *F* with a force of *Fm* dynes. Similarly, the south pole is acted upon by an equal and opposite force. This pair of equal, parallel, and oppositely directed forces, not in the same line, constitutes a force couple, the moment of which is given by the product of one force by the perpendicular distance

between the lines of action. The moment of this deflecting couple is

(1) $$L = Fml \cos \theta.$$

This couple will deflect the needle until its effect is overcome by the influence of the magnetic field (H) in which it swings, which exerts a force of Hm dynes on each pole. These two restoring forces constitute a force couple of which the moment is

(2) $$L = Hml \sin \theta.$$

It will be shown in Chapter IV that the magnetic field is directly proportional to the strength of current in the coils. The magnetic field H is the resultant field due to the horizontal component of the earth's field, together with that of any control magnet which may be in use with the galvanometer. Strictly, the torsion of the suspension fiber is also acting against the deflecting couple, but effort is made to reduce this

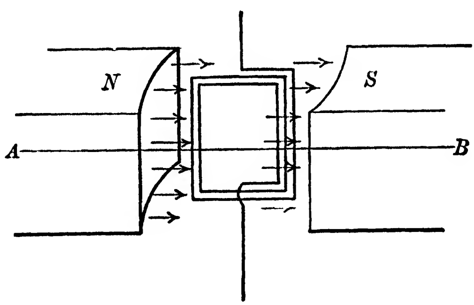

FIG. 7.

to a small value by proper selection of the fiber. It is considered only in precise work of a certain kind.

For the suspended-coil type imagine the current to be flowing down in the right-hand side of the rectangular coil as shown in Fig. 7. A cross-section of Fig. 7 through AB is shown in Fig. 8, page 22, the current flowing away from the reader at a,

and toward the reader at b. The arrows show the direction of the magnetic fields about each coil and about the permanent magnet NS. Since magnetic fields in the same direction tend to repel one another, while those in opposite directions tend to attract, the resultant torque, due to the reaction of the two fields, will cause the coil to rotate in a clockwise direction.

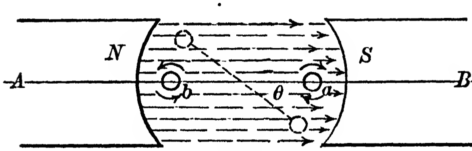

FIG. 8.

The value of the torque is found as follows. Assume the coil to be l cm. long, and b cm. broad, and to have a current of I C. G. S. units flowing through its turns, which lie in a field of value H due to the permanent magnet. The side thrust on one wire in the field H is given by

$$f = IHl \text{ dynes}$$

and on one side of the coil of n turns it is

$$F = IHln \text{ dynes.}$$

The moment of this force is $(b/2)IHln$. Since there are two sets of wires corresponding to the two sides of the coil, the full torque L is given by

$$L = 2\frac{b}{2} IHln.$$

The product bl may be replaced by a, the area of the coil, whence

$$L = IHan. \tag{3}$$

Equation (3) is only true in case the angle θ is very small, for as rotation takes place the effective lever arm is shortened, and the value of the torque becomes

$$L = IHan \cos\theta. \tag{4}$$

17. Suspension and Control. The movable part of the galvanometer may be suspended on carefully ground pivots in jewel bearings, or in the sensitive instruments, by means of a fiber of some material which is as free from torsion as possible. Unspun silk is frequently used, while an excellent fiber can be made by drawing into a fine thread a bit of melted quartz. These quartz fibers can be drawn exceedingly fine, and their tensile strength is relatively high. They are also free from troublesome elastic after effects.

In the case of the moving coil instruments, in which the suspension must carry the current to and from the coil, very fine phosphor bronze or steel ribbon is used above and a spiral spring of similar material below. These ribbons are made by rolling a wire 0.0015–0.0040 inch in diameter into a flat ribbon.

In order that the movable system may return to its position of equilibrium after the current has ceased to flow, there must be a controlling torque.

Referring to Fig. 6, it will be seen that the control is here due to the magnetic field in which the needle swings, the forces being represented by the vertical arrows at the poles of the needle. In the moving-coil instruments, the control is due entirely to the torsion in the suspending metal ribbon and the lower spring.

18. Damping. When deflected by a current, the suspended system, controlled as it is by a restoring torque which increases with the angle of deflection, will vibrate with progressively decreasing amplitudes about its final position before coming to rest. Similarly, when the current ceases, there will be an oscillatory motion of the suspended system about its position of equilibrium. The final position of rest is only reached after all the energy imparted to the system has been given up. Since it is tedious to wait for the gradual dying away of this motion, artificial means are provided for absorbing the energy

in order to bring the system to its final position as speedily as possible.

This process is called ***damping.*** Damping is usually accomplished either by friction of the air against a light vane attached to the moving parts, or by the generation of induced

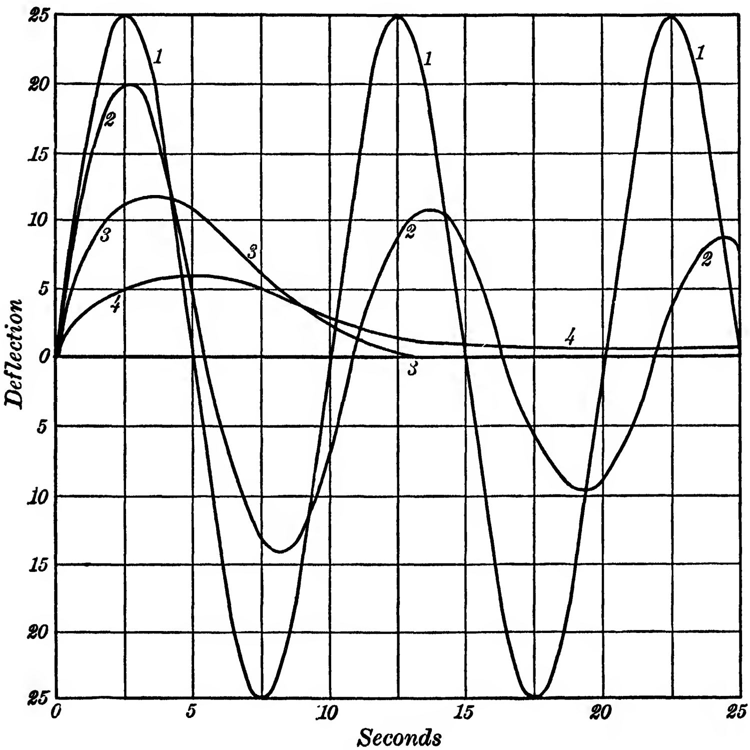

FIG. 9.

currents due to the relative motion of conductors and magnetic fields, according to Lenz's law.

If a system could be imagined entirely devoid of damping, it would continue to vibrate indefinitely after a displacement, as shown in Fig. 9, curve 1. As the damping increases, the progressive decrease in amplitude becomes greater, as in curve 2, until finally a degree of damping is reached for which the

motion just loses its periodic character and becomes *aperiodic*, as shown in curve 3, Fig. 9. The critical value of the damping for which this transformation takes place is also the value for which the return of the system to its zero position is the quickest, with no overswing to the other side. As the damping increases beyond this critical value, the return to zero becomes slower, and this condition is less favorable for rapid work. See curve 4, Fig. 9.

19. Mirror and Scale Methods of Reading. The simple and direct method of attaching a light pointer to the moving part of the galvanometer, and reading the deflection as this pointer swings over a graduated circular scale, is used when great accuracy is not required, and when the moving parts are relatively massive, so that the additional mass of the pointer is negligible.

For sensitive instruments and precise work a small, light mirror is fastened to the moving parts, and a ray of light is made to serve as a pointer, as described below.

When used in this way, the instrument is called a ***reflecting*** galvanometer.

The lamp and scale. In Fig. 10, SS' represents the scale at a distance d in front of the mirror, which is, in this case, concave. A luminous source, such as the filament of an incandescent lamp, or a Nernst lamp glower, is mounted at L, just above or below the scale. The parts are adjusted so that the image of the filament formed by the mirror shall fall at the zero of the scale, and if the scale is translucent, the position of the line of light may be read from the side opposite the mirror.

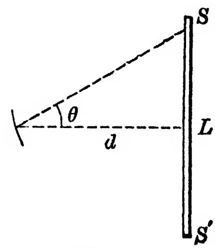

FIG. 10.

The telescope and scale. In Fig. 11, SS' represents the scale, T the telescope, and m a plane mirror fastened to the

moving part of the galvanometer. The portion of the scale at P is reflected by the mirror into the telescope, the objective lens o forming a real image of the scale at x. At this point are placed two crossed threads, usually fibers of spider's web, and these must be in the focal plane of the objective o, as well as of the eyepiece e. The cross threads will then be clearly seen superposed on the image of the scale. Any motion of the mirror will then cause an apparent displacement of the scale with respect to the cross threads.

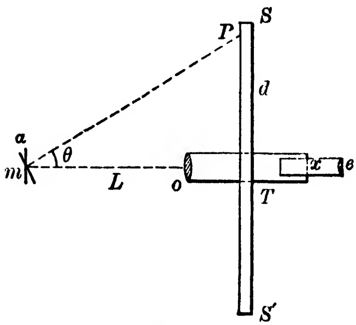

FIG. 11.

If a ray of light is reflected from a mirror, making an angle θ with a normal drawn to the surface of the mirror, and if the mirror is rotated through some angle α, it can be shown that any change in α produces a change just twice as great in θ (Fig. 11). It follows that $\tan 2\alpha = \tan\theta = d/L$, where L is the distance from mirror to scale. Since for small angles $\tan 2\alpha$ may be put equal to $2\tan\alpha$, we may write

$$\tan\alpha = \frac{d}{2L}. \tag{5}$$

Hence it is obvious that for a uniformly graduated straight scale, the scale readings are not proportional to the angles of deflection of the mirror, but rather to tangents of twice these angles. For very small deflections, scale readings may be assumed proportional, but for large deflections, greater than about five degrees, the angles should be computed.

Some galvanometers have curved scales of constant radius, in which case scale readings are proportional to angles of rotation.

Frequently the galvanometer is used in zero methods, in which case a short straight scale with arbitrary graduations is sufficient.

Part II. Descriptions of Special Types of Galvanometers

20. Characteristics. The common galvanometers available for use in the laboratory will be here discussed with reference to the five points mentioned above: (*a*) how the torque is set up; (*b*) suspension; (*c*) control; (*d*) damping; (*e*) method of reading.

21. Suspended Needle Galvanometers. Two examples of this type will be considered.

The ***Thomson*** or ***Kelvin galvanometer*** is named from Sir William Thomson (Lord Kelvin), who perfected it. The torque is set up by the reaction of the field about the needle with the field about the coils. The suspension is a silk or quartz fiber. The control is chiefly magnetic. Damping is effected by means of an air vane. Readings are made with the lamp and scale, or with the telescope and scale.

It is obvious that the sensibility increases as the controlling force is weakened. Hence, to reduce the control as much as possible, use is made of the *astatic* system. This consists of two similar magnetic needles mounted one above the other on the same support, but with their poles reversed in direction. Surrounding each needle is a coil of many turns, the two coils being connected in series and with the direction of their fields reversed. With this arrangement the turning effect of the feeble current is doubled. Were the two needles identical as to magnetic moment, the effect of the earth's field would be exactly neutralized, and the system would stand indifferently in any position. If, however, there is a slight outstanding difference in the magnetic moments of the two needles, which difference can be made as small as desired, the controlling force can be made very small.

For a sensitive galvanometer of this type it is necessary that there should be a large number of turns of wire in the

coils, that the turns should be close to the needle, that the magnetic poles should be strong, that the fiber torsion should be small, and that the control should be weak.

The ***tangent galvanometer*** takes its name from the law that the current strength through the coils is proportional to the tangent of the angle of deflection. This instrument will be described further and its theory given in Chapter IV. The five conditions for the Kelvin galvanometer may be assumed to hold for this one also, except that the control is chiefly due to the earth's field alone.

22. Suspended-coil Galvanometers. D'Arsonval Type. In the various forms of instrument of this type the torque is due to the reaction of the field about the movable coil with the field about the permanent magnet. The suspension is a metal ribbon above and a coiled spring below, both serving to carry current. Control is due to the torsion of the suspension.

Damping is most effectively brought about by induced currents. These may arise either in the metal frame upon which the coil is wound or in the wire turns of the coil itself. In the latter case a tap key connected across the terminals serves to short-circuit the coil through a low resistance. If the coil is formed without a metal frame, a closed rectangle of thick copper wire may be attached to the coil by spring clips. As the coil rotates in the field, current is induced in these moving metal parts, and the associated magnetic field reacts with that of the fixed magnets. The forces thus called into play tend to oppose the motion of the system. The suspended-coil galvanometer is practically free from disturbances due to stray fields from power circuits or from electric machinery.

By properly shaping the pole pieces of the magnets, and adding a cylindrical soft-iron core, the magnetic field in the narrow air gap in which the coil swings may be made uniform,

as well as radial, as shown in Fig. 12. In this case the deflections of the coil are directly proportional to the current strengths through a wide range of angular swing. This principle has been developed and applied in the direct-reading ammeter and voltmeter.

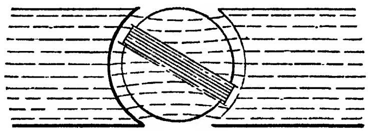

Fig. 12.

Unless extreme care is taken during the process of manufacture, there will be more or less magnetic impurities in the coil, which will tend to give an unsymmetric deflection on the two sides of the zero point. In general, as the sensibility of the galvanometer is increased, its zero-keeping quality decreases. Most metal ribbon suspensions may be expected to show a slight shift of the zero after a full-scale deflection. Differences in deflections on the two sides of the zero are due chiefly to a non-uniform radial field, or to some slight displacement of the coil from its symmetric position in the field.

23. Other Types. In addition to the galvanometers described above, there are many others of special design for special work. We shall mention only four of these.

Alternating-current galvanometers for commercial frequencies are usually of the electrodynamometer type. (See Fig. 72.) The action depends upon the reactions of magnetic fields in fixed and movable coils.

The ***vibration galvanometer*** is an alternating-current instrument which depends on resonance. It will be described in Chapter IX, in connection with its applications.

High-frequency galvanometers are used for measuring alternating currents where the frequency may vary from 100,000 to 2,000,000 cycles per second. Such instruments are in constant use in wireless telegraph circuits, and they must be capable of measuring a wide range of current values. These

values may be as great as 200 or 300 amperes, and as small as a few milliamperes or even a few microamperes.

The action of these instruments usually depends on the heating effect of the current, which may be measured (*a*) by expansion, causing a stretch or sag in a tense wire; (*b*) by thermoelectric effect, causing a deflection on a sensitive potential galvanometer; or (*c*) by a change in resistance, measured by means of a Wheatstone bridge.

It must be understood, however, that measurements of the various electric quantities of these high frequencies cannot be carried out by the ordinary methods suitable for direct currents. Satisfactory calibration of such instruments is only possible in specially equipped laboratories.

The ***Einthoven string galvanometer*** consists essentially of a fine conducting filament carrying current, stretched between the poles of a powerful electromagnet. When current passes through the filament, it is displaced transversely by the reaction of the magnetic fields, and this displacement is greatly magnified by means of a powerful microscope. This type represents the highest sensibility, which may exceed that of a good d'Arsonval by 3000 fold. (See § 29.) A current sensibility of 10^{-13} amperes per scale division has been obtained.

24. Galvanometer Specifications. The complete investigation of any galvanometer involves a study of the following factors: (1) principles of its action, (2) diagrams and descriptions of its parts, (3) resistance, (4) period, (5) damping, (6) accuracy of its return to zero after deflection, (7) sensitiveness, (8) calibration curves, (*a*) ballistic constant (Chapter VII).

25. Laboratory Exercise I. *A study of galvanometer types.* Examine carefully such galvanometers as may be available in

the laboratory, especially the Kelvin and the d'Arsonval table and wall patterns.

In all these instruments note:

(1) General type.
(2) Name of maker and serial number.
(3) Exterior appearance.
(4) Interior appearance.
(5) Method of suspension.
(6) Method of control.
(7) Method of damping.
(8) Method of reading, plane or concave mirror.

In the report discuss the various instruments examined with reference to the points above mentioned. Make sketches showing the working features of the type forms.

Using the hypothesis of lines of force, make drawings to show how the moving system is turned. State whether the instruments examined are aperiodic, and discuss the conditions which affect the periodic time of the suspended system.

State the conditions which govern the sensitiveness of the different types.

Part III. Shunt Circuits

26. The Theory of Shunts. It frequently happens in electrical testing that the galvanometer available for the work in hand is too sensitive, in which case the current passing would cause a deflection beyond the range of the scale, and probably damage the instrument. A convenient and much-used method for safeguarding the galvanometer is that of providing a shunt or by-pass across the terminals, the resistance of the shunt being so adjusted as to permit more or less of the total current to pass through it, thus leaving a safe and measurable fraction of the total current to pass through the galvanometer.

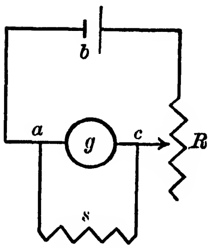

Fig. 13.

Consider a galvanometer of resistance g, provided with a shunt of resistance s (Fig. 13). Let I be the total current flowing and i_g and i_s the currents through g and s, respectively. Let V be the potential difference between the galvanometer terminals. The total resistance between a and c is given by

(6) $$R = \frac{gs}{g + s},$$

which is seen to be a fraction, $s/(g + s)$ of the resistance of the galvanometer alone.

By Ohm's law the potential drop in either of the paths is

(7) $$V = i_g g = i_s s,$$

whence

(8) $$\frac{i_g}{i_s} = \frac{s}{g}.$$

By the law of composition in proportion, (8) may be written

(9) $$\frac{i_g}{i_g + i_s} = \frac{s}{g + s}.$$

Remembering that $i_g + i_s = I$, (4) becomes

(10) $$\frac{i_g}{I} = \frac{s}{g + s}.$$

Then

(11) $$i_g = I \frac{s}{g + s},$$

and

(12) $$I = i_g \frac{g + s}{s}.$$

The factor $(g + s)/s$ is called the multiplying factor or multiplying power of the shunt, because it is the quantity by which the galvanometer current is multiplied in order to give the total current in the line.

It is apparent that the application of the shunt decreases the resistance between a and b, and hence the total current I is increased. Hence, the total current is not a constant for all values of s. However, the relations given in (7) and in (12) are perfectly general relations between the galvanometer current and the *then existing* total current.

In order to study the effect of a shunt upon the total current, it is important to examine the conditions upon which the value of the total current depends. This will be done for two extreme cases: I. when g is small compared to the total circuit resistance; II. when g is large, or practically the only resistance in the circuit.

CASE I. *When g is small compared to R.* Assume a circuit as in Fig. 13. Let E represent the E. M. F. of the battery, b its internal resistance, and R a variable control resistance. Writing Ohm's law for the entire circuit *before* s is connected,

(13) $$I_1 = \frac{E}{b + R + g}.$$

Similarly, *after* s is connected

(14) $$I_2 = \frac{E}{b + R + \frac{gs}{g + s}}.$$

D

Dividing (13) by (14), we have

$$\frac{I_1}{I_2} = \frac{b + R + \dfrac{gs}{g+s}}{b + R + g}. \tag{15}$$

The condition for a constant total current is that the ratio I_1/I_2 shall be equal to 1. The value of this ratio approaches unity only when $gs/(g+s)$ approaches g. This is only possible when the factor $s/(g+s)$ approaches unity, and writing this factor in the form

$$\frac{1}{1 + \dfrac{g}{s}},$$

we see that the ratio approaches unity as g decreases. From equation (15) it is seen that as g approaches zero, the ratio I_1/I_2 approaches unity, and it is then apparent that when g becomes negligibly small compared to the rest of the circuit resistance, the influence of the applied shunt on the total current is also negligible. This is equivalent to saying that in order to keep the total current constant, it is necessary to have R very large compared to g. This case arises when it is desired to compare values of the currents through the galvanometer, with and without the shunt. For this case I must not be appreciably changed by the application of the shunt.

From equation (12) it is seen that the ratio of total current to galvanometer current is equal to the multiplying factor of the shunt. If the desired factor is n, then

$$n = \frac{g+s}{s},$$

or

$$ns = g + s,$$

whence

$$s = \frac{g}{n-1}. \tag{16}$$

From equation (16) it is seen that if the desired multiplying factor is n, the necessary shunt value is determined by dividing the galvanometer resistance by $n - 1$.

It is convenient to have the multiplying factor a decimal multiple, 10, 100, or 1000. This is readily attainable if s be given values 1/9, 1/99, 1/999, respectively, of the galvanometer resistance. This requires a shunt box or set of shunt coils especially adjusted for the galvanometer with which it is to be used. In the Kelvin type, the resistance of the galvanometer coils is constant, and a shunt box once adjusted is thereafter trustworthy, due allowance being made for temperature changes if necessary.

The use of such a shunt box is based upon the assumption that the total current remains constant. This assumption may be made rigidly true if series resistances are introduced into the circuit, of such values as to compensate for the decrease in resistance due to the application of the shunt. Such an arrangement is called a ***constant-current shunt box.*** The compensating-series resistance coils are automatically added to the circuit as any desired shunt is applied to the galvanometer.

With suspended-coil galvanometers, however, the resistance of the metal suspension is considerable, and, inasmuch as it is difficult to replace a suspension without altering the galvanometer resistance, a satisfactory adjustment of a shunt box is impossible. With moving-coil galvanometers, modern practice makes use of the universal shunt, which will be described in § 27. Its use will be illustrated in the laboratory exercise of § 70.

CASE II. *When g is large compared to R.* In this case we may assume that $R+b$ is negligible as compared to g (Fig. 13); that is, g is the only resistance in the circuit. Returning to equation (10) of Case I, we may neglect R and b since they are very small. We may then write

$$\frac{I_1}{I_2}=\frac{\dfrac{gs}{g+s}}{g}=\frac{s}{g+s},$$

whence we have

$$I_2 = I_1 \frac{g+s}{s}. \tag{17}$$

From (7), however, it is seen that the total current is the galvanometer current multiplied by $(g+s)/s$, whence

$$I_2 = i_g \frac{g+s}{s}. \tag{18}$$

It is obvious from a comparison of (17) and (18) that the current i_g through the shunted galvanometer is the same as the original current I through the galvanometer before the shunt was applied. The application of the shunt has left the galvanometer current unchanged. This means that in this case the application of the shunt has lowered the total resistance of the circuit and thereby increased the total current to such a degree that the fraction of it now passing through the galvanometer is *as great as the original line current.* In this case a shunt is useless.

To illustrate this case we may connect a high-resistance voltmeter across a storage battery. A shunt of any value placed across the terminals, although greatly increasing the current drawn from the battery, has very little effect on the current through the voltmeter, and hence its reading is practically unchanged.

The two cases considered above will be recognized as representing extreme conditions, and any actual case, with finite values of g, s, and R, will fall somewhere within the limits discussed. The actual effect on the total current for any practical case can be ascertained by substituting the known values in equation (15).

EXERCISE

Calculate the percentage deviation between I_1 and I_2 for the conditions $b = 1.0$, $g = 100$, $s = 10$, $R = 10$ and $10{,}000$.

27. The Ayrton Universal Shunt. A form which may be used with any galvanometer, irrespective of its resistance, is the so-called ***universal shunt*** designed by Ayrton. The complete circuit is shown in Fig. 15, and a simplified form for explanation is shown in Fig. 14. In Fig. 14, the galvanometer of resistance g is permanently shunted by a high resistance of value R.

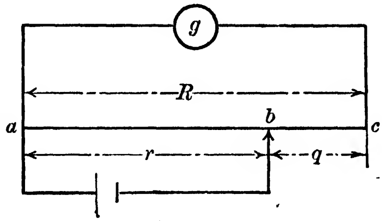

FIG. 14.

The battery circuit is connected at a and b, b being a movable contact corresponding to the rotating switch arm in Fig. 15. The resistance between a and b is denoted by r, and that between b and c, by q. Let I represent the total current flowing from the battery, and assume for the present that it is constant in value, irrespective of variations in r. Let i_1 represent the current through the galvanometer when b is at c, and i_2, i_3, etc., the galvanometer currents respectively for different positions of b. From equation (11) we have

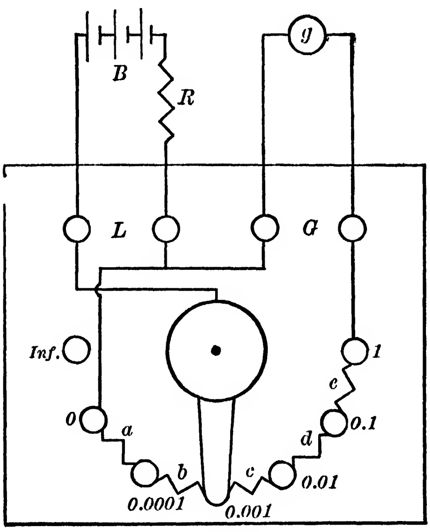

FIG. 15.

$$i_g = I\frac{s}{g+s},$$

and this may be adapted to the circuit of Fig. 14 by replacing s by r and g by $(R - r + g)$. Then we have

$$i_g = I\left[\frac{r}{r+(R-r+g)}\right],$$

or

(19) $$i_g = I\frac{r}{R+g}.$$

Assume in the first place that b is at c, then (19) becomes

(20) $$i_1 = I\frac{R}{R+g}.$$

Next, assume that b is at some point such that $r = \frac{1}{10}R$. Then (19) becomes

(21) $$i_2 = I\frac{\frac{1}{10}R}{R+g}.$$

Dividing (21) by (20), we find

$$\frac{i_2}{i_1} = \frac{\frac{1}{10}}{1},$$

whence

(22) $$i_2 = \tfrac{1}{10}i_1.$$

Moreover, since galvanometer deflections are proportional to currents, it follows that

(23) $$\frac{i_2}{i_1} = \frac{d_2}{d_1} = \frac{1}{10}.$$

This shows that the ratio of i_2 to i_1, or that of the currents through the galvanometer for the two cases considered, is independent of the resistance of the galvanometer. Hence the same ratio holds for any value of g. It is necessary here to note that the assumption that I remains constant is by no means always true, inasmuch as the effective resistance in series with the battery is varied through wide limits by changes in r. However, in the practical applications of the shunt box, the shunted galvanometer is always in series with

a high resistance, of the order of $\frac{1}{10}$ megohm or higher, in which case any change in r would have very little effect on I, and for practical purposes the error is negligible.

Applying the foregoing discussion to Fig. 15, it is seen that the total resistance of 4000 ohms corresponds to R, and the particular fraction, measured from the zero end to the switch contact, corresponds to r. The line circuit is connected at L and the galvanometer at G. The five sections a, b, c, d, and e have a total resistance of 4000 ohms. The following table gives the ratios for the various positions.

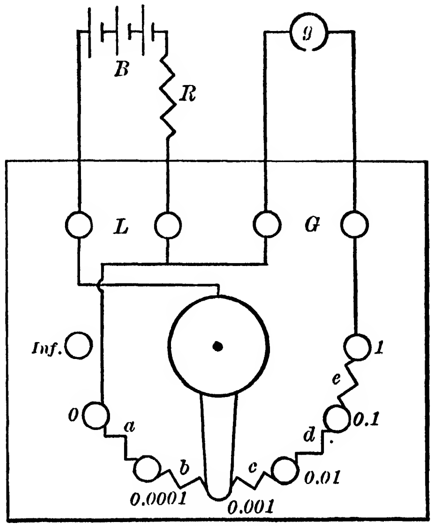

Fig. 15.

Section	Resistance	Switch Position	Ratio i/I
a	0.4 ohm	$\frac{1}{10000}$	$\frac{1}{10000}$
b	3.6 ohms	$\frac{1}{1000}$	$\frac{1}{1000}$
c	36.0 ohms	$\frac{1}{100}$	$\frac{1}{100}$
d	360.0 ohms	$\frac{1}{10}$	$\frac{1}{10}$
e	3600.0 ohms	1	1

Sometimes it will be convenient to use the box for obtaining a fraction of any available voltage. If a certain voltage is impressed across the terminals G, there will be available at L the fractions of the original value given in the third column of the table. The apparatus may then be used as a volt box. (See § 99.)

28. Laboratory Exercise II. *To study the effect of shunts on galvanometer deflections.*

APPARATUS. Battery cell, reversing switch, adjustable resistance, resistance box with low values for s, and a low-resistance galvanometer.

PROCEDURE. (1) Set up the circuit as in Fig. 16, and adjust the galvanometer to a convenient zero.

(2) With K open, adjust R to a high value, making the deflection ten or more scale divisions. Read deflections on both sides of the zero and take the mean. This gives a mean deflection d, which is proportional to the total current flowing.

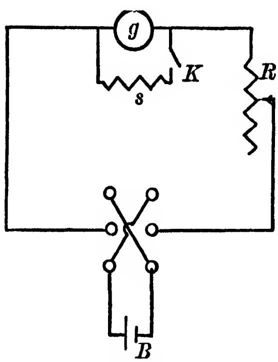

FIG. 16.

(3) Keep R constant, close K, and adjust s until the observed deflection d' has been reduced by about one tenth of that first read.

(4) Continue reducing s by similar steps, about ten in all, until s becomes zero, when also the deflection should be zero. The connecting wires from s to the galvanometer should be as short as possible. Reverse each time, and record values of s, together with the corresponding mean deflections.

(5) Tabulate the observed data, and also corresponding values of d'/d and $s/(s+g)$. State in the report what the experiment teaches.

(6) Plot a curve with values of s as abscissas, and deflections as ordinates, and state what inferences may be drawn from the form of the curve. Draw the asymptote to the curve corresponding to the maximum deflection. Find the value of the shunt such that any desired fraction of the total current, say one fifth, flows through the galvanometer.

PART IV. SENSIBILITY OF THE CURRENT GALVANOMETER

29. Specification of Sensibility. For purposes of specification it is necessary to state precisely the sensitiveness of a galvanometer. Of the several ways in which this may be done, the following are the most useful:

(1) The ***current sensibility*** is the current strength in amperes necessary to cause a deflection of one millimeter when the scale is at a distance of one meter from the mirror. This value of the current strength is called the ***figure of merit***, and galvanometers ordinarily in use have values ranging from 10^{-4} to 10^{-10}. Since for any galvanometer it is the current through its coils, together with the associated magnetic field, which causes the deflection, the figure of merit is the fundamental definition of sensibility. Knowing the applied voltage and the circuit resistances, the other methods of specifying sensibility may be derived from this.

(2) The ***microampere sensibility*** is the number of millimeters' deflection caused by a current of one microampere when the scale is at a distance of one meter from the mirror.

(3) The ***megohm sensibility*** is the number of megohms which must be placed in series with the galvanometer in order that one volt shall cause a deflection of one millimeter when the scale is at a distance of one meter from the mirror. If the galvanometer resistance can be neglected as compared to the high-series resistance, which is usually the case, it will be seen that the megohm sensibility and the microampere sensibility will have the same numerical value.

(4) The ***microvolt sensibility*** is the potential difference in microvolts necessary to cause a deflection of one scale division. This is chiefly used with low-resistance galvanometers and those of a portable type provided with pointers.

(5) The ***voltage sensibility*** is the potential difference in

volts necessary to cause a deflection of one millimeter. This value will always be the product of the resistance of the galvanometer and the figure of merit.

Since sensibility depends on the number of wire turns in the coil or coils, which in turn is directly proportional to the resistance, it follows that high-resistance galvanometers are usually the most sensitive. Portable galvanometers will usually not have a sensibility greater than half a megohm. Wall galvanometers will vary, according to their purpose, from one to fifteen hundred megohms. The sensibility will depend upon several factors such as position and strength of control magnet, torsion of the suspending fiber, number of wire turns in the coils, strength of the magnetic field, and distance from the mirror to the scale. Hence a value of the sensibility of any instrument must be accompanied by a precise statement of the conditions under which it is determined.

30. Current Sensibility Formula. The formula for finding the figure of merit, or the current sensibility of a galvanometer, is derived as follows. With a circuit arranged as in Fig. 16, K being open, it is desired to find what current will give one millimeter of deflection for some stated scale distance. Let the current value required for such unit deflection be F amperes, let E denote the E. M. F. of the battery, and let b denote the internal resistance of the battery. By Ohm's law, for any given deflection d, we may write

$$Fd = \frac{E}{R + b + g},$$

whence

$$F = \frac{E}{(R + b + g)d}. \tag{24}$$

In general, however, it will be necessary to use a shunt with a sensitive galvanometer, which may be done by closing K

and adjusting s to a proper value. The effective resistance of the circuit, including shunt and galvanometer, will then be $sg/(s+g)$, and the total resistance of the entire circuit will be $R+b+sg/(s+g)$. Then, if the E. M. F. of the battery is divided by this total resistance, the quotient is the total current in amperes flowing through the circuit. Since the current through the galvanometer is only a fraction of the total, in order to determine the current effective in producing the observed deflection d, the total current must be reduced in the ratio $s/(s+g)$, as in equation (11). For the shunted galvanometer equation (24) becomes

$$(25) \qquad F = \left[\frac{E}{R+b+\frac{sg}{s+g}}\right]\frac{1}{d}\,\frac{s}{s+g}.$$

In case b is small, as it usually is, it may be neglected.

31. Laboratory Exercise III. *To study the sensibility of a current galvanometer.*

Apparatus. The same as in § 28. Dry cells may be used if a sufficiently high resistance is put in series with them.

Procedure. (1) Connect the circuit as in Fig. 16, and adjust the galvanometer to zero on the scale. Note carefully the distance of the mirror from the scale, as well as any other conditions upon which the figure of merit may depend.

(2) Keep s constant and adjust R, which must be high, so that a full scale deflection is read. Increase R so that the deflection is reduced by about one fifth of the first value, read both right and left deflections, and take the mean. Continue increasing R for five or six values, making the last deflection about two centimeters.

(3) Calculate the values of F for each mean deflection, and tabulate values of R, right and left deflections, mean deflections, and F. Record also the values of b, g, and E. Reduce

the deflections to millimeters. If the scale distance is not one meter, the appropriate correction must be applied to the values of F. In case F is found to vary sensibly from a constant, plot values of current, or reciprocals of total resistance against deflections, and explain the significance of the curves. The galvanometer resistance g may be found directly with the Wheatstone bridge, or by the half-deflection method of § 55. Notice that different procedure is necessary according to the type of battery used, whether non-polarizing or dry cells. The battery resistance may be found by the half-deflection method of § 56. The E. M. F. may be read directly with an accurate voltmeter, or it may be found by the methods described in Chapter III.

(4) From the mean value of F found, calculate the sensibility of the galvanometer according to the definitions (2), (3), and (5) of § 29. Discuss the error introduced by neglecting values of g and b.

32. Laboratory Exercise IV. *To calibrate a galvanometer.* In many experiments in which galvanometers are used, it is necessary to know whether the deflections throughout the entire range of the scale are strictly proportional to the currents producing them. In some types, currents are proportional to sines or tangents of the deflections, while in most of the modern instruments the effort is made to secure strict proportionality over the entire scale.

Arrange a circuit as in Fig. 16, and observe the deflections for ten different values of R.

Plot a curve between values of the deflections and reciprocals of total circuit resistance. This will be the relative calibration curve, and for some instruments will be a straight line. If the actual values of current strength are plotted against deflections, the absolute calibration curve will be traced.

PART V. POTENTIAL GALVANOMETERS. VOLTMETERS

33. The Potential Galvanometer. Deflections of a galvanometer which are proportional to current strengths are also proportional to the potential differences impressed at the galvanometer terminals, since the resistance of the instrument is constant. Such galvanometers may then be used to measure potential differences, and they are called ***potential galvanometers***, or ***voltmeters***.

The potential galvanometer, or voltmeter, is always connected in parallel with the points between which the voltage or potential difference is to be measured. It is readily seen that the application of the instrument to the circuit actually lowers the voltage between the points across which it is applied.

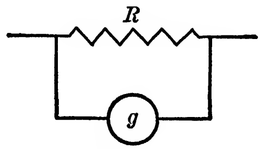

FIG. 17.

Assume a circuit as in Fig. 17, where R is the resistance of a portion of the circuit across which the potential galvanometer of resistance g is connected. Let V_1 be the potential difference across the terminals of R, *before* the galvanometer is applied. Let V_2 be the new potential difference *after* the galvanometer is connected. Before the galvanometer is applied, we have

$$V_1 = I_1 R; \tag{26}$$

and after the galvanometer is applied, we have

$$V_2 = I_2 \frac{Rg}{R+g}. \tag{27}$$

Dividing (26) by (27), we obtain the formula

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} \cdot \frac{R}{\frac{Rg}{R+g}},$$

or

$$\frac{V_1}{V_2} = \frac{I_1}{I_2}\,\frac{R+g}{g}. \tag{28}$$

In order to insure trustworthy readings of the potential galvanometer, the true potential difference between the terminals of R must not be appreciably altered by the application of the measuring instrument, and hence the ratio of V_1 to V_2 should be equal to unity. In any actual case, however, it is readily seen that the potential galvanometer applied as a shunt

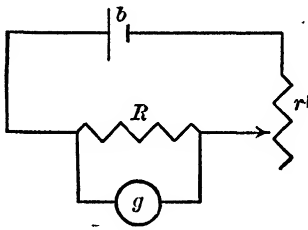

FIG. 18.

across the terminals of R actually lowers the resistance between these points and correspondingly increases the total current flowing. In order to investigate the actual change in the value of the total current we may write Ohm's law for the entire circuit, both before and after applying the potential galvanometer.

Referring to Fig. 18, and letting E represent the E. M. F. of the battery, of internal resistance b, we may write

$$I_1 = \frac{E}{b+R+r}, \tag{29}$$

and, similarly,

$$I_2 = \frac{E}{b+r+\dfrac{Rg}{R+g}}. \tag{30}$$

Dividing (29) by (30), we have

$$\frac{I_1}{I_2} = \frac{b+r+\dfrac{Rg}{R+g}}{b+r+R}. \tag{31}$$

Substituting the value of the ratio I_1/I_2 as given by equation (31) in equation (28), we obtain a formula for the ratio V_1/V_2 which does not involve I_1 or I_2:

(32)
$$\frac{V_1}{V_2}=\frac{\left(b+r+\dfrac{Rg}{R+g}\right)(R+g)}{(b+r+R)g}$$

$$=\frac{bR+rR+bg+rg+Rg}{bg+rg+Rg}$$

$$=\frac{bR+Rr}{bg+rg+Rg}+1,$$

or

(33)
$$\frac{V_1}{V_2}=1+\frac{R}{g}\cdot\frac{1}{1+\dfrac{R}{b+r}}.$$

From (33) it is seen that by making g large as compared to R, V_2 may be made to differ from V_1 by as small an amount as may be desired. A necessary condition for a potential galvanometer is that its own resistance shall be high. The actual value which g must have in order to secure a required minimum variation in V_2 as compared to V_1 is readily computed from equation (33), if the other circuit resistances are known.

In practical voltmeters the value of the internal resistance is usually made not less than 100 ohms for each volt of scale range.

34. The Voltmeter Multiplier. It is often desired from the standpoint of economy or convenience to make a potential galvanometer or voltmeter available over a range greater than that of its own scale.

Assume that the range of the voltmeter V_m (Fig. 19) is 3 volts for a full scale deflection, and that it is desired to measure with it a value of 30 volts. Since the maximum deflection is caused by a potential difference of 3 volts between its terminals bc, the voltmeter obviously cannot be connected directly across ad, for the potential difference between these points is 30 volts. However, if a resistance of value M is placed in series with the voltmeter where M is so chosen that the potential drop through it is 27 volts, then the full scale (or 3-volt point) may be marked 30, and a tenfold increase in scale range will always be indicated so long as this particular resistance is used in series with the instrument.

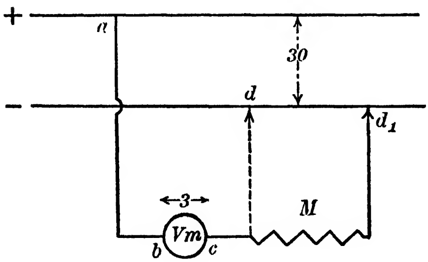

FIG. 19.

Such a resistance is called a ***multiplier***. It may be separately mounted in a suitable case, or it may be inclosed within the case of the voltmeter, its terminals being so arranged that it can be put in series with the voltmeter when the larger range is desired.

The value of M is readily derived as follows. When the voltmeter of resistance g is connected as shown in Fig. 19, a feeble current i flows through it. Then the potential drop in it alone is given by

(34) $$V_1 = ig = 3 \text{ volts.}$$

The potential drop through the voltmeter and the multiplier together is given by the equation

(35) $$V_2 = i(g + M) = 30 \text{ volts.}$$

If the value of the current is the same in both cases, so that the same deflection of the pointer is caused, we may divide (34) by (35) and cancel i, whence

$$\frac{g}{g+M}=\frac{V_1}{V_2}=\frac{3}{30}. \tag{36}$$

This reduces to

$$M=\left[\frac{V_2-V_1}{V_1}\right]g=\left[\frac{30-3}{3}\right]g=9\,g. \tag{37}$$

Thus, if the resistance of the voltmeter is 300 ohms, $M=$ 2700 ohms for a tenfold increase in scale range.

In general, if it is desired to increase the scale range by some factor n, the resistance of the multiplier will always be the voltmeter resistance multiplied by $(n-1)$. The student should compare this case with that of the galvanometer or ammeter shunt, in which case the galvanometer resistance is divided by $(n-1)$.

A desirable uniformity follows from the practice of making the voltmeter resistance 100 ohms for each volt of scale range, in which case multipliers are interchangeable.

35. Voltmeters Used to Measure Current. It has been pointed out that a necessary condition for sensibility in galvanometers is that the movable system must be light. In the suspended-coil type of instrument this means that only very fine wire may be used, and the current-carrying capacity is therefore very small.

On this account an *ammeter* seldom carries through its movable coil the total current to be measured. However, it is really designed as a potential galvanometer, to show the fall of potential across an accurately known low resistance or shunt, through which most of the current passes.

Since for a constant resistance the potential differences are directly proportional to the current strengths, it follows that

the scale of the instrument may be calibrated to read directly in amperes.

The usual arrangement is shown in Fig. 20, where A is the sensitive potential galvanometer. The deflections are proportional to the potential differences between the ends of S, and the scale is graduated to read appropriate units of current strength in the line.

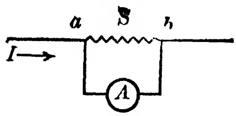

Fig. 20.

A potential galvanometer may be provided with several low-resistance shunts of different values. It is then available for reading current strength over any desired range. These shunts may be contained within the case of the instrument, or they may be separately mounted outside the case, and provided with current and potential terminals.

It is not practicable to use interchangeable shunts with low-resistance galvanometers. Consider an ammeter of 0.01 ohm resistance shunted with a resistance of equal amount. The current through the ammeter is half the total current, hence the scale range is doubled. However, if the shunt is removable or interchangeable, the contact resistances are likely to be too large to be neglected. The contact resistance of a number 16 wire clamped under a binding post with moderate pressure will not be less than 0.0001 ohm, and may be as great as 0.001 ohm. In this case the contact resistance is at least one per cent, and may be ten per cent, of the value of the shunt, and the effective value of the shunt is too great by this amount. Hence it is necessary to make the resistance of the galvanometer high, and use it as a potential galvanometer. In this case the small contact resistances are negligible.

36. Laboratory Exercise V. *Some experiments on Ohm's law.*

Apparatus. Voltmeter, ammeter, resistance box, dry cells, single resistance, and tap key.

Procedure. (1) Connect the voltmeter across the battery

terminals, and record the reading. Insert in series with the voltmeter and battery a resistance box, and adjust this until the reading is reduced to exactly one half its former value. Write Ohm's law for each case, and solve for the voltmeter resistance.

(2) Calculate the value of the multiplier necessary to increase the scale range of the voltmeter tenfold. Check this value by actual trial with the resistance box.

(3) Connect the battery in series with the ammeter, the single resistance, and the tap key. Connect the voltmeter in parallel with the resistance. Read and record several simultaneous values of current and voltage and compute the value of the resistance.

(4) Connect the ammeter and voltmeter in series with the battery, and record the reading of each. Explain the result.

Remember that the ammeter is a low-resistance, direct-reading current galvanometer. The voltmeter is a high-resistance, direct-reading potential galvanometer.

Questions. Two ammeters are connected in series with a battery. Will the readings be alike or different? Two voltmeters are connected in series with a battery. Will they read differently or the same: (*a*) if they have the same resistance? (*b*) if they have different resistances? What is the effect of too great a resistance in an ammeter? What is the effect of too small a resistance in a voltmeter?

EXERCISES

1. Assume a circuit like that in Fig. 18, and let $R = 10$, $b = 2$, and $r = 10$. It is desired that the voltmeter readings shall not be in error by more than one half of one per cent. Find the value of the voltmeter resistance.

2. How can a line voltage of 90 volts be measured with two voltmeters, each of which has a range of 50 volts? The resistances of the voltmeters are 4000 ohms and 6000 ohms respectively.

3. Two voltmeters of 150 volts range and 15,000 ohms resistance are placed in series across a 110-volt circuit. What will be the readings on each of the two instruments? Suppose that the resistances of the voltmeters are 11,500 ohms and 16,000 ohms respectively. What current flows through each instrument, and what is the voltage reading on each?

4. How can a current strength of 90 amperes be measured with two ammeters, each of which has a range of 0–50 amperes? Find the current through each, and the potential difference at the terminals of each, (*a*) if the resistances are each equal to 0.05 ohm; (*b*) if the resistances are 0.045 and 0.055 respectively.

5. An ammeter and a voltmeter are to be used for measuring a resistance by Ohm's law. They may be connected as shown in either (*A*)

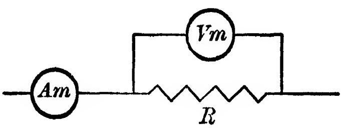

Fig. *A*.

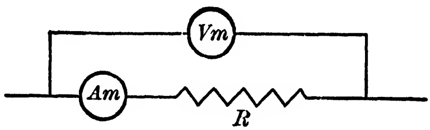

Fig. *B*.

or (*B*). Discuss the relative merits of these two arrangements, especially when *R* has (*a*) a high value; (*b*) a low value.

6. A 4-volt battery has an internal resistance of 1 ohm. It is put in series with a resistance *R* of 3 ohms, across which is connected a voltmeter of 3000 ohms resistance. Find (*a*) the total current; (*b*) and (*c*) the potential difference across *R* before and after the voltmeter is connected; (*d*) the percentage error in the voltmeter reading.

7. In order to secure a multiplying power of ten, a galvanometer of 90 ohms resistance is shunted with 10 ohms. Assume an error of $\pm \frac{1}{2}$ % in the shunt. What is the effect on the value of the multiplying power?

8. A voltmeter of 3000 ohms resistance and an ammeter of 0.2 ohm resistance are connected in series with a battery of 3 volts. Neglecting the internal resistance of the battery, find (*a*) the current which flows in each instrument; (*b*) the potential drop in the voltmeter; (*c*) the potential drop in the ammeter.

CHAPTER II

RESISTANCE AND ITS MEASUREMENT

PART I. GENERAL DEFINITIONS

37. Resistance and Ohm's Law. The relations existing in an electric circuit in which a steady current is flowing between electromotive force, current, and resistance were first accurately studied by G. S. Ohm. He stated these relations in a work which he published in 1827.[1]

The law of the electric circuit which expresses these relations is called ***Ohm's law*** (§ 3). This law states the experimental fact that the strength of current is always directly proportional to the electromotive force, and inversely proportional to the resistance. It may be written in the form

$$I = \frac{E}{R}, \tag{1}$$

where I is the *strength of current*, E is the *electromotive force*, and R is a quantity called the *resistance*, which is characteristic both of the material of which the circuit is made and of the form (length and cross-section) in which this material is disposed. It is found that the resistance is not dependent upon the strength of current flowing nor upon the electromotive force. It depends only upon (a) the nature of the material, (b) the form, size, and physical state of the material, (c) the temperature. If the electromotive force is doubled, the character of the circuit remaining otherwise unchanged, the current is found to be doubled; in other words, the ratio

[1] *Die Galvanische Kette, Mathematisch Bearbeitet.*

E/I is constant and equal to R. The fact that this ratio remains constant shows that R corresponds to a definite physical property of the material. The ***resistance*** may then be defined as that characteristic constant of the circuit which is the constant ratio of E to I.

The basic idea underlying the use of resistance in electric circuits is that of opposition to the flow of current. If the resistance is altered, the current strength is altered in an inverse ratio. This opposition to the flow of current is analogous to friction in mechanics, and it is always accompanied by the production of heat. If an unvarying current is maintained through a metallic conductor by a constant potential difference, the electrical energy is entirely transformed into heat, the amount of which is proportional to the resistance. The rate of transformation of energy is given by Joule's law (Chapter IV, § 111):

$$\frac{H}{t} = i^2R,$$

where i is the current strength, R is the resistance, H is the heat produced, and t is the time.

It follows that resistance may be measured by means of the heat generated, as well as by the ratio of simultaneous values of E and I. This ratio, and hence the value of the resistance, remains constant only so long as the rate of dissipation of the heat produced is equal to the supply rate of the electrical energy, so that the *temperature* of the conductor does not change.

In applying Ohm's law to any *portion of a circuit,* which does not contain an electromotive force, the current is given by the ratio of the potential difference impressed on the terminals of the resistance to the resistance itself. If the law is applied to the entire circuit, the total resistance of the circuit must be used, including the internal resistance of the generator, or of all the generators if there are several. Likewise, the

resultant or net value of all the electromotive forces must be used. If the generators are in series, this resultant value is found by taking the algebraic sum of all the electromotive forces present in the entire circuit.

In the case of some substances, such as electrolytes, in which polarization phenomena appear, there are in general, in addition to the potential difference impressed at the electrodes, internal potential differences which must be taken into account before applying Ohm's law. These internal potential differences, which are opposed to the impressed potential difference, will virtually have the effect of a resistance, and care must be taken that the actual resistance found has been freed from these effects.

It has been found that the ratio of potential difference to current is ***not*** exactly constant for many non-metallic substances. This is true for gases, and also for such substances as rubber, paraffin, and shellac. The same departure from a constant ratio, especially for high voltages, is found for many high-resistance liquids, such as certain oils, benzine, and ether. In these instances it is probable that polarization phenomena are present, as in electrolytes. It is also probable that actual changes in the value of the resistance are caused by the application of the electromotive force.

With the corrections indicated above, Ohm's law has been verified after most careful investigation for currents passing through metals and electrolytes, and with a precision of one part in one hundred thousand. Hence this law is a general principle of fundamental importance for electrical measurements.

38. Units of Resistance. The absolute unit of resistance may be derived from Joule's law. It is that resistance in which the C. G. S. unit of current strength will dissipate one erg of heat in one second. This is, however, not easily real-

ized experimentally, nor is it of convenient size. A multiple of 10^9 is chosen, which is practically represented by the resistance of a mercury column at 0° C., 106.300 cm. long, of mass 14.4521 grams, and of constant cross-section. This practical unit is the ***international ohm*** (§ 5).

For purposes of convenience, resistances are grouped in three ranges, according as their values are *medium, high,* or *low.* Medium resistances are ordinarily measured in *ohms,* and include values from about 1 ohm to 100,000 ohms. Values beyond this range are called *high;* they are most conveniently expressed in *megohms* (§ 12). Fractions of an ohm, especially small fractions, are called *low,* and are most conveniently expressed in *microhms* (§ 12).

39. Resistivity, or Specific Resistance. Experiment has shown that the resistance of a conductor whose cross-section is uniform, and whose substance is homogeneous, is directly proportional to its length, inversely proportional to the area of its cross-section, and is also proportional directly to some constant which has different values for different materials. If R is the resistance of a piece of wire of cross-section a and length l, then we may write

$$R = k\frac{l}{a}, \tag{2}$$

where k is a constant depending on the material. Solving this equation for k, we find

$$k = \frac{Ra}{l}. \tag{3}$$

It follows that for a conductor of unit length and unit cross-section, k is equal to R; hence k may be defined as the resistance between opposite faces of a unit cube of the substance. This value k is called the ***resistivity*** or the ***specific resistance*** of the material. Since the resistance of a sample unit cube

is always small, it is usually expressed in terms of microhms per unit cube. Instead of giving resistivity in terms of resistance, unit length, and unit cross-section, it may be expressed also in terms of the resistance in ohms per foot of wire one mil (0.001 inch) in diameter, or the resistance in ohms per meter of wire one millimeter in diameter.

Another group of units of resistivity is based upon the mass of the sample instead of the volume; for example, the resistance of a piece of uniform wire which has a length of one meter and a mass of one gram, or the resistance of a piece of wire which has a length of one mile and a mass of one pound. The units of resistivity may be summarized as follows:

Volume units { microhm per centimeter^3, ohm per mil-foot, ohm per millimeter-meter.

Mass units { ohm per meter-gram, ohm per mile-pound.

In practice, the mass units of resistivity are preferable to the volume units for the following reasons: (*a*) the measurement of the cross-section is frequently difficult or inaccurate, (*b*) for many shapes this measurement is impossible, (*c*) copper is ordinarily sold by mass rather than by volume and hence the data of greatest value are given more directly.

The mass units of resistivity may be readily converted into volume units if the density of the sample is known. For standard annealed copper at 20° C., the density is 8.89. A wire of this material one meter long and one square millimeter in cross-section has a resistance of 0.1724 ohm. If the wire is one meter long and one gram in mass, its resistance is 0.15328 ohm.[1] The figures will vary slightly for different samples. A

[1] Much data of value will be found in *Copper Wire Tables*, U. S. Bureau of Standards, *Circular* No. 31.

certain sample of copper has the following values for its resistivity when expressed in the various units:

900.77	pounds per mile-ohm,
0.1577	ohm per meter-gram,
1.7726	microhms per centimeter^3,
10.663	ohms per mil-foot.

40. Conductivity. The reciprocal of the resistivity is called conductivity and its unit is the ***mho***, or ***reciprocal ohm, per centimeter cube.*** For many reasons, especially in engineering practice, conductivity is specified instead of resistivity. Apparatus of the bridge type has been arranged for measuring directly in terms of arbitrary standards the conductivity of such materials as copper rods and wires, and steel rails. The samples used must be short for reasons of economy, and the methods of measurement follow closely those given below for low resistances.

41. Temperature Coefficient of Resistance. The electrical resistance of all substances is found to change more or less with changes in the temperature. All pure metals and most alloys have their resistance increased with rising temperature, while carbon and many electrolytes show a decrease in resistance with increasing temperature.

Experiment has shown that the resistance of any conductor at a temperature $t°$ C. is given by the formula

$$R_t = R_0 + R_0 \alpha t + R_0 \beta t^2 + \cdots, \tag{4}$$

where R_0 is the resistance of the sample at zero, α is the change in 1 ohm when the temperature changes from 0° to 1° C., and β is a measure of the variation per degree in α. The factor α is called the ***temperature coefficient.*** The value of β is very small; for copper it is 0.0000012; hence it

may be neglected in all but the most precise work. The simplified formula for metallic conductors will then read

$$R_t = R_0(1 + \alpha t). \tag{5}$$

The temperature coefficient is a function of many factors, such as the process of purifying the metal, the degree of its purity, and the methods of drawing and annealing during manufacture. All pure metals have nearly the same value of the temperature coefficient, approximately 0.4 of one per cent per degree C. If values of resistance are plotted against corresponding temperatures on the absolute scale, the curves approach straight lines, with a general trend toward the origin, indicating that at the absolute zero the conducting metals lose their resistance. It has recently been shown by Onnes, in experiments with conductors surrounded by helium boiling under diminished pressure, that at a temperature of three or four degrees above the absolute zero the resistance does practically disappear. In this state Ohm's law no longer holds, and when current flows through the conductor at this temperature, there is neither a generation of heat nor a fall of potential.

42. The Formula for Temperature Coefficient. If the values of resistances of a conductor at two temperatures other than zero are known, the value of α may be derived in the following way. Let t and t' be two temperatures for which the corresponding values of resistances are R_t and $R_{t'}$. Then, from equation (5), we may write

$$R_t = R_0(1 + \alpha t), \tag{6}$$

$$R_{t'} = R_0(1 + \alpha t'). \tag{7}$$

Dividing (6) by (7), we find,

$$\frac{R_t}{R_{t'}} = \frac{1 + \alpha t}{1 + \alpha t'}. \tag{8}$$

Clearing of fractions and solving for α, we have

$$\alpha = \frac{R_{t'} - R_t}{R_t t' - R_{t'} t}. \tag{9}$$

43. Current Control by Means of Resistance. One of the most frequent needs in the electrical laboratory is the control of current strength, and this is readily accomplished by means of resistances, either fixed or variable in value, included in the circuit. The variable resistances are grouped under two heads, ***resistance boxes*** and ***rheostats***. A third group includes ***standard coils*** of fixed values. The common resistance materials and the various controlling devices above mentioned will be taken up in order.

44. Resistance Materials. For standard coils and resistance boxes it is important that the material used should have the following qualities:

(*a*) *Permanence*, so that a coil once adjusted to a given value may be relied on for a long period of time;

(*b*) *Small temperature coefficient*, so that changes in temperature may affect the resistance in a small degree;

(*c*) *Large resistivity*, so that a high resistance may be assembled without too great bulk;

(*d*) *Small thermoelectric effect* against copper or brass, so that variations in temperature between different parts of the circuit may not cause troublesome thermal currents.

Laboratory standards of precision are sometimes made of mercury in glass, but these are not readily portable, and they are exceedingly fragile, requiring the highest technical skill in their assembly and use.[1]

Early materials for resistance coils were german-silver and platinum-silver alloys, but these had a high-temperature coefficient. Hence, other alloys were sought which would

[1] See U. S. Bureau of Standards, *Bulletin*, vol. 12, p. 375.

more nearly meet all the conditions mentioned above. A large number of copper-nickel alloys have been produced which are useful because they have high resistivity, and in general, low-temperature coefficients; but they are not available for precision work because of their high thermoelectric effects. In the alloy called ***manganin***, however, this objection is almost entirely overcome, as the thermoelectromotive force between manganin and copper does not exceed 1.5 microvolts per degree C. The table herewith given shows the composition of some of the most-used resistance materials, together with the resistivity and temperature coefficient.

Properties of Resistance Materials[1]

Substance	Composition	Resistivity microhms—cm³.	Temperature Coefficient
Aluminum		3.2	0.0039
Copper (annealed)		1.7	0.0042
Iron (pure)		9.96	0.0062
wire		10.0–15.0	20° C. 0.0052 800° C. 0.015
Steel soft-hard		15.9–46.0	0.0042–0.0016
transformer plate		11.1	
silicon-steel		51.0	0.004–0.006
Mercury		95.8	0.00088
Ia Ia	Cu — Ni —	50.0	0.00001
Manganin	Cu 84% Ni 12% Mn 4%	42–47	12° + 0.0001 30° — 0.00002
German silver	Cu 50% Zn 30% Ni 20%	30.0	0.0033
Arc lamp carbon		4000–6000	— 0.0003

[1] Complete tables of data on these properties will be found in the various electrical handbooks; also in Landolt-Börnstein, *Physikalisch-Chemische Tabellen.*

45. Resistance Boxes. Resistance boxes are groups of coils of wire arranged compactly, so that single coils, or any series combination of them, may be introduced into the circuit by manipulation of switches or plugs. These coils may range in value from a few tenths or hundredths of an ohm up to a tenth of a megohm. They are usually non-inductively wound with fine wire of small current-carrying capacity. They are used only with feeble currents, usually small fractions of one ampere.

What constitutes a safe current for any particular box can only be determined by some knowledge of the way the box is made, and the size of the wire used in its coils. Usually the maker selects the wire so that the rate of transformation of electric energy into heat is practically constant for all the coils, and the carrying capacity is usually rated in terms of the watts dissipated, without overheating.

The watt load on any coil is readily found from the formula E^2/R, where E is the voltage across the coil and R is its resistance. The current, impressed voltage, and total circuit resistance must be so adjusted that the safe minimum for the box in hand shall not be exceeded. In general, this will be about one watt, although some resistance boxes on the market will dissipate four watts without overheating. The common type of wood spool boxes would have the paraffin coating softened at two watts; complete safety requires that the temperature shall not rise more than a few degrees above that of the room.

In any event, especially if a storage battery or dynamo is the source of power, some estimate should be made of the electromotive force across the coils, and of the probable resistances in circuit, and the current should be so controlled that the power supply does not exceed one watt, unless the reasons for allowing larger values are approved by those in charge of the laboratory.

The accuracy of adjustment of the coils in resistance boxes varies greatly with the kind of box selected. For most purposes one fifth to one tenth of one per cent is sufficient. If greater precision is desired, the coils are wound on metal spools and the whole inclosed in a metal case with perforated sides, to permit the use of an oil bath whose temperature may be controlled accurately.

In order to vary the number of coils which are at any time included in a circuit, some form of rotary dial switch, or simple plug switch will be used. The latter is illustrated in Fig. 21. When the plug P is withdrawn, current can pass through the coil r, which is connected to the brass blocks a and b. With the plug inserted the coil is short circuited by a path of practically zero resistance.

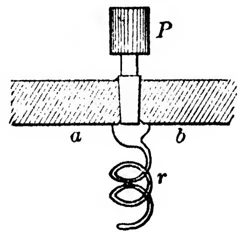

FIG. 21.

The use of plugs is legitimate only when they fit accurately into their sockets, and when the contact surfaces are clean. The contact surface of the plugs should not be touched with the hand, nor should the plugs be laid carelessly on the work table. Idle sockets are sometimes provided to receive the plugs when not in use; otherwise they should be laid on a sheet of clean paper, or placed in a clean box kept for that purpose. A thin film due to oxidation tends to prevent perfect metallic contact, and a slight twisting motion is desirable as the plug is inserted.

Plugs should not be rubbed with abrasives, but may be cleaned by the use of a cloth moistened with a dilute solution of oxalic acid. The resistance-box tops are usually made of hard rubber, and should be protected from dust, moisture, and prolonged exposure to sunlight.

The resistance of a clean and well-fitting plug contact is under 0.0001 ohm. With all the plugs in place, the resistance

across the top of an average resistance box will be between 0.001 and 0.002 ohm.

Resistance-box coils should be free from inductance and capacity as nearly as possible. The former may be largely eliminated by winding the wire double, so that the magnetic fields of the two wires may annul each other. Capacity may be effectively reduced by reversing the direction of winding in alternate layers.

46. Rheostats. The name ***rheostat*** is applied to a device of smaller range than a resistance box, but with a larger current-carrying capacity. It may be adjustable continuously or by steps, and it is usually not important to know the actual resistance values which correspond to the various settings. It is rated, as is a resistance box, in terms of the watts dissipated without overheating, and the safe maximum current is readily inferred from the temperature of the coils, which should not be hot to the touch. Rheostats are commonly wound with manganin, advance, or german silver; and they may be wound even with iron wire if variations in resistance with temperature are not important. Another useful form is made with carbonized cloth in disks, or with carbon blocks pressed together by means of a screw, operated by a hand wheel. This arrangement affords a continuously variable resistance of extreme fineness of adjustment. Tanks of liquid, such as water solutions of copper sulphate or common salt, when provided with suitable electrodes, make useful continuous rheostats.

47. Standard Resistance Coils. Standard coils of a single fixed value are used for precision work, and are usually made in the form shown in Fig. 22. A unit of this type consists of a coil of manganin wire, wound non-inductively on a brass tube, carefully insulated from it and covered with a coating of a protective varnish. The ends are attached to curved ter-

minals arranged to dip into mercury cups, or more frequently they are provided with separate current and potential binding posts. For work of commercial accuracy these coils may be used in air; but for work of great precision they are immersed in a suitable oil bath, and are provided with devices

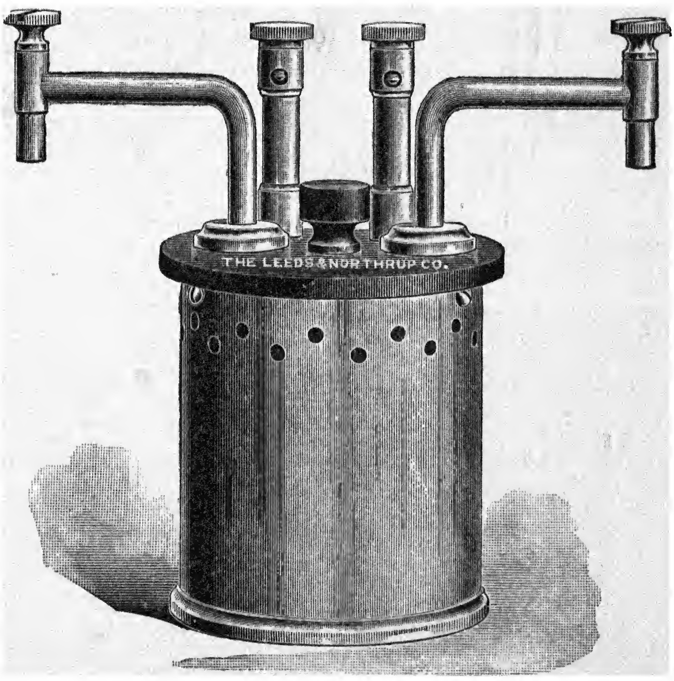

FIG. 22.

for controlling the temperature. Such standard coils of any desired degree of precision up to one or two parts in a hundred thousand may be purchased; their range extends from 100,000 ohms to 0.0001 ohm, or even to 0.00001 ohm. These standards vary greatly in current-carrying capacity, depending on the purpose for which they are made. Some knowledge of this capacity should be obtained from the maker in order to avoid overheating.

48. Measurement of Resistance. Aside from the direct measurement of resistance by means of an ammeter, a voltmeter, and Ohm's law, two fundamental methods may be employed. Methods of the first kind are called ***comparison methods***; in these, the resistance to be measured is compared with some previously determined standard.

Methods of the second kind are called ***absolute methods.*** In these, the resistance is determined in absolute measure without reference to any existing standards.

The first method is the one commonly used; absolute methods will not be considered in the present chapter. The reference standards used will be carefully calibrated resistance boxes, or single-valued coils.

PART II. MEASUREMENT OF RESISTANCES OF MEDIUM VALUES

49. The Wheatstone Bridge. Of the comparison methods for measuring resistance, the ***Wheatstone bridge*** is more used than any other, especially for values of medium range. The method depends upon the fact that in a branched circuit (Fig. 23) the potential drop from *a* to *d* must be the same over both branches. It then follows that for any point *b* chosen on the upper branch *abd*, there must be a corresponding point *c* on the lower branch *acd*, at which the potential is the same. There is, then, no difference of potential between these points, and a galvanometer connected between these points will indicate no deflection.

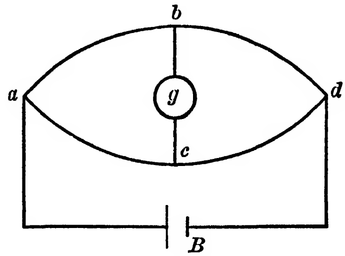

FIG. 23.

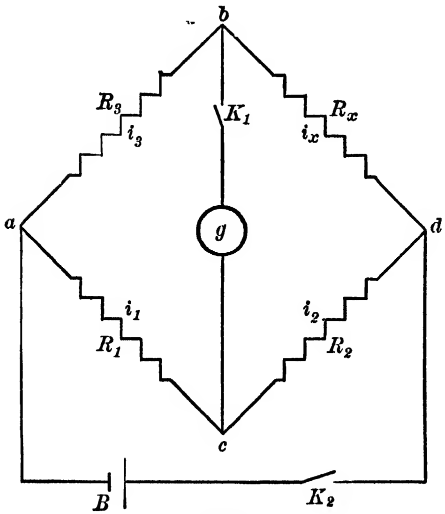

FIG. 24.

The circuit may take the form shown in Fig. 24, where three of the resistances are known and one of them, R_x, is to be determined. The current strength in each of the arms is represented by i_1, i_2, i_3, and i_x, respectively. An adjustment of the resistance values may be made such that the galvanometer shows no deflection. When this has been done, the potential drop along *ab* is the same as that along

ac, and also the potential drop along *bd* is the same as that along *cd*. These relations may be written in the form

(10) $$i_1R_1 = i_3R_3.$$

(11) $$i_2R_2 = i_xR_x.$$

Dividing (10) by (11), we find

(12) $$\frac{i_1R_1}{i_2R_2} = \frac{i_3R_3}{i_xR_x}.$$

Since no current is passing through the galvanometer, $i_1 = i_2$ and $i_3 = i_x$. Then (12) becomes

(13) $$\frac{R_1}{R_2} = \frac{R_3}{R_x},$$

or

(14) $$R_x = R_3\frac{R_2}{R_1}.$$

The pairs of junction points, *ad* and *bc* (Fig. 24), are called the *conjugate points* of the bridge circuit. In general, the positions of battery and galvanometer are interchangeable. However, if the resistance of the galvanometer is greater than that of the battery, which is usually the case, the galvanometer should be connected between the junction points of the highest two and the lowest two resistances.

50. The Meter Bridge. In one common form of the Wheatstone bridge the arms R_2 and R_1 are replaced by a straight homogeneous wire of uniform cross-section. This form is called the ***slide-wire bridge***, or ***meter bridge***. Such a bridge is represented in Fig. 25. It consists essentially of a wire *DH*, one meter long, stretched over a graduated scale, and with its ends soldered to massive brass straps *SS′*. With a known resistance at R_3 and the resistance to be measured at R_x, a galvanometer across *BE*, and a battery across *AC*, we have essentially the arrangement shown in the simple diamond form, Fig. 24. A point can now be found on the wire, *E*, at

which, if contact is made, no current flows through the galvanometer. We may then write

(15) $$\frac{R_x}{R_3} = \frac{R_2}{R_1}.$$

Since for a uniform wire resistances are proportional to lengths, the ratio of the resistances of the two parts of the

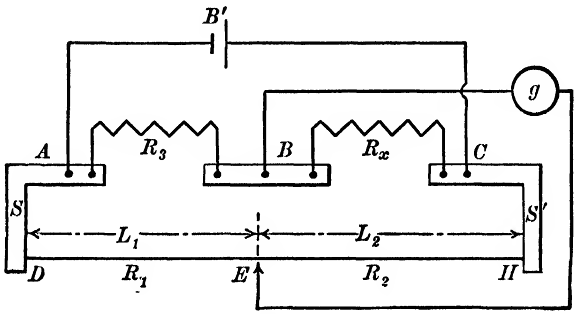

FIG. 25.

wire to the right and left of E is the same as the ratio of their lengths. The actual values of R_2 and R_1 are not then necessary and (15) may be written

(16) $$\frac{R_x}{R_3} = \frac{L_2}{L_1},$$

or

(17) $$R_x = R_3 \frac{L_2}{L_1}.$$

Were the apparatus and method free from error, equation (17) would at once give the value of the resistance required. Certain errors are inevitable, however. The most important of these are the following:

(1) The tapping edge which makes contact at the point E, Fig. 25, may not be exactly in line with the pointer by means of which the position is read on the scale.

(2) In order to form the Wheatstone bridge proportion R_3 ought to include, besides the resistance of the coil itself, that of the connecting wires and brass plates from A to B, and also the extra resistances introduced at the points where connections are made. Similarly R_x ought to include the corresponding resistances from B to C; R_1 those from A to E, and R_2 those from C to E. These extra resistances are usually very small in the case of R_3 and R_x, but for R_1 and R_2 they frequently are not negligible, because the bridge wire may not have been soldered to the plates exactly at the ends of the scale.

We will consider a method for avoiding or minimizing these errors. Suppose that the tapping edge at E touches the wire at a point which is d cm. nearer H than the scale indicates, and suppose that a length e_1 cm. of the bridge wire would have to be added to the length of L_1 in order to correct for extra resistances from A to D and at D. Further suppose that a length e_2 cm. of the bridge wire would have to be added to the length of L_2 in order to correct for the extra resistances at that end of the bridge. Then, by the law of the bridge

$$\frac{R_x}{R_3} = \frac{L_2 - d + e_2}{L_1 + d + e_1}. \tag{18}$$

This equation contains four unknown quantities, and it is probable that some of them can be eliminated if another equation can be written. This can be done if we interchange R_3 and R_x, and obtain another balance at some point E', probably very close to E. Let DE' be called L'_1 and $E'H$ be called L'_2. Then the law of the bridge gives

$$\frac{R_x}{R_3} = \frac{L'_1 + d + e_1}{L'_2 - d + e_2}. \tag{19}$$

From the law of composition in proportion it is known that if

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f},$$

then

$$\frac{a}{b}=\frac{c+e}{d+f}.$$

Applying this law to (18) and (19), we have

$$(20)\qquad \frac{R_x}{R_3}=\frac{L_2-d+e_2+L'_1+d+e_1}{L_1+d+e_1+L'_2-d+e_2}=\frac{L_2+L'_1+e_1+e_2}{L_1+L'_2+e_1+e_2}.$$

It is seen that the quantity d no longer appears in the equation. Hence by taking two readings with R_x and R_3 interchanged, the so-called tapping error has been eliminated. A further simplification of the formula may be made by writing $100-L_1$ for L_2, and $100-L'_1$ for L'_2. Equation (20) then becomes

$$(21)\qquad \frac{R_x}{R_3}=\frac{100-L_1+L'_1+e_1+e_2}{L_1+100-L'_1+e_1+e_2}=\frac{100-(L_1-L'_1)+e_1+e_2}{100+(L_1-L'_1)+e_1+e_2}.$$

The quantities e_1 and e_2 will be small as compared to 100 cm. if the bridge has been carefully made, and from (21) it is seen that when $(L_1-L'_1)$ is small, the presence of the term (e_1+e_2) makes very little difference in the ratio R_x/R_3. In this case e_1+e_2 may be neglected without sensible error, and (21) becomes

$$(22)\qquad \frac{R_x}{R_3}=\frac{100-(L_1-L'_1)}{100+(L_1-L'_1)}.$$

In order to make $(L_1-L'_1)$ small, it is only necessary to choose R_3 near R_x in value, so that the contact point E is near the middle of the scale.

51. Laboratory Exercise VI. *To measure a resistance with the meter bridge.*

Apparatus. Meter bridge, portable or other sensitive galvanometer, tap key, one or two dry cells, adjustable resistance box, and the samples to be measured.

Procedure. (1) Arrange the circuit as in Fig. 25. Select some value of R_3 which you judge to be of the same order of

magnitude as the resistance to be measured. Tap the sliding contact lightly at opposite ends of the wire and note whether there is a reversal of the galvanometer deflection. If no reversal occurs, there is some fault in the circuit which must be sought for and corrected. If a reversal does occur, note the scale reading and calculate roughly the value of R_x to the nearest ohm. Set R_3 at this value, and again seek the point at which reversal occurs. This should now be near the middle of the wire.

(2) Having found this position E, let the notebook sketch show the relative positions of R_3, R_x, L_1, and L_2, and record the values of R_3 and L_1. Without altering the value of R_3, interchange R_3 and R_x and again locate the balance point. Record the new value of L'_1. Also note and record the distance through which the slider can be moved from the balance point before the least observable deflection occurs on the galvanometer, and state the probable precision of the settings.

Repeat each reading several times, making independent settings each time. The balance point should be located with the attention on the galvanometer and not on the scale, in order to avoid being influenced by the previous setting.

(3) To obtain the value of R_x substitute in equation (22).

It may be observed that the galvanometer deflects when the tapping contact is pressed, even though the battery key is open. This means that thermal electromotive forces are acting at some of the contacts. The average of readings taken with the terminals of the battery reversed will be free from errors due to this cause. The battery key should invariably be pressed before the key in the galvanometer circuit, in order to insure the current rising to its full value before the galvanometer is connected. Otherwise there may be a slight deflection due to induction in some part of the circuit. The keys should be kept closed only as long as may be necessary to take readings, in order to prevent any heating effects.

52. The Box Bridge. Another common form of the Wheatstone bridge is the ***box bridge***, or ***post-office bridge***, so-called

because it was used at an early date by the British Post and Telegraph Office. A top view of the circuit of one form of this bridge is shown in Fig. 26. The resistance to be measured is connected at R_3; R_1 and R_4 are called the ratio arms, and R_2 is called the rheostat arm. A little study of the circuit will show that

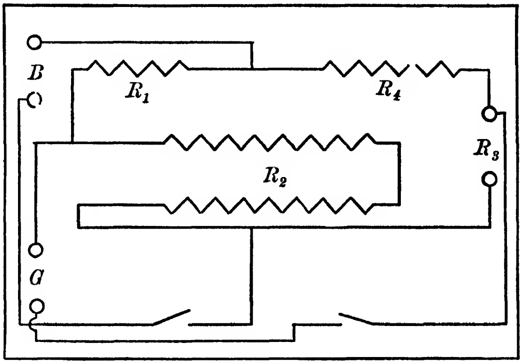

FIG. 26.

when the ratio of R_1 to R_4 is unity, the bridge will be balanced when R_2 is equal to R_3. In any case, when the resistances have been adjusted so that there is no galvanometer deflection, we have

$$\frac{R_1}{R_4}=\frac{R_2}{R_3}. \tag{23}$$

If $R_1 = R_4$, obviously $R_2 = R_3$. However, if R_1 equals 1/10, or 1/100, or 1/1000, of R_4, then R_3 must be respectively 10, 100, or 1000 times R_2 in order that a balance may exist. With the highest value in R_2 equal to 11,110 ohms, and the ratio R_1/R_4 equal to 1/1000, a resistance in R_3 as high as 11,110,000 ohms can be measured. In case the ratio arms can be reversed, the bridge equation becomes

$$\frac{R_4}{R_1}=\frac{R_2}{R_3},$$

and if R_4 is 1000 and R_1 is 10, then for a balance R_2 will be 100 times R_3. The smallest value in R_2, which is usually one ohm, is then equivalent to 0.01 ohm in R_3. If the highest ratio

of R_4 to R_1 is 1000 to 1, the smallest fraction measurable in R_3 with one ohm as the least step in R_2, is 0.001 ohm.

Box bridges commonly have ratio arms with a ratio of 1000 to 1, and rheostat arms with a range from 1 to 1000, or higher. This gives a very large theoretical range, but practically the method is limited, and its greatest value is in the middle range from one ohm to perhaps a megohm. For higher values than this an increased potential difference is necessary, and insulation troubles appear. Moreover, a high degree of precision is required in the ratio coils. For high resistances, however, advantage may be taken of the law of parallel circuits, and a known resistance may be connected in parallel with the unknown high resistance, the combined value of the pair being thus made low enough to fall well within the precision limits of the bridge.

For low values of less than one ohm, the method gives less accurate results because of the errors due to contact resistance.

Detailed descriptions of the great variety of box bridges now in use are not practicable in a textbook. When confronted with an unfamiliar type, the student should first make a sketch of the simple diamond form of the bridge circuit, and then one of the actual connections of the box bridge in hand. In this way it is easy to identify the ratio arms, rheostat arm, and the unknown resistance, together with the proper points of connection for the battery and the galvanometer.

53. Laboratory Exercise VII. *To measure a resistance with the box bridge.*

Apparatus. Box bridge, portable or other sensitive galvanometer, one or two dry cells, and the samples to be measured.

Procedure. (1) Connect the resistance to be measured to the line terminals, and the battery and galvanometer to their respective terminals. Make the ratio arms equal. With the rheostat arm equal to zero, press the battery key firmly, then

cautiously and lightly tap the galvanometer key and note the direction of the galvanometer deflection. Then make the rheostat arm as large as possible, or withdraw the infinity plug if there is one, and again tap the keys as before and note the direction of the deflection. If the circuit has been correctly set up, the two deflections will be in opposite directions. If this is not the case, the fault must be found and remedied, as it is useless to attempt to secure a balance unless the deflection reverses in the above test.

It is not necessary to wait for the galvanometer to come to rest except during the final adjustment. The galvanometer key should be tapped with a quick motion, so that a lack of balance is indicated without the risk of a too violent fling of the suspended system.

(2) Having found the approximate resistance with an even ratio, step at once to the highest ratio which the box will allow, or which the measurement requires, and find the value of the rheostat arm necessary for a balance. Find the value of the unknown resistance and confirm this by several repetitions, also with different settings of the ratio arms.

Questions. (1) What will be the effect of using more or less battery cells?

(2) What governs the choice of values in the ratio arms?

(3) How would you adjust the ratio arms in order to measure (*a*) a resistance higher than the range of the rheostat arm? (*b*) a resistance lower than the smallest step in the rheostat arm?

54. Laboratory Exercise VIII. *To verify the laws of series and parallel resistance combinations.*

Apparatus. Three or more samples to be measured and box bridge with accessories.

Procedure. (1) Measure in the prescribed way the value of the resistances singly and then in various series and parallel groups.

(2) Calculate the equivalent resistance for each of these groups, and compare the result with the value found from measurement. Express the percentage deviation of the single values and show the effect of these deviations on the computed results.

(3) Tabulate all data and results. Prove in full the formulas used.

55. Galvanometer Resistance. A precise determination of the resistance of a galvanometer is best made by means of the box bridge. Connect the galvanometer to the line terminals of the bridge, taking care that the suspended system is arrested, or supported in such a manner that it will not be deflected by the current which passes through it.

Frequently, however, an approximate value will suffice, in which case some modification of the *half-deflection method* is convenient. Two methods of procedure will be described.

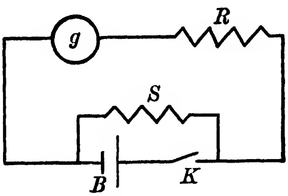

FIG. 27.

CASE I. For finding the approximate resistance of a galvanometer which has a straight-line calibration curve, connect as in Fig. 27, using a gravity battery because of its freedom from polarization. Fix R at some value R_1, such that the galvanometer gives a full-scale deflection, the battery being shunted, if necessary, with a low resistance. Ohm's law for the entire circuit may be written in the form

$$(24) \qquad I_1 = Fd_1 = \frac{E}{R_1 + g + \dfrac{BS}{B+S}}.$$

If the resistance R is now increased to some value R_2, such that the galvanometer deflection d_2 is equal to one half of d_1, then

(25) $$I_2 = F\frac{d_1}{2} = \frac{E}{R_2 + g + \dfrac{BS}{B+S}}.$$

Dividing (25) by (26), we find

$$2 = \frac{R_2 + g + \dfrac{BS}{B+S}}{R_1 + g + \dfrac{BS}{B+S}},$$

or

(26) $$g = R_2 - 2\,R_1 - \frac{BS}{B+S}.$$

If B is small as compared to g, it may be neglected. Moreover, if S is low enough so that R_1 may be made equal to zero, then, by (26), $g = R_2$.

Case II. When the battery to be used is one that polarizes rapidly, the foregoing method is useless, as the low resistance shunt would permit the cell to run down. In this case arrange a circuit as in Fig. 28, in which the battery is in series with a high resistance AC, and let the potential difference between A and P be that used in the galvanometer circuit. Let V represent the potential difference between A and P, and let r be the resistance of this portion of the circuit. The position of P may be so chosen that a full-scale deflection results with $R = 0$. The current through the galvanometer is then given by the formula

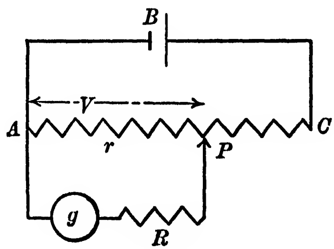

Fig. 28.

(27) $$I_1 = Fd_1 = \frac{V}{r+g}.$$

If R is now increased to some value R_1, such that the galvanometer deflection is reduced to one half its former value, we have

$$I_2 = F\frac{d_1}{2} = \frac{V}{r + g + R_1}. \tag{28}$$

Dividing (27) by (28), and assuming that V remains constant,[1]

$$2 = \frac{r + g + R_1}{r + g},$$

or

$$g = R_1 - r. \tag{29}$$

56. Battery Resistance. The internal resistance of a battery will be more fully discussed in Chapter III. Often for a battery of fairly high resistance and constant electromotive force, such as the gravity battery, a method similar to that described in the preceding article will be convenient. Connect as in Fig. 27, but with the shunt across the galvanometer instead of across the battery. Make R some value, say R_1, such that the deflection d_1 is about the full scale. Increase R to some value R_2, such that the corresponding deflection d_2 is just one half of d_1. Then, by reasoning similar to that used in deriving equation (26) it can be proved that

$$B = R_2 - 2\,R_1 - \frac{sg}{s + g}. \tag{30}$$

If the galvanometer is not shunted, and if R_1 can be made zero, we have

$$B = R_2 - g. \tag{31}$$

In case g is shunted with a very low value so that $sg/(s + g)$ is negligible, and if $R_1 = 0$, we have

$$B = R_2. \tag{32}$$

[1] Changes in R will change the value of V. The student should work out this relation carefully for any particular case in which the method is used. In general, the method will be used only when approximate results for g will suffice.

PART III. METHODS FOR MEASURING LOW RESISTANCES

57. Inadequacy of the Wheatstone Bridge. Although the various forms of the Wheatstone bridge are capable of great precision when applied to the measurement of resistances of medium values, they are not suited for small resistances of the order of fractions of an ohm because of contact errors. Hence other methods are required which are free from these errors, and which will enable a resistance of the order of 0.001 ohm to be measured with the same precision as that attained with a Wheatstone bridge on a 1000-ohm coil. By far the most widely used method for measuring low resistances is the one devised by Lord Kelvin, and known as the Thomson or Kelvin bridge. Its development may be traced through four stages, and will be considered in the four articles that follow.

58. The Kirchhoff Bridge. The circuit shown in Fig. 29 represents a device which is not now in use, but which contains the fundamental idea from which present-day methods have arisen. It depends upon the use of a differential galvanometer, which is simply a double instrument, symmetrically arranged, so that equal currents through the two coils will leave the equilibrium of the suspended system unchanged. If such a galvanometer has its respective pairs of terminals connected across the standard resistance s and the unknown resistance x, and if a constant current is maintained through s and x in series by the battery B, there will be no deflection when the two resistances are equal.

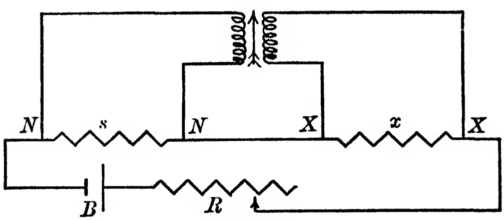

FIG. 29.

59. A Direct-deflection Method. In this method the two resistances may be compared by means of any sensitive galvanometer, arranged as in Fig. 30. This galvanometer is connected to the middle points of a double-pole, double-throw switch D, and can be successively connected across the terminals of s or x. The corresponding deflections will be shown to be proportional to the resistances respectively.

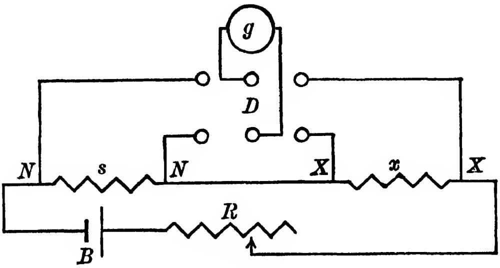

FIG. 30.

Let V_s and V_x be the potential differences respectively between the terminals NN of the standard resistance s, and the terminals XX of the unknown resistance x, when a suitable current is maintained through them in series by the battery B. When the switch connects the galvanometer across the terminals of s, the current through the galvanometer is given by the equation

(33) $$i_1 = \frac{V_s}{g}.$$

Similarly, when the switch connects the galvanometer across the terminals of x,

(34) $$i_2 = \frac{V_x}{g}.$$

Dividing (33) by (34), we have

(35) $$\frac{i_1}{i_2} = \frac{V_s}{V_x}.$$

But

(36) $$\frac{i_1}{i_2} = \frac{d_1}{d_2};$$

hence it follows that

(37) $$\frac{V_s}{V_x} = \frac{d_1}{d_2}.$$

If the currents through s and x are represented by i_s and i_x, then, by Ohm's law, we have

$$i_s = \frac{V_s}{s}, \tag{38}$$

and

$$i_x = \frac{V_x}{x}. \tag{39}$$

If the galvanometer resistance is relatively high, so that an inappreciable current is diverted from the main circuit when it is placed in parallel with s or x, then i_s may be considered equal to i_x. Hence, equating (38) and (39), we find

$$\frac{V_s}{s} = \frac{V_x}{x},$$

or

$$\frac{V_s}{V_x} = \frac{s}{x}. \tag{40}$$

Combining (40) and (37), we have

$$\frac{s}{x} = \frac{d_1}{d_2},$$

whence

$$x = s\frac{d_2}{d_1}. \tag{41}$$

The unknown resistance is then readily calculated when s is a standard of known value. A high value of the galvanometer resistance becomes of less importance as the values of s and x become more nearly equal. When they are equal, the same current is diverted through the galvanometer in each case, and i_s is then rigorously equal to i_x. It is important, then, to select a standard of as nearly as possible the same value as the sample to be measured. In such measurements large errors may enter, unless changes in resistance with temperature are taken into account.

60. A Zero Method. The arrangement shown in Fig. 31 may be used to compare an unknown low resistance with a standard. This method assumes a continuously adjustable

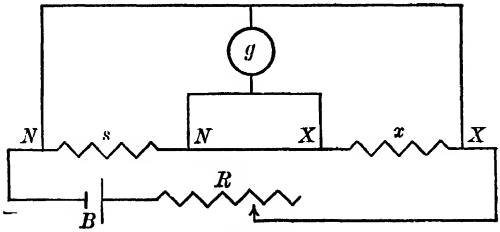

FIG. 31.

standard resistance, which is difficult to realize in practice. It depends upon the equalization of the potential drops on the two sides of the galvanometer, in which case the deflection is zero.

61. The Double Bridge. As a direct development of the method described in § 60, four resistances are added to the circuit, as shown in Fig. 32, and in this form it is usually known as the ***Kelvin bridge*** or the ***Thomson bridge***.

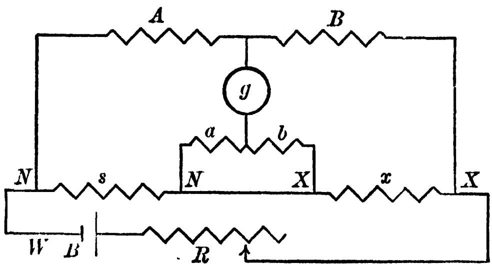

FIG. 32.

Let s represent a known standard resistance, and x the resistance of the sample to be measured. The resistance coils A, B, a, and b are fixed in value so that

$$\frac{A}{B} = \frac{a}{b}. \tag{42}$$

It will be proved below that when the various resistances are adjusted so that the galvanometer shows no deflection, we have the relation

$$\frac{s}{x}=\frac{A}{B}=\frac{a}{b}.$$

When the resistances in the bridge have been so arranged that no current passes through the galvanometer, the current i_1 in A equals that in B. Similarly, the current i_2 in a equals that in b, and the current i_3 in s equals that in x. Then,

$$si_3 + ai_2 = Ai_1. \tag{43}$$

$$xi_3 + bi_2 = Bi_1. \tag{44}$$

Dividing both equations by their right-hand members, we have

$$\frac{si_3}{Ai_1}+\frac{ai_2}{Ai_1}=1. \tag{45}$$

$$\frac{xi_3}{Bi_1}+\frac{bi_2}{Bi_1}=1. \tag{46}$$

From (42) $a/A=b/B$; hence a/A may be substituted in (46) for b/B. Then, subtracting (46) from (45), we find

$$\frac{si_3}{Ai_1}-\frac{xi_3}{Bi_1}=0, \tag{47}$$

or

$$\frac{s}{A}=\frac{x}{B},$$

whence

$$\frac{s}{x}=\frac{A}{B}, \tag{48}$$

which is the relation desired. The values of A and B may be made very large, and in comparison with them the resistances of the lead wires and contacts may be neglected.

The Kelvin bridge may be built up from resistance boxes available in the laboratory. However, the usefulness of the

method has led to the development of a convenient and self-contained box of coils, designed to yield results with a minimum of time and labor. Such a set is shown in Fig. 33 and

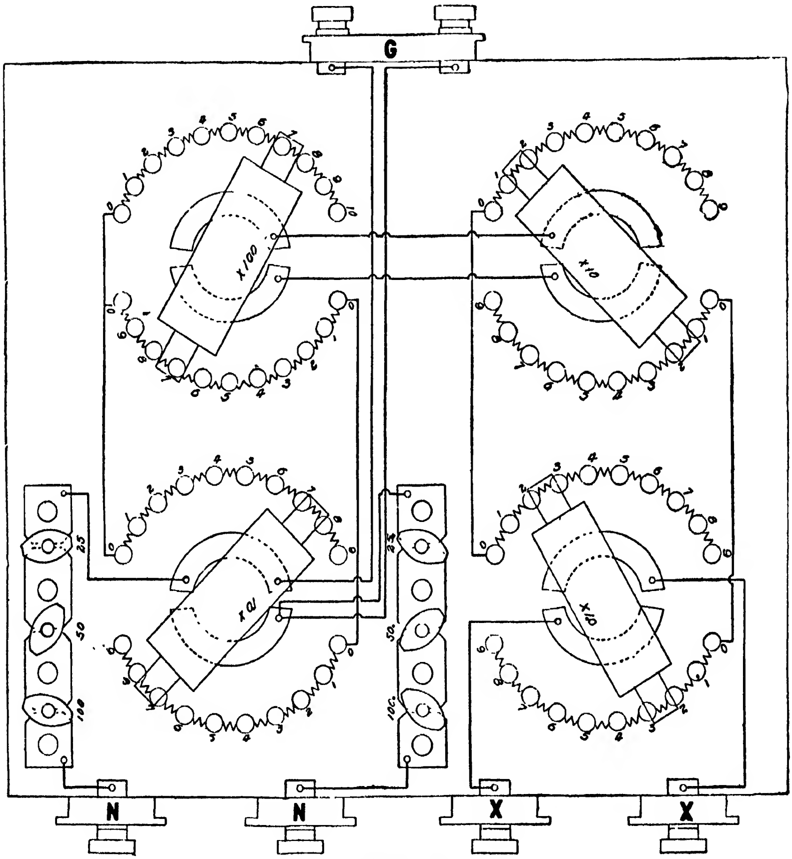

FIG. 33.

includes in a single case all those parts of the circuit in Fig. 32 which are above *NN* and *XX*, exclusive of the galvanometer.

In the group of coils under switch x 100, there are twenty coils of 100 ohms each, ten in each semicircle. In the group

marked x 10, there are eighteen coils of 10 ohms each, nine in each semicircle. In the group marked x 1 there are eighteen 1-ohm coils, and in the group marked x 0.1 there are eighteen 0.1-ohm coils. In each case there are the same number of coils in the upper as in the lower semicircles. The contact brushes of the rotating switches over the semicircles in each group are insulated from one another. It is readily seen that as any switch is rotated, the resistance effective in the upper and lower semicircles of any dial group is always the same.

In measuring a resistance with this apparatus, a working battery *WB*, Fig. 32, is put in series with a properly chosen standard resistance s, and the resistance to be measured x. A control rheostat should be included in this circuit. The terminals of the standard resistance are connected to the binding posts *NN*, and those of the resistance to be measured to *XX*.

The effective resistance of all four of the upper semicircles taken in series corresponds to B of Fig. 32, while that of the four lower semicircles corresponds to b. The group of three plug controlled resistances at the left in Fig. 33 corresponds to A, and that near the middle of the diagram corresponds to a. The rotation of any one of the four dial switches to the right or to the left will increase or decrease, respectively, the values of B and b by equal amounts. Since A and a will always be equal, it is seen that the ratio A/B will always be equal to a/b, which is the fundamental condition upon which the operation of the bridge is based. Inasmuch as this is a zero method, its operation is not affected by fluctuations in the working-battery current.

The method is widely used in the comparison of low resistances. It is available also for calibrating standard coils, for measuring the variation of resistance with changes in temperature, and for finding the conductivity of bars and wires in the form of short samples.

62. Laboratory Exercise IX. *To measure a low resistance ith the Kelvin bridge, and to find the temperature coefficient of esistance for a sample of wire.*

APPARATUS. Standard resistance of low value, samples to e measured, oil bath with heater, Kelvin bridge, galvanometer, few cells of storage battery, tap key, and connecting ires.

PROCEDURE. (1) Connect in series with the battery and a ontrol rheostat the standard resistance s, and the sample to e measured x. Connect the terminals of s to the points arked NN on the bridge and the terminals of x to the points X. Include a tap key k in series with the galvanometer, hich is connected at G, Fig. 33.

(2) Introduce equal resistances at A and a, and with all the al switches on zero, tap k and note the direction of the throw. et the switch x 100 on 10 and again tap k. If the galvanomer does not deflect in the opposite direction, it is necessary interchange the wires at either NN or XX. If reversal curs, a setting of the four dials may be found such that there no deflection when k is closed.

(3) Using the notation of § 61, the upper halves of all the als will read the value of B, and the lower halves of the als will read the value of b, which values in this apparatus ill always be the same. Calculate the value of x from equan (48).

(4) With the sample immersed in oil or water, make seval measurements at carefully determined temperatures, and lculate the value of the temperature coefficient from equation), § 42. The bath must be stirred so that the temperature uniform throughout.

63. Laboratory Exercise X. *To measure a low resistance the fall of potential, direct-deflection method, and to find the sistivity of a sample of wire.*

APPARATUS. One or two low-resistance standards, samples to be measured, double-pole double-throw switch, reversing switch, galvanometer, one or two constant battery cells, resistance box, and clamps.

PROCEDURE. (1) Fasten the clamps to the edge of the table and secure the ends of a sample of wire 15 to 30 cm. long in the clamps, stretching it tight. Arrange the circuit as in Fig. 30, and include a reversing switch between the galvanometer and the switch D.

Connect current and potential wires to the double binding posts on the clamps, and adjust R until suitable deflections are observed. Connections should be so made that galvanometer deflections are on the same side of the scale when the switch D is thrown over.

(2) Take reversed galvanometer deflections across x and s, repeating several times for each sample.

(3) Calculate from equation (41) the value of the resistance of the sample.

(4) Measure the length and diameter of the wire used and calculate the resistivity in microhms per centimeter cube, for each sample furnished, using equation (3), § 39.

Note the resistance of the galvanometer used. Is it high enough to avoid sensible error?

Discuss the effect on the results obtained if the galvanometer resistance were increased or decreased. State the probable precision attained in the apparatus used.

For many commercial purposes a moderate degree of precision suffices and a millivoltmeter may be used instead of the galvanometer. The ratio of the readings will give the ratio of the resistances.

64. Measurement of Low Resistances by Ohm's Law. A simpler method than the foregoing, and one which is sufficiently accurate for a great deal of routine testing, depends on Ohm's law, and upon the direct measurement of current and potential difference with ammeter and millivoltmeter.

Consider a circuit arranged as in Fig. 34, in which x is the resistance to be measured. Let I and V represent the readings of the ammeter and millivoltmeter, respectively, and let r be the resistance of the millivoltmeter. Writing Ohm's law for the portion of the circuit between a and b, we have

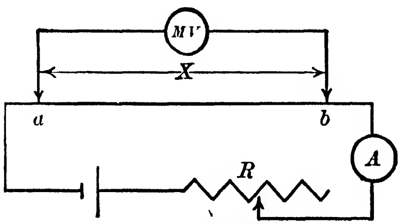

FIG. 34.

$$I = \frac{V}{\frac{xr}{x+r}},$$

from which the value of x is found to be

$$x = \frac{V}{I - \frac{V}{r}}. \tag{49}$$

If r is relatively large, the term V/r may be neglected, and a direct application of Ohm's law gives the value of the unknown resistance x.

65. Laboratory Exercise XI. *To measure a low resistance with ammeter and millivoltmeter.*

APPARATUS. Ammeter, millivoltmeter, one or two storage cells, control rheostat, and samples to be measured.

PROCEDURE. (1) Arrange the circuit as in Fig. 34, with the contact points a and b at the terminals of the sample to be tested. Adjust R so that the current and potential difference are easily readable on the instruments.

(2) Compute the value of x for several pairs of values of I and V, from equation (49), both with and without the factor V/r, and show the percentage accuracy attained.

PART IV. MEASUREMENT OF HIGH RESISTANCE

66. High-resistance Methods. It has been pointed out in § 52 that the Wheatstone bridge is not available for the measurement of high resistance. The procedure most commonly employed for such measurements involves the direct observation of deflections. Hence it is not a zero method. Moreover, since the high resistances measured are usually those of insulation, great precision is not sought.

67. A Direct-deflection Method. The following is a rapid and fairly accurate method for finding the value of a high resistance. It requires a galvanometer of known figure of merit or a sensitive milliammeter, a voltmeter, a sufficiently high electromotive force, and the sample to be tested. The circuit is arranged as in Fig. 35. The galvanometer becomes an ammeter when its figure of merit is known and may then be used to measure the current passing through the circuit. By Ohm's law, we have

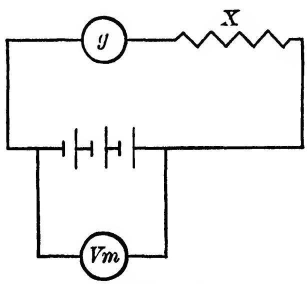

FIG. 35.

$$i = Fd = \frac{V}{g + x}, \tag{50}$$

where F is the figure of merit of the galvanometer, g is its resistance, V is the terminal potential difference of the battery, and x is the resistance to be measured. From (50) we find

$$x = \frac{V}{i} - g. \tag{51}$$

If g is small compared with x, it may be dropped for approximate results.

68. A Voltmeter Method. A high resistance can be measured also by means of a circuit arranged as shown in Fig. 36. A battery of 100 volts or upwards is connected in series with the high resistance x. The voltmeter is first connected across the battery terminals CA, and its reading V is observed. The contact point is then changed from A to B, and again the reading of the voltmeter V is observed. Writing Ohm's law for the two cases, we have

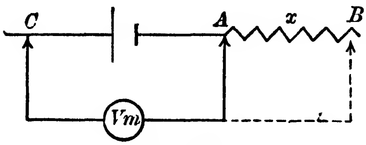

FIG. 36.

$$i_1 = Fd_1 = \frac{E}{g}, \tag{52}$$

and

$$i_2 = Fd_2 = \frac{E}{g+x}, \tag{53}$$

where F is the figure of merit of the galvanometer, g is its resistance, and E is the electromotive force of the battery. The internal resistance of the battery is negligible in comparison with the high resistance in series with it. Dividing (52) by (53), we find

$$\frac{d_1}{d_2} = \frac{g+x}{g}. \tag{54}$$

Since deflections on the voltmeter are proportional to potential differences, we have

$$\frac{d_1}{d_2} = \frac{V_1}{V_2} = \frac{g+x}{g}, \tag{55}$$

or

$$x = g\left[\frac{V_1}{V_2} - 1\right]. \tag{56}$$

This method will not yield results of high precision, but it is convenient and requires only simple equipment. It is much used in practice for rapid and approximate determinations

It is applied, for example, in the case of finding the insulation resistance of a wiring circuit, as shown in Fig. 37. The distributed resistances between the two sides of the line and the ground G are represented by x_1 and x_2, respectively. With the instruments as shown, the value of x_1 may be found. In order to find x_2, the point a must be changed to the other side of the line.

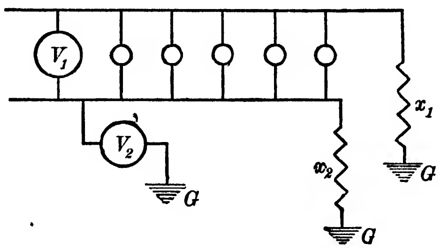

FIG. 37.

69. The Method of Substitution. As a general method of good precision for measuring the insulation resistance of wires and cables, or other high resistances, the following method is frequently used. With a circuit arranged as in Fig. 38, let R_1 be some known resistance of one tenth megohm or higher, and let R_2 be the unknown. The galvanometer must be one of high current sensibility, and the battery should be capable of supplying a constant potential difference of from one hundred to five hundred volts. With the switch K raised to the point u, the current through the galvanometer is given by the equation

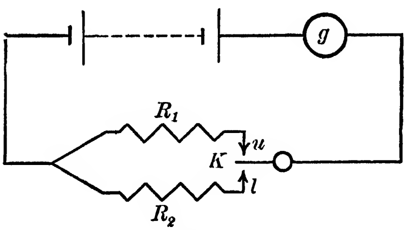

FIG. 38.

(57) $$i_1 = Fd_1 = \frac{E}{R_1 + b + g},$$

where E and b are the electromotive force and resistance of the battery, respectively. With the switch K on l, the galvanometer current is given by the equation

$$i_2 = Fd_2 = \frac{E}{R_2 + b + g}. \tag{58}$$

Dividing (57) by (58), and neglecting the resistance of the battery as small, we have

$$\frac{d_1}{d_2} = \frac{R_2 + g}{R_1 + g}. \tag{59}$$

If g is negligible as compared to the other resistances, we may write

$$R_2 = R_1 \frac{d_1}{d_2}. \tag{60}$$

Frequently, however, the available standard R_1 is so small compared with R_2 that the two deflections are not readily comparable. For example, if the value of R_1 is one tenth megohm, while that of R_2 is 100 megohms, then d_1 is one thousand times as great as d_2. For such a case the deflections would not be of the same order of accuracy. In order to avoid difficulties of this kind, and to keep the values of the deflections nearly the same, the galvanometer is shunted. The foregoing method as used with the Ayrton shunt is illustrated in Fig. 39, and the formula will now be developed. Suppose that for some position a of the adjustable contact arm, the value of r is R/a. Similarly, for some position b, the value of the resistance to the left of b is R/b. Calling I_1 and I_2 the currents in the main circuit for the upper and lower positions of the switch, respectively, and using equation (11), § 26, we may write expressions for the corresponding currents through the galvanometer in the form

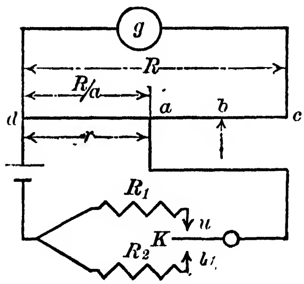

FIG. 39.

$$i_g = Fd_1 = I_1\left[\frac{R/a}{g + R/a + (R - R/a)}\right] = I_1\left[\frac{R/a}{g+R}\right], \tag{61}$$

and

$$(62)\ i'_g = Fd_2 = I_2\left[\frac{R/b}{g + R/b + (R - R/b)}\right] = I_2\left[\frac{R/b}{g + R}\right].$$

Since R_1 and R_2 will always be large, the currents through them will not be affected appreciably by a change in the value of the shunt ratio. Therefore the effective potential difference of the battery is essentially unchanged. We may then write equations (61) and (62) in the form

$$(63) \qquad i_g = Fd_1 = \frac{E}{R_1}\left[\frac{R/a}{g + r}\right],$$

and

$$(64) \qquad i'_g = Fd_2 = \frac{E}{R_2}\left[\frac{R/b}{g + r}\right].$$

Dividing (63) by (64), we have

$$\frac{d_1}{d_2} = \frac{b}{a} \cdot \frac{R_2}{R_1},$$

whence

$$(65) \qquad R_2 = R_1\left[\frac{a}{b} \cdot \frac{d_1}{d_2}\right].$$

The quantities a and b here represent the fractional parts of R corresponding to the positions of the contact arm. It must be kept in mind that these fractions do not represent the ratio of the galvanometer current to the total line current, but the ratio of the two galvanometer currents corresponding to the positions a or b and c. The ratio of currents through the galvanometer for positions a or b and c is a constant, and is independent of the galvanometer resistance.

In all these high resistance measurements we are dealing essentially with materials other than metals. Nevertheless, it is assumed that Ohm's law holds. This is approximately true for ordinary insulating materials or for surface leakage. However, if there is electrostatic capacity present, as in con-

densers or long cables, special care must be given to the interpretation of the results.

Moreover, in the case of insulating substances it is often found that the deflection which occurs when the electromotive force is first impressed across the high resistance does not remain constant, but falls off at first rapidly, then more slowly, the variation becoming inappreciable only after a long interval, frequently several hours. This phenomenon is called **electrification**, and is due to certain changes which take place within the body of the substance. The effect is less marked as the temperature rises.

In order that measurements of high resistance may have definite significance, they must be accompanied by statements of the experimental conditions, including impressed voltage, time of electrification, and temperature. A representative specification for cables is as follows:

Each conductor shall show, after twenty-four hours' immersion in water, an insulation resistance of not less than 300 megohms per mile, with not less than 100 volts applied for one minute, at a temperature of 75° Fahrenheit.

For further details on cable testing see § 165, Chapter VIII.

70. Laboratory Exercise XII. *To measure a high resistance by the substitution method.*

APPARATUS. A standard resistance of one tenth megohm or more, sensitive galvanometer, Ayrton shunt, well-insulated switch, and an E. M. F. of 100 volts or more.

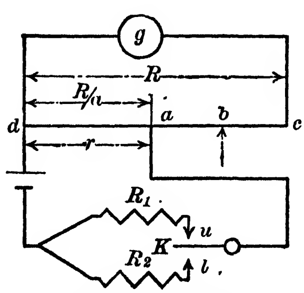

FIG. 39 (repeated).

PROCEDURE. (1) Arrange the circuit as in Fig. 39, with a high-insulation switch in the battery circuit. In case the E. M. F. is taken from a dynamo, it is well to include a carbon filament lamp in

the circuit in order to prevent an excess of current in case of an accidental short-circuit. The resistance of this lamp may be known and allowed for, or it may be neglected if the resistance to be measured is high enough. It is always preferable to use a well-insulated battery for the E. M. F.

(2) The maximum current will pass through the galvanometer when the switch is on point 1, Fig. 15, § 27, and the first position to try is then on the other side, beginning on the zero point, and setting successively on *b*, *c*, *d*, etc., until a readable deflection occurs.

In using the Ayrton shunt it must be kept in mind that the ratios given hold only when the deflections for the respective positions of the switch are compared with the deflection which would occur with the switch on point 1. The multiplying factors for the other positions are then 10, 100, 1000, etc. This differs from the ordinary shunt practice, where the multiplying factor gives the ratio between total current and galvanometer current. In the Ayrton shunt the galvanometer is always in parallel with the combined resistances of all the steps.

(3) Read and record the deflections for the standard resistance and for the samples furnished.

(4) Calculate the values of the unknowns by substituting the proper values in equation (65). If insulation resistance of wire or cable is being found, express the result in megohms per mile.

All parts of the circuit must be well insulated, and since the E. M. F. used will, in general, be high, the utmost precaution must be exercised to avoid a dangerous short-circuit. In case porcelain or glass insulators are being tested, the resistance of one of them may be too high to be measured conveniently. A group of ten or a hundred are then connected in parallel, the resistance of a single one being inferred from the result.

71. The Loss of Charge or Leakage Method. For some kinds of testing where the resistance of cables, dielectric resistance of condensers, etc., is to be measured, the loss of charge method is preferred. This method involves the use of the ballistic galvanometer, and it will be described after this instrument has been studied. See Chapter VIII, § 165.

PART V. THE MEASUREMENT OF LIQUID RESISTANCE

72. Resistance of Electrolytes. With respect to electrical resistance, liquids may be divided into two classes:

I. Good conductors.

II. Poor conductors, commonly called non-conductors.

Under the first class two groups are recognized: (*a*) mercury and other fused metals; (*b*) electrolytes, which are, in general, water solutions of salts and acids.

Of the liquids classed as good conductors, those in group (*a*) offer no difficulty so far as the measurement of their resistance is concerned. The Wheatstone bridge or any fall of potential method yields accurate results.

Under group (*b*), however, special difficulties are encountered. An electrolyte conducts by virtue of the decomposition of the

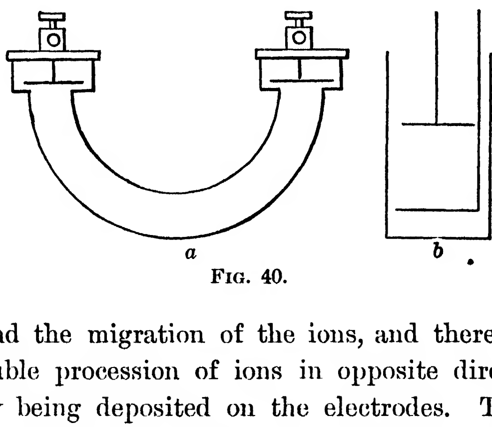
a *b*

FIG. 40.

solution and the migration of the ions, and there is, in general, a double procession of ions in opposite directions, the ions finally being deposited on the electrodes. This sets up a counter electromotive force of polarization which affects measurements as though an extra resistance had been added. Hence, determinations based on Ohm's law are inaccurate.

The Wheatstone bridge may be used, however, by putting the sample of the electrolyte to be measured in a suitable tube or vessel provided with platinum electrodes (Fig. 40), connecting this as the unknown arm of the bridge, and impressing an

alternating instead of a steady E. M. F. A sensitive telephone receiver will replace the galvanometer, and will give a minimum sound when the bridge is balanced. If the alternating current is supplied from the secondary of a small induction coil from which the condenser has been removed, the polarization during one half of the cycle will be annulled by the E. M. F. of opposite sign during the following half cycle, and a balance may be obtained from which the true resistance of the sample is found.

The electrodes should be coated with a deposit of platinum black, which greatly increases the effective area, thus reducing the surface density of such residual charges as are present. The size and style of the containing tubes will depend upon the conductivity of the solution to be measured. A poorly conducting electrolyte should be measured, in general, in a short tube with a large electrode area.

The tube selected is first calibrated by measuring in it the resistance of a sample of a standard solution of known conductivity, such as a normal solution of potassium chloride. After carefully cleansing the tube, the sample of the electrolyte of unknown conductivity is measured in the same way.

The resistivity of any substance is given by the equation (3) of § 39. If the length of the column of electrolyte is denoted by L, the resistivities in the two cases are given by the equations

$$k_1 = R_1 \frac{a}{L}, \tag{66}$$

$$k_2 = R_2 \frac{a}{L}, \tag{67}$$

where k_1 is the resistivity of the standard solution, k_2 is that of the unknown, R_1 and R_2 are the measured resistances respectively, and a/L is the ratio of the area of cross-section to the length of the column of electrolyte.

Since this ratio is a constant for the given containing cell,

the unknown resistivity is found by dividing (66) by (67) and solving for k_2; this gives

$$k_2 = k_1 \frac{R_2}{R_1}. \tag{68}$$

The conductivity is the reciprocal of resistivity and is expressed in terms of the reciprocal ohm or the mho (§ 40). A table of conductivities for potassium chloride at different temperatures is given in the appendix.

If polarization effects are avoided, Ohm's law holds throughout a wide range of values for electrolytes, from highly conducting salt solution to poorly conducting water. The conductivity of pure water has been measured in a vacuum and found to be 0.4×10^{-6} mhos per centimeter cube. Water in contact with the atmosphere has a conductivity about twice as great, and this cannot be reduced because of the tendency to dissolve carbonic acid and ammonia from the air, as well as certain substances from the glass vessel in which it is usually kept. While the amount of these impurities is very minute, the effect on the conductivity of the water is comparatively very large.

Precise measurements require that the applied alternating wave of E. M. F. shall have the form of a sine curve, with equal positive and negative amplitudes, and that the bridge arms shall be free from capacity and inductance.

Except for the highest accuracy, the above method, due to Kohlrausch, is satisfactory. Such measurements are of particular interest in physical chemistry, in affording information as to the degree to which any dissolved substance has been dissociated.

73. Laboratory Exercise XIII. *To find the conductivity of an electrolyte.*

Apparatus. Wheatstone bridge, sensitive telephone receiver, small induction coil, resistance box, one or two dry cells, containing cell, and thermometer.

A convenient form of Wheatstone bridge, known as the ***Kohlrausch bridge***, is shown in Fig. 41. A uniform manganin wire nearly five meters long is wound in ten turns on a marble cylinder. This is covered with a protecting hood which revolves on a vertical spindle, threaded with a

Fig. 41.

pitch exactly equal to that of the winding of the wire. A spring brush fastened to the inside of the hood is always in contact with the wire, and is connected through the spindle to the binding post on the front of the base. The two ends of the wire are attached to the two binding posts at the back. Total turns are read on the glass scale, and fractions of a turn are read on the graduated circle against a vertical line etched on the glass plate. Ratios of lengths on either side of the contact point, and hence also ratios of the resistances of wire segments, are readily determined.

Procedure. (1) Draw the usual diamond diagram of the Wheatstone bridge, properly label the arms, and write the corresponding formula.

The entire arrangement of the circuit, with the Kohlrausch bridge inserted, is shown in Fig. 42, p. 100.

(2) Connect the containing cell E in series with the known resistance and with the bridge wire, as in Fig. 42. Attach the telephone and the secondary of the induction coil to the proper terminals. Usually it is preferable to connect the telephone receiver between A and B, although either of the conjugate

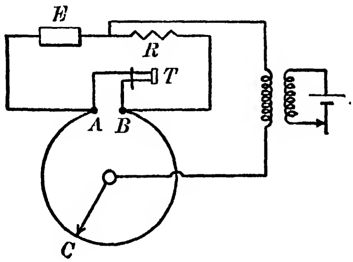

FIG. 42.

arrangements may be used. Compare this circuit with the diagram previously drawn, and write the formula required for this actual arrangement.

(3) Measure a sample of the standard solution, adjusting R and the position of the contact on the bridge wire until the minimum sound is found near the middle of the wire. Take the temperature of the sample.

(4) Rinse out the cell and refill it with the electrolyte of unknown conductivity. Measure its resistance as above and take its temperature.

(5) Calculate the resistivity from equation (68) and find the conductivity of the samples at the observed temperature.

(6) If the balance point is not sharply defined, the electrode surfaces probably need replatinizing. These surfaces may be rinsed with clear water after using, but they should not be touched with the fingers nor wiped with a towel.

An increase in sensitiveness may be effected by putting a short-circuiting key across the telephone terminals and repeatedly making and breaking the circuit for any particular setting of the movable contact. The ear is more sensitive to sudden changes than to gradual ones.

CHAPTER III

ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

PART I. SOURCES OF ELECTROMOTIVE FORCE

74. There are three well-known classes of apparatus for transforming energy of other kinds into electrical energy. Of these, the ***thermoelectric generator*** is of use only for relatively feeble currents. Its phenomena and some of its applications will be discussed in Art. 81. The ***dynamo generator*** depends upon principles which will be discussed in Chapter VI.

In the following articles some of the more common types of ***battery cells*** will be described, as these are especially important in connection with electrical measurements. The function of a battery cell is to transform chemical energy into electrical energy. Two classes of cells will be considered, ***primary batteries***, which are renewed by the addition of fresh chemicals, and ***secondary batteries***, or ***accumulators***, which are renewed or charged by means of a current which is applied in a direction opposite to that yielded by the cell when in service, and which produces in the cell the necessary chemical changes.

75. The Gravity Cell. The gravity cell has for its negative electrode a zinc plate, suspended near the top of the containing jar, and for its positive electrode a plate of copper at the bottom of the jar. This plate is covered with a layer of copper sulphate crystals, and over it for a few inches is a water solution of the same salt. Above this, and surrounding the zinc plate, is a solution of zinc sulphate. A sharp line of separa-

tion should appear between the two solutions. Due to its greater density the copper sulphate solution tends at first to remain below the zinc sulphate. However, diffusion will occur unless the battery is supplying current. This type yields the most constant electromotive force of any of the ordinary primary cells. Owing to its freedom from polarization, it is useful for closed circuit work. Its E. M. F. is 1.08 volts, and its internal resistance may vary from half an ohm to several ohms.

To set up a gravity cell for experimental purposes, first fill the jar half full of water and stir in about 50 grams of $ZnSO_4$. Place the copper in position at the bottom of the cell and hang the zinc at the top. By means of a funnel tube introduce a saturated solution of $CuSO_4$ at the bottom of the cell, taking every precaution against mixing the two solutions. The blue should appear at the bottom, and gradually rise, carrying the white $ZnSO_4$ solution above it. Continue introducing $CuSO_4$ solution until the zinc plate is well covered by the $ZnSO_4$. In case the fluids mix they must be emptied out and a fresh trial made. When through using a gravity cell, if it is desired to keep it set up, close its circuit through a resistance of a few hundred ohms, in order to prevent diffusion of the solution. If it is not to be kept set up, pour out the solution into the waste jar, rinse the parts in clean water, and wipe them dry with a towel. These battery solutions should not be emptied into the ordinary laboratory sinks.

76. The Leclanché Cell. The Leclanché cell has a voltage of 1.4, a low internal resistance, and rapid polarization. It is useful chiefly for open-circuit work. The elements are zinc and carbon in a solution of salammoniac. The chemical action is complex, a double chloride of zinc and ammonium being formed, with liberation of hydrogen and ammonia gas at the carbon plate. To diminish polarization effects, a paste con-

taining black oxide of manganese is used, sometimes packed in a porous jar about the carbon electrode, and sometimes formed into solid bars, which are held close to the carbon plate by rubber bands. Recovery from polarization is rapid when thus treated.

77. The Dry Cell. The so-called dry cell is a modification of the Leclanché cell, in which the solutions are absorbed in the materials of the cell, thus rendering it portable in any position, with no liquids to be spilled. The E. M. F. is 1.5–1.6 volts, and when fresh, its internal resistance is low, so that with an external resistance of 0.01 ohm, a current of 25 or 30 amperes can be drawn from it.

The anode is a sheet of zinc in cylindrical form, which is the outer casing or container for the cell. This zinc shell is lined with several layers of porous paper heavily impregnated with ammonium chloride solution. Within this paper is a granular mixture of carbon, manganese dioxide, and ammonium chloride saturated with zinc chloride and ammonium chloride solutions. Packed securely within this mixture and at the center of the cell is a fluted carbon rod, and the whole is sealed air tight with pitch or hard wax.

Polarization is rapid when large currents are drawn from it, but the manganese dioxide is a strong oxidizing agent, and the return to nearly normal voltage is rapid after use.

The internal resistance of this type of cell increases rapidly with use and age, partly due to the drying out of the contained moisture and partly due to the reduction of the manganese dioxide. There is also a deposit on the zinc of non-soluble impurities and products of secondary reactions. Particularly in the dry cell, the so-called internal resistance is a compound quantity, which is dependent upon many factors, and which varies with the current output, the age, and the temperature. Ordinary tests, such as those described in §§ 85–86, are of little

significance with dry cells. They should be subjected to service tests, based on the actual conditions of use.[1]

Where galvanometer deflections are to be observed, constant potential cells are necessary. For zero methods, however, other types will often answer. Dry cells in series with a high resistance may be regarded as constant potential cells for many purposes.

78. The Edison-Lalande Cell. Modifications of the original Lalande cell, usually sold under the name ***Edison-Lalande***, have an E. M. F. of 0.8–0.9 volt, and a low internal resistance of only a few hundredths of an ohm. The negative electrode is amalgamated zinc, and the positive electrode consists of a plate of compressed copper oxide, the surface of which has been reduced to the metallic state. These plates are immersed in a twenty per cent solution of sodium hydroxide. A thin layer of heavy mineral oil is poured over the top to prevent evaporation and creeping. After the cell has been in operation for a short time the E. M. F. becomes practically constant.

79. Standard Cells. It is convenient to have at hand certain voltaic or electrochemical standards of E. M. F. Two such standards, the ***Clark cell*** and the ***Weston cell***, can be prepared with a high degree of constancy and trustworthiness.

The Clark cell has zinc or zinc amalgam for its negative, and mercury for its positive electrode, the zinc being surrounded by a saturated solution of $ZnSO_4$, while a paste of mercurous sulphate is above the mercury. The E. M. F. of a cell of this sort is 1.434 volts at 15° C. Between 10° and 25° its variation is about 0.00115 volt per degree, the E. M. F. decreasing with a rise in temperature.

The E. M. F. of these cells will vary with the concentration of the solutions and with the circumstances of manufacture. Hence a certificate should accompany each cell when purchased.

[1] For standard methods of testing dry cells see TRANS. AM. ELECTROCHEMICAL SOCIETY, vol. 21, 1912, p. 275.

This cell, as originally prepared, was not sufficiently portable. Moreover, its E. M. F. did not follow accurately the changes in temperature. An improved form was devised by Carhart. In this form the zinc sulphate solution is saturated at 0° C., and its temperature coefficient is somewhat less than in the earlier forms. The value of its E. M. F. for any temperature $t°$ is found from the formula

(1) $E_t = 1.440 - 0.00056(t - 15)$ volts.

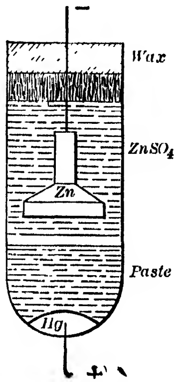

FIG. 43.

The interior arrangement of a ***Carhart-Clark cell***[1] is shown in Fig. 43.

The ***Weston cell***, or ***cadmium cell***, takes its name from the use of cadmium instead of zinc as in the Clark cell. It was first suggested by Weston in 1891. It is preferably made in the H form, as shown in Fig. 44. Platinum wires are sealed into the tubes and make contact with mercury on the positive side and with a cadmium-mercury amalgam on the negative side. Above the mercury is a layer of a thick paste, made by intimately mixing metallic mercury, mercurous sulphate, and a saturated solution of cadmium sulphate. Above the cadmium amalgam on the negative side is a layer of cadmium sulphate crystals, and over all, filling both sides of the tube, is a saturated or nearly saturated solution of cadmium sulphate. The open ends of the tube are closed with corks and sealed with wax. When these cells are made according to precise specifications[2] they can be

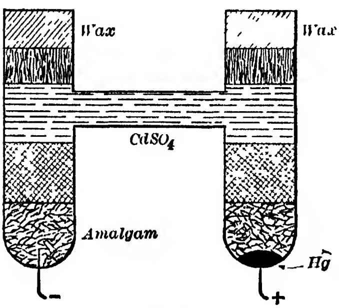

FIG. 44.

[1] Specifications for setting up these cells are given in the U. S. BUREAU OF STANDARDS BULLETIN, vol. 4, p. 1, 1907.

[2] U. S. BUREAU OF STANDARDS BULLETIN, vol. 4, p. 1, 1907.

reproduced with a variation of only a few parts in 100,000. Temperature changes have but slight influence on this type of cell, a change of ten degrees C. either way causing a change in E. M. F. of less than five parts in 10,000.[1] For this reason the cadmium cell has practically displaced all other types. It is portable and may be sent through the mails or otherwise transported without ill effects.

Formerly the international volt was defined as a stated fraction of the E. M. F. of a cell of the Clark type, but its evaluation depended upon the use of a known resistance and a known current. The London Conference (1908), although defining the international volt in terms of the ampere and the ohm, recommended the adoption of the Weston cell as a substandard of voltage; its value is now taken as

1.0183 volts at 20° C.

Standard cells are not intended to supply current, and they must always be used in such a way that there is no danger of polarization taking place. They should be used only in series with a high resistance (10,000 ohms), or for zero methods where the potential difference is compensated.

EXERCISE

Will a standard cell show its rated voltage when connected to a commercial voltmeter? Explain. What would be the effect on the cell of this experiment?

80. Secondary Batteries. When a direct current is passed through an electrolytic cell, decomposition products are deposited on the cathode. In general, if the source of current is removed and the circuit is again completed, there will be a flow of current in the reverse direction, due to the polariza-

[1] The precise formula from which the E. M. F. of a Weston cell is calculated for any temperature $t°$ is

$$E_t = E_{20} - 0.0000406(t-20) - 0.00000095(t-20)^2 + 0.00000001(t-20)^3.$$

See U. S. Bureau of Standards, *Circular No.* 29.

tion of the electrodes. In this way an exhausted battery can have its active material renewed by electrolytic deposition. In a class of batteries called ***storage cells***, or ***accumulators***, this is an efficient process.

A common type of accumulator consists of lead plates perforated with many apertures, into which the active material is compressed. This is usually a paste made by mixing certain lead salts (red lead, Pb_3O_4, and litharge, PbO) with sulphuric acid. If plates thus prepared are immersed in a twenty per cent solution of sulphuric acid, and a current is sent through the cell, hydrogen passes to the cathode and reduces the paste to spongy metallic lead. The SO_4 ions pass to the anode, and a higher oxide of lead, PbO_2, is formed. The rapid evolution of hydrogen at the cathode is evidence of complete transformation of the material, and the cell is then said to be charged. If the charging current is now cut off and the cell connected to a circuit, current will be found to flow from the cell in a direction opposite to that during the process of being charged. Such cells have a discharge voltage of about 2.2, and a low internal resistance of the order of a few hundredths of an ohm. Moreover, their behavior when in good condition is very constant, and they are indispensable for many kinds of electric testing. The disadvantages of this type of cell are its great weight and its requirement of regular and systematic attention during charging and use.

In the ***Edison storage cell***, the positive plate consists of nickel oxide packed in perforated steel tubes, several of which are mounted side by side in a steel frame. The negative plate consists of iron oxide held in a somewhat similar way. These plates are immersed in a twenty per cent solution of caustic potash, and are sealed into a container of welded sheet steel. These cells have an average voltage of 1.2, which is a disadvantage as compared with the lead type of cell. However, they are rugged, constant in their behavior, and but slightly

affected by extreme temperatures. Because of their smaller weight, they are preferred for many purposes to other types.

The storage cell does not store electricity, but by means of the current supplied to it chemical changes are set up which renew the active material necessary for the continued operation of the cell.

81. Thermoelectricity. In 1826 Seebeck discovered that a current of electricity flows in a circuit consisting of two different metals, when a difference of temperature is maintained at the two points of contact, or *junctions.* This is commonly explained by saying that at the junction thermal energy is transformed into electrical energy, and this point is regarded as the seat of an E. M. F.

In the following table some common metals are so arranged that when any two are chosen for the circuit, current flows across the hot junction from any metal to one standing lower in the list.

Bismuth	Lead
Nickel	Silver
Platinum	Iron
Copper	Antimony

Of the pure metals the bismuth-antimony pair yields the greatest thermoelectromotive force, that is, these elements have the greatest thermoelectric power. However, certain alloys such as german silver, advance, and platinum with iridium or rhodium, are frequently used for one of the materials. Expressed in microvolts per degree of temperature difference, the thermoelectromotive force of a pair of iron-german silver junctions is about 25, that of an iron-advance pair is about as great, and that of a copper-advance pair is about 40. The purity and the physical state of these materials is an important factor in the measured values.

The thermoelement is made by soldering two pieces of the hosen metals together, with their ends soldered respectively o pieces of copper wire. The junctions AA, Fig. 45, are kept t some constant temperature, usually that of melting ice, and he junction B is ieated. The galanometer will hen indicate the assage of a curent. Knowing

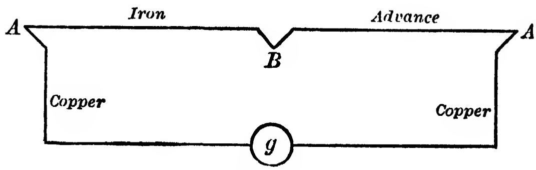

FIG. 45.

he sensibility of the galvanometer and the resistance of the ircuit, the effectiveness of the arrangement in microvolts per legree can be calculated. The presence at the junction of an ntermediate metal or alloy like solder will not affect the value f the E. M. F., because whatever effect is developed at one oint of contact is annulled at the other.

Many and varied forms of thermoelements find application n the measurement of temperatures where mercury-in-glass hermometers would be too massive to respond quickly to mall temperature changes, or where it would be impossible r inconvenient to introduce them. The chief advantage of hermoelements is found in their small mass and their quick esponse to changes of temperature. The range over which hey may be used is limited only by the temperature at which he metals oxidize, or for some metals, the temperature at vhich the effect changes sign. For example, in the iron-copper lement the effect changes sign at about 270° C. For a range rom liquid air temperatures, − 190° C. to 200° or 300° C., opper-advance or iron-german silver is used. For higher emperatures, upwards of 1700° C., a thermocouple of platinum nd a platinum-rhodium alloy is used. The E. M. F. may be neasured by means of a potential galvanometer[1] or by a po-

[1] Since the resistance of the thermocouple and its circuit is usually low, he galvanometer should also be of low resistance.

tentiometer method. Any thermoelement may be calibrated with a galvanometer so that temperatures can be read directly.

82. Laboratory Exercise XIV. *To calibrate a galvanometer used with a copper-advance thermoelement.*

Apparatus. Galvanometer, thermocouples with hot and cold baths and thermometers, one accumulator cell, voltmeter, two resistance boxes, and a tap key.

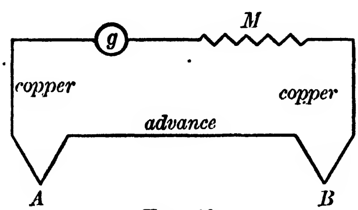

Fig. 46.

Procedure. (1) Arrange the circuit as shown in Fig. 46, keeping the junction *B* at the temperature of melting ice. Heat the junction *A* by means of a flame under the oil bath, and take simultaneous readings of temperatures and galvanometer deflections. A multiplier of required amount to keep the deflection on the scale for the highest temperature used (about 200° C.) should be put in series with the galvanometer. Take eight or ten readings over approximately equal intervals of temperature change, from room temperature to 200° C.

(2) Plot deflections as ordinates and temperature readings as abscissas on a sheet of cross-section paper.

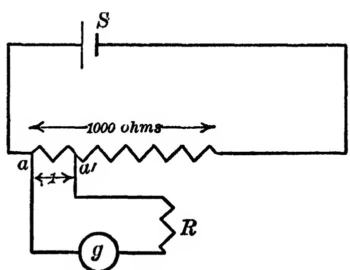

Fig. 47.

(3) To calibrate the galvanometer in microvolts per scale division, connect the apparatus as in Fig. 47. A storage cell *S* is put in series with a resistance of 1000 ohms. Traveling contacts at *aa'* will enable a potential difference one thousandth of that of *S* to be applied to the galvanometer in series with the variable resistance *R*. The voltage of the cell may be taken with a voltmeter. Take deflections, five or more in

number, over the full-scale range. Calculate the potential difference at the galvanometer terminals in microvolts.

(4) Use the same sheet as in (2) above, and lay off a scale of microvolts along the axis of abscissas. Plot deflections against microvolts. From the two curves thus plotted, calculate the potential differences developed at the various temperatures, and plot a third curve showing the relation between potential difference and temperature.

83. Electromotive Force and Terminal Potential Difference. In general, whenever two plates of different metals are dipped in an electrolyte, a difference of potential is found to exist between them. If dilute sulphuric acid is used as the liquid, the potential relations of a few familiar substances are given in the following table. Selecting any two of the substances, current is observed to flow through a wire connecting them outside of the battery, from any one, to one which stands lower in the table.

	↑ Carbon	As battery poles, these are positive downward.
	Mercury	
	Copper	
	Iron	
As substances, these are	Cadmium	
electropositive upward.	Zinc	↓

The terms ***positive*** and ***negative*** as applied to the poles of a battery must not be confused with ***electropositive*** and ***electronegative*** as applied to the substances.

The seat of the E. M. F. is at the surface of contact between the electropositive substance (zinc) and the liquid, and the negative pole is the exposed part of the electropositive plate. In a simple voltaic cell (Fig. 48) charges of opposite sign are found at the poles. If the poles are connected by a conductor, these charges are removed.

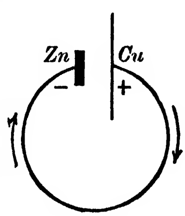

FIG. 48.

However, the action of the cell restores them and the potential difference is maintained. There is, then, some process going on within the cell which tends to maintain this transfer of charge from the negative pole to the positive pole *within* the cell, and from positive to negative *outside.* This is called the ***electromotive force***, or E. M. F. of the cell.

In the external conductor, as well as in the liquid of the cell, there will be a fall of potential in the direction of the current flow, and at the interface between the zinc plate and the liquid there will be a rise in potential, due to a transfor-

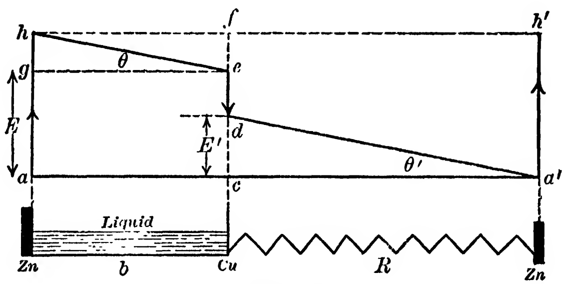

FIG. 49.

mation of chemical energy. The potential-resistance relations throughout the entire circuit are illustrated in Fig. 49.[1] At the zinc-liquid surface there is a sudden rise in potential ah. Through the liquid resistance b there is a fall of potential fe. Also, there is a fall of potential ed at the liquid-copper surface, and a further drop dc through the external resistance R. In this figure, vertical distances represent potential differences, and horizontal distances represent resistances; hence, it is clear that the slope of the lines he and da' is measured by the ratio of potential difference to resistance respectively. This slope is then equal to the current flowing, and since the cur-

[1] Figure 49 represents a special case where the line ga', parallel to he, intercepts the line ah at the point g, so that $gh = fe$. In general this line will not cut ah at g, but at some point which may be called g'. The student should draw other similar figures with R much larger and much smaller than the battery resistance.

rent is constant in all parts of the circuit, the angles θ and θ' are equal. The maximum effective potential difference E, or the total E. M. F., is given by the equation

$$ah - ed = cd + ef, \tag{2}$$

while the potential difference effective in the external part of the circuit, or E', is represented by cd.

From similar triangles it then follows that

$$\frac{ef}{cd} = \frac{ac}{ca'};$$

or, by composition,

$$\frac{cd + ef}{cd} = \frac{ac + ca'}{ca'} = \frac{aa'}{ca'};$$

whence

$$\frac{E}{E'} = \frac{R + b}{R} \tag{3}$$

or

$$E' = E\frac{R}{R + b}. \tag{4}$$

The quantity E' is called the ***terminal potential difference*** and it is seen that E' will approach E as b diminishes in value, or as R becomes so great that in comparison with it b may be disregarded.

When a potential galvanometer is connected across the terminals of a cell which is not delivering current, the observed reading E is the E. M. F. of the cell, or the open-circuit potential difference. If, without removing the galvanometer, a resistance R, Fig. 50, is connected across the battery terminals, a current will flow, and the galvanometer reading E' will be less than before. This decrease will go on as the value of R is made less, until when $R = 0$, that is, when the cell is short-circuited, the galvanometer will show no deflection whatever. The potential galvanometer shows, for any value

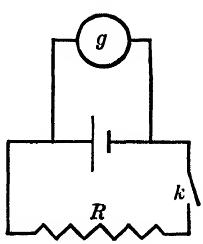

FIG. 50.

of R, the *then existing potential difference at the cell terminals,* and this may vary from the value of the E. M. F. of the cell to zero. For any value of R the current flowing is given by Ohm's law in the form

$$i = \frac{E}{b + R}, \tag{5}$$

or

$$E = ib + iR. \tag{6}$$

The term iR is equal to E', whence

$$E' = E - ib. \tag{7}$$

Solving equation (7) for b, the internal resistance of the cell, we find

$$b = \frac{E - E'}{i}; \tag{8}$$

and since $i = \frac{E'}{R}$, equation (8) may be written in the form

$$b = \left[\frac{E - E'}{E'}\right]R. \tag{9}$$

Since galvanometer deflections are proportional to potential differences, equation (9) may be written in the form

$$b = \left[\frac{d_1 - d_2}{d_2}\right]R. \tag{10}$$

The preceding equations hold not only for the voltaic cell, but also for any other form of generator. The battery resistance will not be constant for all values of current, and in stating the value of the internal resistance, the current output must be specified.

The E. M. F. may be expressed in terms of the work done in conveying a unit charge once around the entire circuit. Referring to Fig. 49, the work done by the cell at the interface between the zinc plate and the liquid is proportional to line *ah*. On the same scale, *fe* represents the work done by the current against the ohmic resistance of the electrolyte.

The current does work proportional to *ed* at the liquid-copper surface, and a further amount of work proportional to *dc* is done in the external resistance *R*. For one such cycle the battery must supply an amount of energy proportional to the difference between *ah* and *ed*, or $cd + ef$. The work available for the external circuit is only that which is proportional to *cd*.

The entire circuit, including the battery resistance, may be regarded as divided into small portions. Then across the terminals of each portion there will be a certain potential difference, or potential drop, which will vary for any given portion with the value of the current flowing. The algebraic sum of all these differences of potential over the entire circuit will give the value of the E. M. F.

In order to avoid confusion, it is customary to use the term *potential difference*, or *potential drop*, with reference to certain limited portions of the circuit, and to reserve the expression *electromotive force* for that maximum value of the terminal potential difference which the generator yields when measured with no current flowing. Invariably when a current is flowing, some part of the E. M. F. is required to overcome the effective internal resistance of the generator, and the available potential difference of the terminals is always less than the E. M. F. by this amount.

84. Battery Resistance. It is important to note that the quantity *b*, which has been called the internal resistance of the battery, is not a constant, but varies more or less with the current drawn from the cell. Some batteries (*e.g.* the gravity type) show a decreasing resistance with an increase in current. Some have a high polarization (*e.g.* the dry cells), which tends to increase with the current output. The back E. M. F. of polarization opposes the E. M. F. of the cell, and the resultant or effective potential difference is decreased. The effect is

the same as that of an increased internal resistance, and the values of b calculated from equation (8) are apparently increased.

The temperature coefficient of the internal resistance is large, and the total ampere-hour output of the cell also affects its value. The quantity b is an important one, and it is treated as a resistance; properly it should be called the *effective resistance* of the cell. A complete study of a battery cell involves the determination of several factors which will be considered again in § 89.

85. Laboratory Exercise XV. *To study the variation in the potential difference at the battery terminals, and to find the internal resistance of the battery cell by the galvanometer method.*

Apparatus. Potential galvanometer with shunt and high-series resistance, reversing switch, resistance box, tap key, and one gravity cell.

Procedure (1). Arrange the circuit as in Fig. 51. Adjust the galvanometer to zero and choose such values for the shunt S and the series resistance R' that the deflection d_1 will be about full scale with K open. The series resistance should be at least a thousand ohms.

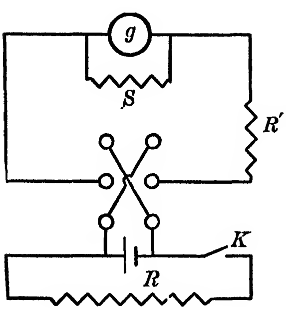

Fig. 51.

(2) Make $R = 200$ ohms, close the key K, and read the deflection d_2. Make R smaller, decreasing by ten steps until zero is reached, choosing the steps so that the deflections decrease by approximately equal intervals, and read the deflection for each step. The connecting wires from R to the battery should be as short as possible. Why?

(3) Tabulate values of R, right and left galvanometer deflections, and mean deflections. The deflection d_1 is proportional

to the E. M. F. of the cell. The other deflections are proportional to the respective values of the terminal potential difference.

(4) Plot a curve with values of R as abscissas, and deflections as ordinates, d_1 being the maximum value of the series. State what inference may be drawn from this curve. Locate on the y-axis a point at $d_1/2$. Project this point on the curve and read the corresponding value of R. Show that this is approximately the battery resistance.

(5) Calculate from equation (10) the value of the battery resistance, choosing several different values of d_2. If this is not a constant, account for its variation.

86. Laboratory Exercise XVI. *To measure the internal resistance of a battery by the voltmeter-ammeter method.*

APPARATUS. Voltmeter, ammeter, battery to be tested, control resistance, and tap key.

PROCEDURE. (1) Arrange the circuit as in Fig. 52. R is a control rheostat, the resistance of which need not be known. With K open, read E on the voltmeter, which gives the E. M. F. of the cell. Close K, having adjusted R so that the range of the ammeter is not exceeded, and again take the voltmeter reading E', simultaneously reading the current i.

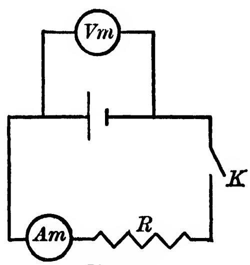

FIG. 52.

(2) Take several sets of E and i values, and calculate the value of b from equation (8). In case the battery consists of more than one cell, for example, a storage battery, the value of b determined above is the sum of the internal resistances of the several cells. The mean value for one cell is, however, readily calculated.

(3) If storage batteries are under test, E may be the potential difference measured while the battery is being charged, the charging current i being read at the same time. Then,

with the charging current cut off, E' is the terminal potenti difference. The internal resistance is then given by equation (8 The difference between the two voltages is that required to se the current i through the internal resistance of the battery.

87. Laboratory Exercise XVII. *To compare electromoti forces by the condenser method.*

APPARATUS. Ballistic galvanometer, standard condense three-way discharge key, standard cell, and battery to l tested. The theory of the ballist galvanometer and condenser will l given in subsequent chapters.

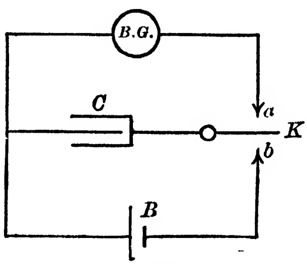

FIG. 53.

PROCEDURE. (1) Arrange the ci cuit as in Fig. 53, with a standa cell at B. Charge the condenser l pressing the key to b, then dischar it by raising the key to a, and rea the deflection d_1.

(2) Replace the standard cell by the cell to be tested a repeat (1), reading the deflection d_2.

(3) When the condenser is charged, the quantity is giv by the equations

$$Q_1 = Gd_1 = CV_1, \tag{11}$$

and

$$Q_2 = GD_2 = CV_2, \tag{12}$$

where Q_1 and Q_2 are the quantities stored in the condens when charging potentials V_1 and V_2 are impressed. The co responding deflections are d_1 and d_2 respectively, and G is t constant of the galvanometer. Then, from (11) and (12), we ha

$$\frac{V_1}{V_2} = \frac{d_1}{d_2}. \tag{13}$$

(4) Calculate the value of the unknown E. M. F. fro equation (13).

88. Laboratory Exercise XVIII. *To measure the internal resistance of a battery by the condenser method.*

APPARATUS. As in Laboratory Exercise XVII, § 87, together with a resistance box and a tap key.

PROCEDURE. (1) Arrange the circuit as in Fig. 54. With k open, press K to b for an instant and then raise it to a. Read the deflection on the galvanometer d_1.

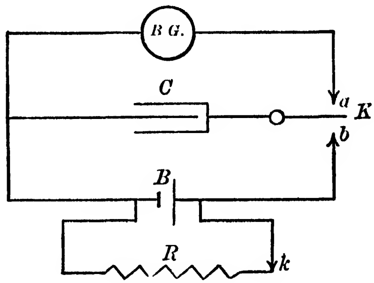

FIG. 54.

(2) With k closed, set R at some low value, and repeat the procedure of (1), reading the deflection d_2. Repeat for several different values of R.

(3) Calculate the value of b from equation (10). Each deflection used in the formula should be the mean of several observations. Tabulate all data and results.

This method is most reliable when the battery is one which does not polarize rapidly, and which has a high internal resistance. If a standard condenser of low value is used, the charge flowing into it will not appreciably polarize the cell.

89. Test of a Primary Cell. The two chief characteristics of a battery cell are its E. M. F. and its internal resistance. In general, that cell is most useful in which the E. M. F. is high and the internal resistance low. There are three different tests to which a battery must be subjected in order to investigate systematically and completely its quality and usefulness.[1] These are: (1) the *time test*, (2) the *life test*, and (3) the *efficiency test*.

The *time test* shows: (a) the decrease in E. M. F. due to polarization, together with the rate of this decrease when the

[1] For standard methods of testing dry cells, see PROC. AM. ELECTROCHEMICAL SOCIETY, vol. 21, p. 275, 1912.

cell is kept for a given period on closed circuit through a specified resistance; (*b*) the rate and extent of recovery from polarization; (*c*) the terminal potential difference when the cell is closed through a fixed resistance.

The *life test* is quite the same as the foregoing, except that the polarization is allowed to continue until the E. M. F. has been reduced to at least half of its initial value, after which the recovery is observed for a similar period.

The *efficiency test* shows: (*a*) the ratio of the quantity of electricity obtained from the cell by the consumption of a given mass of zinc to the quantity necessary to deposit the same mass in an electrolytic cell; (*b*) the ratio of the energy available in the external circuit to that dissipated within the cell itself as heat. If these last two tests are carried out, they will work the cell to exhaustion. If the first of them is carried over a period of perhaps an hour, it will give an accurate indication of the intrinsic worth of the cell. Hence it is the more common, and is frequently the only test made.

90. Laboratory Exercise XIX. *To make a time test of a battery cell.*

Apparatus. As in Laboratory Exercise XVIII, § 88, together with a watch.

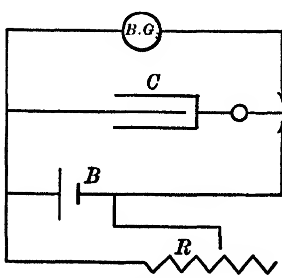

Fig. 55.

Procedure. (1) Arrange the circuit as in Fig. 55. If a Leclanché cell is used, make R about five ohms; if a dry cell is used, make R about half an ohm. With k open, charge and discharge the condenser by successively throwing K to b and a. The observed deflection will be proportional to the E. M. F. of the cell.

(2) The E. M. F. of the cell may be expressed in volts by means of a *calibration curve* for the galvanometer, which is

prepared as follows. Before beginning the test on the battery *B*, put in its place a standard cell, and note the galvanometer deflection *d*, using the same value of the capacity as that which will be used throughout the test. On a sheet of cross-section paper, scale off one axis in volts and the other in deflections, and locate the point corresponding to *d* and the E. M. F. of the standard cell. This will be one point on the calibration curve. If the deflections are strictly proportional to quantities of electricity, and hence to charging potentials, the curve will be a straight line passing through the origin. From this curve any value of the deflection may be read directly in volts. The scales should be so chosen that at least hundredths of a volt can be read.

(3) Observing the exact time, close *k* and immediately charge and discharge *C* as before, observing the deflection d_2. The terminal potential difference can then be taken from the calibration curve. Keeping *k* closed, again read d_2 after an interval of two minutes. After four minutes, open *k* for a very short time, just long enough to manipulate the key *K*, and observe another value of d_1, which gives the open circuit voltage at that time. After six minutes take a reading of d_2 as before, and after eight minutes with *k* open for an instant, take another reading for d_1.

Continue in this way for one hour, taking readings of the open-circuit voltage every four minutes, in order to get the decrease due to polarization. Alternate with these, also at four minute intervals, readings for the terminal potential difference. The key *k* will be left firmly closed except at the appropriate four-minute intervals, when it is opened for a second or two in order to secure the open-circuit readings. A convenient form for *k* is a spring tap key, making contact on the upper points.

(4) At the end of the hour, open *k* and continue the readings as before, in order to determine the rate and the extent of the recovery from polarization. For the first quarter of an

hour of recovery readings should be taken every two or three minutes. For the rest of the period, or until the recovery curve ceases to rise, longer intervals will suffice. Avoid closing the circuit containing the battery under test until quite ready to begin counting time.

(5) Calculate from equation (10) the internal resistance of the cell for ten or more sets of values of E and E'. Also calculate the same number of values of the current strength by dividing the respective values of the terminal potential difference by the external resistance R.

(6) Tabulate in full the values of time, d_1, d_2, open-circuit voltage, terminal potential difference, internal resistance, and current.

Tabulate, also, values of time, deflections, and voltages during the recovery period.

(7) On a sheet of squared paper choose a suitable time scale along the x-axis, and along the y-axis arrange three scales of suitable range for voltage, resistance, and current. Plot five curves: (1) open-circuit E. M. F., (2) recovery, (3) terminal potential difference, (4) internal resistance, (5) current. It is customary to start the recovery curve at the last reading of the polarization curve, running it toward the left, above the polarization curve.

Discuss the curves and state what inferences may be drawn as to the excellence of the cell.

Part II. Potentiometers

91. General Principles and Simple Circuit. Perhaps no single instrument is capable of more general application in the electrical laboratory than the ***potentiometer***, which is, as its name implies, a device for measuring potential differences. A simple form of potentiometer circuit is shown in Fig. 56.

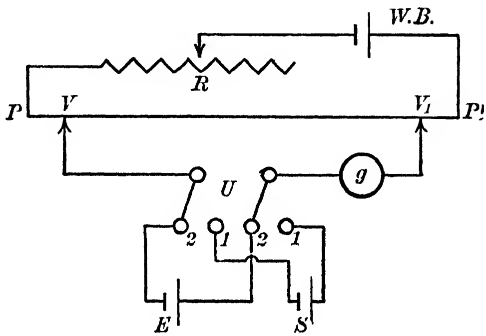

Fig. 56.

A uniform, homogeneous wire PP', usually a meter or more in length, is stretched over a graduated scale, and connected in series with a constant battery WB, called the working battery. An adjustable resistance R is introduced for the purpose of controlling the potential difference between P and P'. This resistance, as well as the wire, must be so chosen as to carry the necessary current without sensible heating. A double-pole, double-throw switch U enables either a standard cell S or the test cell E to be connected to the points VV_1.

Let us suppose (1) that the potential difference between P and P' is greater than that of the cell S, and (2) that the circuit is so arranged that the poles of the working battery and S are opposed. Then, if the switch U is on the points 1, 1, it will be possible to find two points V and V_1 on PP', such that the fall of potential between them is just equal to the potential difference at the terminals of S. In this case no

current will flow through the galvanometer g, and the circuit is said to be compensated or balanced.

If the current flowing through the wire is constant, the potential difference between V and V_1 is proportional to the resistance of the wire included between these points, and if the wire is uniform and homogeneous, the resistance is proportional to the length. Hence, the distance d_1 between V and V_1, read on the scale under the wire, may be assumed to be proportional to the value of the potential difference at the terminals of the standard cell S.

If the switch U is thrown across the points 2, 2, any other potential difference, such as E, may be balanced against the

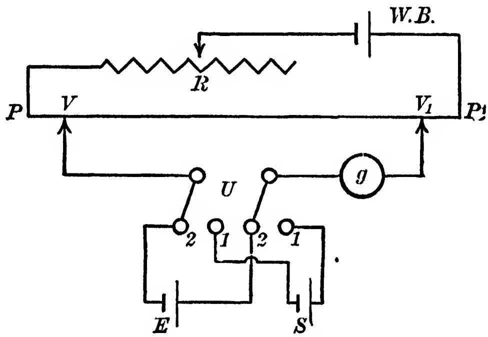

FIG. 56 (repeated).

fall of potential along some other length of the wire d_2. Then we have the relation

$$\frac{E}{S} = \frac{d_2}{d_1}, \tag{14}$$

whence the value of E is found to be

$$E = S\frac{d_2}{d_1}. \tag{15}$$

The method may be simplified and the apparatus made to read directly by the following procedure. Fix V at the point P, and graduate the scale into 1500 parts, with the zero at P

and the 1500 mark at P'. Assume that we are using a Clark cell, for which the voltage, corrected for temperature, is 1.432. Set the point V_1 at the scale division 1432, taking care that the conditions 1 and 2, as given above, are fulfilled. Adjust R until the galvanometer shows no deflection. Then the potential drop along the wire, which is directly proportional to the distance from P as read on the scale, is just equal to the known voltage of the standard cell. Assuming the constancy of this adjustment, values of potential difference may be read directly from the scale.

The great utility of the potentiometer lies in the fact that it may be used to measure current strength and resistance, as well as potential difference. The strength of a current is determined by finding the potential drop between the terminals of a standard resistance while the current is flowing through it. A resistance is measured by comparing the potential difference across the unknown resistance with that across a known resistance, both carrying the same unvarying current.

It is thus seen that all of these measurements are really referred to the E. M. F. of a standard cell and a standard resistance. These two quantities have been so thoroughly studied that the greatest confidence is felt in their correctness and permanence, provided that temperature corrections are properly applied.

A distinct advantage of the method is that the standard cell is so used that at the moment of balance no current whatever is being drawn from it. Hence precise results uninfluenced by polarization can be obtained. Furthermore, at the instant of balance, the lead wires do not carry any current, so that errors due to potential drop or contact resistance do not occur.

92. A Resistance-box Potentiometer. The long wire of the simple potentiometer may be replaced by a pair of resistance boxes, in which case a greater degree of precision may be attained.

The circuit of Fig. 57 shows such an arrangement. The working battery *WB* is placed in series with two resistance boxes R_1 and R_2, each of 10,000 ohms or more. The sum of R_1 and R_2 must be kept constant. A standard cell *S* with its poles opposed to the working battery is connected in series

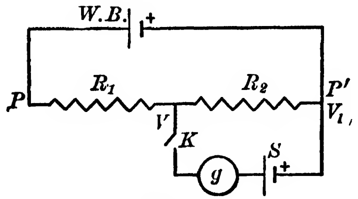

FIG. 57.

with a galvanometer across the terminals of R_2. If the E. M. F. of the standard cell is less than the potential drop along PP', some value of R_2 can be found for which the potential drop measured by iR_2 is just equal to the E. M. F. of *S*. For this value of R_2 there will be no current through the galvanometer. For every change made in R_2 an equal compensating change must be made in R_1.

It is convenient to use two boxes with the same range for R_1 and R_2, and also to start with all the plugs out of R_1 and with $R_2 = 0$, increasing R_2 until a balance is found. A high resistance of 10,000 ohms should be placed in series with *S*, and the key *K* should be tapped cautiously, until it is seen that the deflection is not going to exceed the range of the scale. When a balance is nearly reached, the high resistance may be removed.

With suitably chosen resistance boxes, a high degree of precision is possible. The double adjustment is tedious, however, and more convenient arrangements are found in the commercial types, which are described in §§ 94 and 96.

93. Laboratory Exercise XX. *To measure electromotive force with the simple potentiometer and with the resistance-box potentiometer.*

Apparatus. Storage battery and control rheostat, long wire on baseboard with scale, standard cell, cell to be tested, three resistance boxes, galvanometer, connecting wire, and tap key.

Procedure. (1) Arrange the circuit as in Fig. 56, with the standard cell in series with the galvanometer. Tap the contact key at extreme ends of the wire and note whether the galvanometer reverses. If no reversal occurs, interchange the terminals of the standard cell, or increase the voltage at PP'. Find the point on the wire for which no deflection occurs, and read this position on the scale. Replace the standard cell by the cell to be measured and again read the position of the contact for no deflection. The ratio of these two readings will give the ratio of the electromotive forces of the two cells.

(2) Make the potentiometer direct reading as explained in § 91, and again measure the E. M. F. of the test cell.

(3) Note carefully by how much the position of the contact point can be shifted without disturbing the balance, and state the probable precision of the settings. Each determination should be the mean of several readings.

(4) In place of the long wire, connect two similar resistance boxes (Fig. 57), and remove all the plugs from one box, say R_1. With the galvanometer and standard cell connected across R_2, tap the key and note the deflection. Make R_1 low and R_2 high, again tap the key, and note whether the galvanometer reverses its deflection. If a reversal does not occur, the circuit is not properly arranged. Adjust R_1 and R_2 until the deflection is zero when the key is tapped, and read R_2. Replace the standard cell by the test cell and repeat the procedure. The ratio of the values of R_2 will give the ratio of the electromotive forces of the two cells.

(5) As in (3) above, note how much R_2 must be changed in order to cause the least observable deflection on the galvanometer. After calculating the unknown E. M. F., discuss the precision of the results by the various methods.

94. The Wolff Potentiometer. The circuit of the ***Wolff potentiometer*** is shown in Fig. 58, and a conventionalized diagram of it is shown in Fig. 59. In order to understand this circuit, refer again to Fig. 56, and fix clearly in mind the following points:

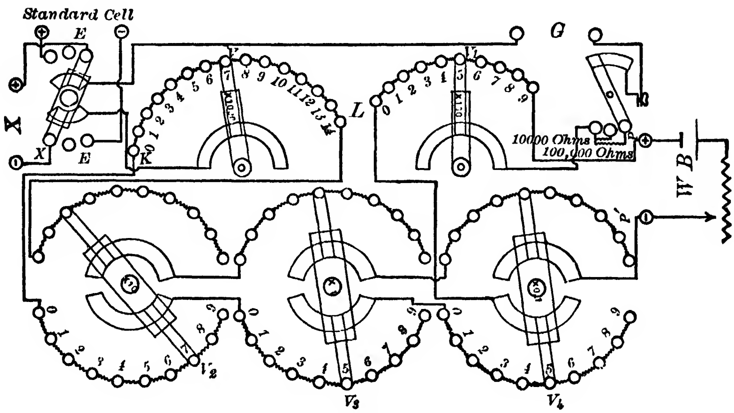

FIG. 58.

(1) The potential drop between P and P' must be greater than the potential difference to be measured.

(2) The terminals of the same sign must be connected to the same side of the circuit.

(3) However the contact points VV_1 are moved, or wherever they are applied along PP', the total resistance of PP', and also the current strength through PP', must remain constant.

In Fig. 59, WB is the working battery, R is a control resistance, and PP' can be readily traced. The upper and lower contact points, represented by the arrowheads, are connected mechanically, but they are electrically insulated from each other. When any one of the switches V_2, V_3, or V_4 is moved to the right, the effective resistance between V and V_1 is diminished. The total resistance PP' is kept constant, however, since a compensating increase is effected by the slid-

ing of the upper arrowheads along the resistance wire. On the other hand, if any one of the switches V_2, V_3, or V_4 is moved to the left, the effective resistance between V and V_1 is increased; but the total resistance PP' is kept constant by a compensating decrease along the upper branch. With this

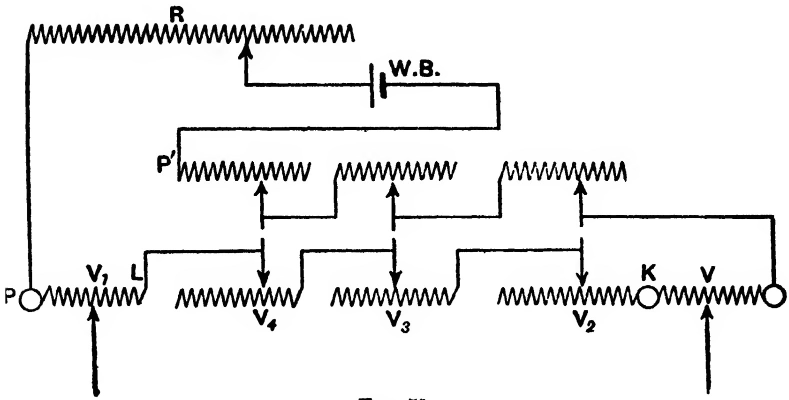

Fig. 59.

arrangement, we can keep PP' constant, while varying the potential difference between V and V_1 through any value from zero to the maximum potential drop along PP'.

The circuit of this potentiometer, as actually arranged for use, is shown in Fig. 58. The corresponding points in Figs. 58 and 59 are similarly lettered. The working battery and galvanometer will be connected as indicated, with the standard cell and the unknown potential difference at E and at X, respectively. Either E or X may be thrown into the circuit by the switch, as desired. In series with the working battery is a control resistance adjustable to one tenth ohm or less. Any sensitive galvanometer will suffice, although for rapid work a critically damped d'Arsonval is most convenient. The working battery should be one or more storage cells in good condition, and carefully insulated. Dry cells may also be used, inasmuch as the resistance in series with them is high.

95. Laboratory Exercise XXI. *To compare electromotive forces with the Wolff potentiometer.*

APPARATUS. Potentiometer, storage battery or other cells, control resistance, galvanometer, standard cell, and cell to be tested.

It will be seen by reference to Fig. 58 that the resistance of the potentiometer coils in series with the working battery is made up of fourteen 1000-ohm coils, nine 100-ohm coils, nine 10-ohm coils, nine 1-ohm coils, and nine 0.1-ohm coils, making a total of 14,999.9 ohms.

PROCEDURE. (1) Connect a two-volt storage battery and a control resistance to the working battery terminals PP', with a standard cell and the test cell at E and X, respectively. Insert the galvanometer at the place indicated. Put the battery switch on E and the galvanometer switch on the 100,000-ohm point.

Note the temperature of the standard cell and compute the correction, if any, setting the dial switches V, V_1, V_2, V_3, and V_4 to read the corrected value. Adjust the control resistance until the galvanometer deflection is small when the key is tapped.

Reduce the high resistance to 10,000 ohms, or, if necessary, to zero, in which case the switch rests on the left-hand point. Then continue the adjustment of the control resistance until there is no deflection when the key is pressed.

When this adjustment has been made, the dials read the standard cell voltage directly, because the working current through PP' has been adjusted to such a value that the potential drop along one of the 1000-ohm coils is 0.1 volt, along one of the 100-ohm coils is 0.01 volt, etc., each of the other dials reading the next figure in turn.

(2) Set the battery switch at X and adjust the dial switches until the galvanometer shows no deflection, using the high resistance as before until near a balance. The value of the

control resistance, galvanometer, standard cell, one or more standard resistance coils, rheostat, and ammeter to be calibrated.

PROCEDURE. (1) The arrangement of the apparatus is as shown in Fig. 62. The battery *B* is sending current through an adjustable rheostat *R*, an ammeter *Am*, and a standard resistance *r*, which must have a sufficient current-carrying capacity so that overheating will not occur. The terminals of *r* will be connected to the test circuit of the potentiometer. For some value of the current as read on the ammeter, the potential difference at the terminals of *r* will be measured in terms of a standard cell. This value of the potential difference divided by the known value of *r* will give the value of the current flowing. If this is not in agreement with the ammeter reading, the error of the instrument is apparent.

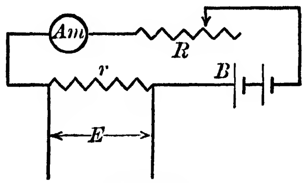

FIG. 62.

(2) Investigate in this way the scale of the ammeter at four or five points.

Plot a correction curve, that is, a curve showing the relation between the observed ammeter readings and the instrumental corrections.

In general, the current through a ten-ohm standard coil should not exceed 1/10 ampere; through a 1-ohm coil, 1 ampere; and through a 1/10-ohm coil, 5 amperes. These values are reasonable if we assume that the coil is open to the air. When oil baths are used, the current capacity is much higher.

Ammeters of the moving-coil type are subject to various errors, chiefly due to transportation or accident, temperature effects, or local magnetic fields. Hence, they require frequent calibration. For this purpose, the potentiometer method is well suited.

99. The Volt Box. When a constant current is flowing through a resistance, the potential drop between any two points is directly proportional to the resistance included between these points. If a potential difference is applied at AA' (Fig. 63), the fraction of it which exists across AB is one tenth as great, provided that the resistance between A and B is one tenth of R. Accordingly, any desired fraction of the impressed voltage may be secured by adjusting the contact point B. This exact ratio only holds when no current is drawn from the derived circuit.

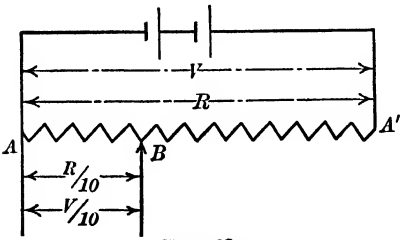

Fig. 63.

Any good resistance box provided with sockets and traveling plugs can be used as a volt box, but it is frequently convenient to have special designs for special purposes. Two such special volt boxes are shown in the accompanying illustrations. In Fig. 64, the resistances of a, b, and c are respectively 200, 1800, and 18,000 ohms, the total resistance being 20,000 ohms. With the switch on 1, the voltage at P is one tenth of that impressed at V, while with the switch on 2, the voltage at P is one hundredth of that at V. This is the arrangement of the volt box used with the type K potentiometer.

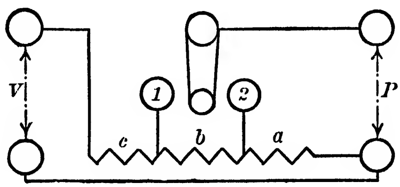

Fig. 64.

A slightly different arrangement is shown in Fig. 65. The resistances of a, b, c, d, and e are respectively 0, 100, 900, 9000, and 90,000 ohms. Hence, with K connected at p, the derived voltage at P is one tenth of that impressed at V. The contact K may be moved to q or r, in which case the voltage at P is .01 or .001 of that impressed at V. This is the cir-

cuit for the volt box used with the Wolff potentiometer. It differs from the other type in that the switch controls the position of the impressed voltage terminals instead of the derived voltage terminals.

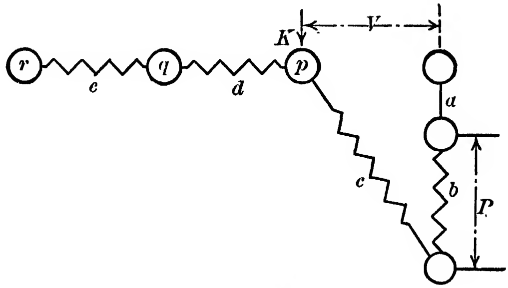

FIG. 65.

QUESTION. Suppose a voltmeter is placed across P (Fig. 65), in order to measure the derived voltage. Will the volt-box ratios yield strictly accurate results? Are the ratios strictly accurate when used with a compensation scheme, as in the potentiometer?

100. Laboratory Exercise XXIV. *To measure a high voltage with the potentiometer, and to calibrate a voltmeter.*

APPARATUS. Potentiometer, constant working battery and control resistance, galvanometer, standard cell, volt box, voltmeter, and suitable source of E. M. F.

PROCEDURE. (1) The voltage to be measured is impressed across the terminals V, Fig. 64 or Fig. 65, and the terminals P are connected to the potentiometer test circuit. The potentiometer reading multiplied by the factor of the volt box will give the desired voltage.

(2) Take readings for three or more points on the voltmeter scale, and plot a curve showing the relation between true volts and scale readings. This is called a calibration curve. When great precision is desired, it is better to plot the errors of the scale as ordinates against observed volts.

101. The Comparison of Resistances with the Potentiometer. The resistance to be measured is connected in series with a standard resistance, an unvarying current is passed through both of them, and potential wires are taken from the terminals of each in succession to the test circuit of the potentiometer. For a constant working battery, the potentiometer readings will be respectively proportional to the resistances, whether or not the instrument has been adjusted to read volts directly. For precise comparisons the standard resistance and the resistance to be measured must be immersed in oil baths, and the temperature must be carefully controlled and read.

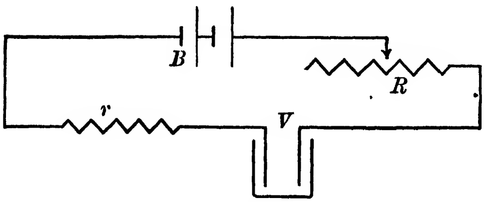

FIG. 66.

For measuring high resistances or those of medium value, this method offers no advantage over the Wheatstone bridge. For small resistances, however, the advantage is great, since it is a zero method and contact resistances are avoided.

The accuracy of a standard resistance may be checked by using a circuit arranged as in Fig. 66. A constant battery *B* sends current through the resistance to be tested *r*, and through a silver voltameter *V* (§ 113) in series with it. Potential wires are taken from the terminals of *r* to the test circuit of the potentiometer, and the strength of the current is found from the mass of silver deposited on the cathode. The resistance of *r* is then calculated from Ohm's law.

CHAPTER IV

ELECTRIC CURRENTS

102. Current Strength. The phenomenon of current is the phenomenon of the flow of electric charge. ***Current strength*** is defined as the *time rate of flow of charge* along a conductor. If the current is constant, the charge which passes in time t seconds is given by

(1) $$Q = it,$$

whence

(2) $$i = \frac{Q}{t}.$$

However, when it is desired to examine in a general way all possible phases of a changing state of flow, it is necessary to introduce instantaneous values. If dQ and dt represent small increments of charge and time, respectively, the instantaneous value of the current strength is given by the formula[1]

(3) $$i = \frac{dQ}{dt}.$$

In any event, it is by means of the current that energy is transferred from the generator through the circuit, and liberated in one form or another, depending on the devices and equipment used. Hence, the measurement of current is a fundamental one in electric work. Since it is not easy to measure directly the simultaneous values of charge and time

[1] It is here understood that the current is the same at the same time everywhere throughout the circuit; but in a large class of problems dealing with variable currents, this is not the case. For example, in circuits containing capacity, such as transmission lines, account must be taken of the rate of variation of the current strength with distance along the conductor.

in order to find their ratio, other methods are sought, in which current strength is quantitatively associated with other phenomena which can be measured more readily. There are four such phenomena which always accompany the flow of current through a circuit, upon each of which methods of measurement are based:

(*a*) The ***fall of potential*** through a constant resistance of known value, included in the circuit.

(*b*) The ***force action of the magnetic field*** surrounding the current, on other magnetic fields.

(*c*) The ***heating effect***, which appears when the terminal device transforms the energy of the current into heat.

(*d*) The ***electrolytic effect***, which occurs when the current causes a deposit of ions on the cathode of an electrolytic cell.

103. Fall of Potential. Whenever a current flows through a conductor of constant and known resistance, there is a definite value of the potential difference at its terminals, which may be measured with a voltmeter. The value of the current is readily found by Ohm's law, and is

$$i = \frac{E}{R}. \tag{4}$$

This is one of the simplest methods of measuring current. The scale of the voltmeter is really showing a deflection which is proportional to the current. An ammeter, if of the shunt type, is really a sensitive voltmeter which gives a deflection proportional to the fall of potential through a standard resistance.

In many circuits the product of current strength and resistance iR is of great importance. For convenience it is called the ***potential drop***, or the ***iR drop*** along the circuit.

104. The Magnetic Effect of the Current. The electric current through a conductor is always accompanied by a mag-

netic field in the region surrounding the conductor. A magnetic pole placed in this field will be acted on by a force. By means of the action of this force on a magnetic needle the presence of a current in a conductor can be ascertained. Since the force is proportional to the current strength, it also affords a direct measure of the current strength. This magnetic force action does not depend upon the kind of conductor, but is present alike with metallic and with electrolytic conductors.

The deflection of the magnetic needle when brought near to a conductor through which current is flowing was first observed by Oersted in 1819. This observation was quickly followed by the discovery that the force action is at right angles to the conductor, and in a plane perpendicular to the axis of the conductor.

Laplace assumed that the magnetic field strength dF, due to a current i, through a short element of length ds, is proportional directly to that length and to the strength of the current, and inversely to the square of the perpendicular distance from ds. He expressed this relation in the formula

$$dF = k\frac{i\,ds}{l^2}. \tag{5}$$

The unit of current strength is so chosen that $k = 1$.

The relation expressed in equation (5) cannot be verified directly for short elements of the conductor, because steady currents can only be thought of as flowing in complete circuits. If the expression for dF is integrated with proper regard for the geometric form and the extent of the conducting path, the derived results are fully confirmed by experimental tests. Such tests were first performed by Biot, Savart, and Ampere. Indeed, the magnetic field due to a long, straight wire was established by Biot and Savart experimentally before the general law of equation (5) was formulated.

A conductor carrying a constant current is to be thought of

as surrounded by a magnetic field, the lines of force being represented by concentric circles which lie in planes at right angles to the axis of the conductor. The direction of these lines is clockwise as one looks along the conductor in the direction in which the current flows, and the strength of the field, or the force on a unit pole, is determined by integrating the equation (5).

105. The Magnetic Field due to a Long Straight Wire. The wire WW', Fig. 67, is assumed to be carrying a current i, in a direction vertically downward, the magnetic force at P being toward the reader. Let the perpendicular distance from P to the wire be denoted by r, and let l denote the distance from P to the short element of the wire ds. If this element were at right angles to l, the force dF_P at P would be given by equation (5); but, since the effective length of the element is $ds \cos\theta$, we have instead

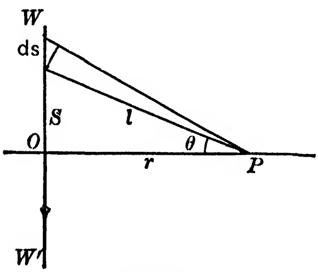

FIG. 67.

(6)
$$dF_P = \frac{i\,ds}{l^2}\cos\theta.$$

The total force at P, due to that part of the wire above 0, is given by integrating both sides of (6) between the limits zero and infinity. The result must be doubled in order to include the effect of that part of the wire below 0. Remembering that $\cos\theta = r/l$, and that i is a constant, we may write

(7)
$$F_P = 2\,i\int_0^\infty \frac{r}{l^3}\,ds.$$

Substituting for l its value $(r^2+s^2)^{\frac{1}{2}}$, we find

$$F_P = 2\,ir\int_0^\infty \frac{ds}{(r^2+s^2)^{\frac{3}{2}}},$$

whence[1]

(8) $$F_P = \frac{2\,i}{r}.$$

The work w done in moving a unit magnetic pole once around a conductor carrying a current i is readily determined by multiplying the force as given in equation (8) by the length of path. Since the length of the circular path is $2\,\pi r$, we have

(9) $$W = \frac{2\,i}{r} \cdot 2\,\pi r = 4\,\pi i.$$

This result will be expressed in ergs when i and r are expressed in C. G. S. absolute units.

106. The Magnetic Field Strength at a Point in the Axis of a Circular Current. Let the points A and B, Fig. 68,

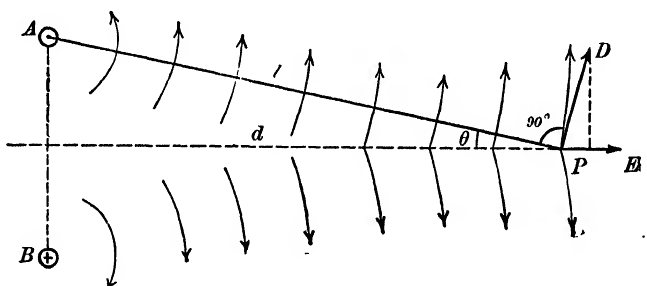

Fig. 68.

represent the intersections with the paper of a circular loop of wire, whose plane is normal to the plane of the paper. If the current is flowing in at B and out at A, the lines of force will be represented by the concentric arcs. At P the force will be in a direction PD, tangent to the arc, and at right angles to AP. From equation (6), § 105, the force at P due

[1] The integration is as follows:

$$\int_0^\infty \frac{ds}{(r^2+s^2)^{\frac{3}{2}}} = \left[\frac{s}{r^2(r^2+s^2)^{\frac{1}{2}}}\right]_0^\infty = \left[\frac{1}{r^2\left(\frac{r^2}{s^2}+1\right)^{\frac{1}{2}}}\right]_{s=\infty} = \frac{1}{r^2}.$$

to a short element ds, at right angles to the line AP, is given by the equation

(10) $$dF = \frac{i\,ds}{l}.$$

The component dF_a of this force along the axis is given by the equation

$$dF_a = \frac{i\,ds}{l^2}\sin\theta = \frac{i\,ds}{l^2}\frac{r}{l} = \frac{irds}{l^3},$$

or

(11) $$dF_a = \frac{irds}{(r^2+d^2)^{\frac{3}{2}}}.$$

This being the force at P in the direction PE, due to an element of length ds, the total force at P in the same direction due to the entire loop is given by integrating both sides of (11) around the circle; whence we have[1]

(12) $$F_a = \frac{2\,\pi i\,r^2}{(r^2+d^2)^{\frac{3}{2}}}.$$

If the loop is made up of n turns instead of one, the total force F_p at P in the direction of the axis is

(13) $$F_p = \frac{2\,\pi n\,ir^2}{(r^2+d^2)^{\frac{3}{2}}}.$$

The component ED will be annulled by an equal component due to an element ds on the opposite side of the loop, and these components at right angles to the axis annul one another for every position about the axis. The only effective force is that along the axis as given by equations (12) and (13).

If the point P is moved back to the center of the loop, d becomes zero; hence, the force F_c at the center is

(14) $$F_c = \frac{2\,\pi ni}{r}.$$

[1] **This integration, since i, r, and d are all constant, is as follows:**

$$F_a = \frac{ir}{(r^2+d^2)^{\frac{3}{2}}}\int_0^{2\pi r} ds = \frac{ir}{(r^2+d^2)^{\frac{3}{2}}}\,2\,\pi r = \frac{2\,\pi i r^2}{(r^2+d^2)^{\frac{3}{2}}}.$$

107. The Single-coil Tangent Galvanometer. Let the direction of the magnetic meridian be represented by the line *NS*, Fig. 69. The points *A* and *B* represent the intersections with the plane of the paper of a circular coil of wire of *n* turns, whose plane lies in the magnetic meridian. A magnetic needle *ns*, of length *l*, is suspended by a fiber attached at *o*, which is at right angles to the plane of the paper. The horizontal component of the earth's field is denoted by *H*, and the magnetic force due to the current is denoted by *F*.

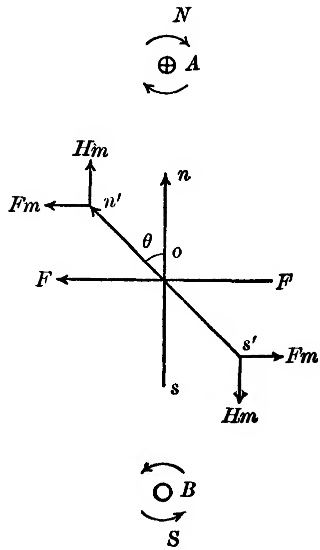

FIG. 69.

The magnetic strength of the pole of the needle is denoted by *m* in C. G. S. units. There are two equal, oppositely directed forces *Fm* which act on the two ends of the needle, and tend to turn it into a position parallel to the direction of the field. Moreover, there are two forces *Hm* acting on the poles, tending to restore the needle to its position of equilibrium.[1] The needle is, therefore, under the influence of two couples, the *deflecting* couple and the *restoring* couple. When the moments of these two couples are equal, the needle will take up some definite position, making an angle θ with its original position in the magnetic meridian.

Equating the two moments, we have

$$Fml\cos\theta = Hml\sin\theta, \tag{15}$$

[1] The suspension fiber also supplies a restoring torque, but this is assumed small enough to be neglected in all cases except where the highest precision is required.

whence, dividing by $ml \cos \theta$, we find

$$F = H \tan \theta. \tag{16}$$

Substituting in (16) the value of F given by equation (14), we have

$$\frac{2 \pi n i}{r} = H \tan \theta, \tag{17}$$

whence

$$i = \frac{Hr}{2 \pi n} \tan \theta, \tag{18}$$

which may be written in the form

$$i = K \tan \theta, \tag{19}$$

where $K = Hr/(2 \pi n)$.

This equation gives the value of the current strength in terms of a constant and the tangent of the angle of deflection. For this reason, this form of galvanometer is known as the ***tangent galvanometer***. The current strength is expressed in absolute units when H and r are expressed in absolute units. If i is to be given in amperes (see § 5), the equation becomes

$$i = 10 K \tan \theta. \tag{20}$$

It will be seen that for any given instrument r and n will be constant, while for any assigned location H may be considered constant during the time of using the instrument. In any single-coil tangent galvanometer the length of the needle must be small as compared to the diameter of the coil; moreover, it must be carefully centered, for otherwise, as it is deflected, the poles pass into regions in which the field strength is not constant.

The tangent galvanometer affords a ready means of comparing current strengths, or of measuring them in absolute units. At the present time, with high-grade, direct-reading instruments, potentiometers, and zero methods, it is difficult to

appreciate the importance of the instrument to the electrical laboratory of an earlier period. Its present usefulness lies in the illustration of fundamental principles, rather than in practical measurements.

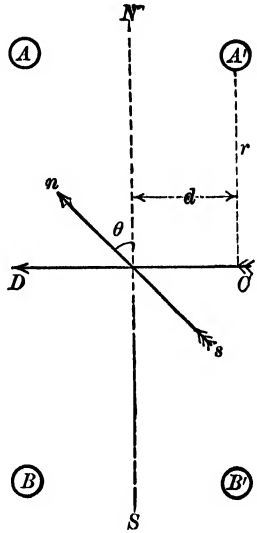

FIG. 70.

108. The Double-coil Tangent Galvanometer. In the case of the single-coil tangent galvanometer it was assumed that the needle was short, and that it did not swing out of a uniform field at any time. By using two coils with their planes parallel, a much longer needle may be used, the field is more nearly uniform, and a greater precision may be attained. The general relations of the two coils *AB* and *A′B′* to the magnetic meridians *NS*, and to the needle *ns*, are shown in Fig. 70. The direction of the resultant field due to the current is *CD*. The plane of each coil is at a distance *d* from the needle. From equation (13) the force at the needle due to one coil is

$$F = \frac{2\pi n i r^2}{(r^2 + d^2)^{\frac{3}{2}}}, \tag{21}$$

where the symbols have the same meanings as in § 106. Since there are two coils in this case, the total force will be

$$F = \frac{4\pi n i r^2}{(r^2 + d^2)^{\frac{3}{2}}}. \tag{22}$$

An arrangement frequently used is that for which $d = r/2$. In that case, the equation (22) becomes

$$F = \frac{4\pi n i r^2}{\left(r^2 + \frac{r^2}{4}\right)^{\frac{3}{2}}} = \frac{32\pi n i}{5\sqrt{5}\, r}. \tag{23}$$

Since $F = H \tan\theta$, we may write

$$\frac{32\,\pi n i}{5\sqrt{5}\,r} = H \tan\theta,$$

whence

(24) $$i = \frac{5\sqrt{5}\,Hr}{32\,\pi n} \tan\theta,$$

or

(25) $$i = K \tan\theta,$$

where

$$K = \frac{5\sqrt{5}\,Hr}{32\,\pi n}.$$

With a large and accurately constructed instrument, r and n are readily determined. It is with instruments based upon extensions of these principles that absolute determinations of current strength are made.

109. The Magnetic Field Strength at the Center of a Long Solenoid. Figure 71 represents a section through a long solenoid AA'/BB', of length L cm., radius r cm., wound with wire

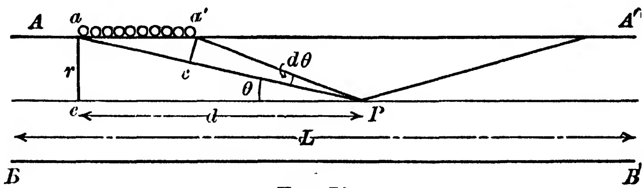

Fig. 71.

of diameter x cm., and with n turns for each centimeter of length. It is desired to find the field strength at a point P in the center of the solenoid. The effect of the n' turns lying in the element of length aa', which is, for the present, considered very short as compared with L, is given by equation (13) in the form

(26) $$F_p = \frac{2\,\pi n' r^2 i}{(r^2 + d^2)^{\frac{3}{2}}}.$$

Since n' is the number of wire turns in a length aa', we have

$$aa' = n'x,$$

provided the wire is closely wound, or

$$n' = \frac{aa'}{x}. \tag{27}$$

Drawing $a'c$ normal to aP, and writing the proportions between corresponding sides of the similar right triangles $aa'c$ and aPe, we have

$$\frac{aa'}{a'c} = \frac{aP}{r}. \tag{28}$$

We know by trigonometry that

$$a'c = a'P \sin d\theta.$$

When $d\theta$ is small, $a'P$ may be set equal to aP, and $\sin d\theta$ may be set equal to $d\theta$. Making these substitutions, (28) becomes

$$aa' = \frac{\overline{aP}^2 d\theta}{r}. \tag{29}$$

Hence, the value of n' in (27) becomes

$$n' = \frac{\overline{aP}^2 d\theta}{rx}. \tag{30}$$

Putting this value of n' in (26), we find

$$F_P = \frac{2\pi r^2 i \overline{aP}^2 d\theta}{(r^2 + d^2)^{\frac{3}{2}} rx}. \tag{31}$$

Moreover, it is evident that we have

$$(r^2 + d^2) = \overline{aP}^2, \quad \frac{r}{aP} = \sin\theta, \quad \frac{1}{x} = n,$$

where n is the number of turns per centimeter on the solenoid. Whence the force at P in terms of the number of turns per centimeter is given by the equation

$$F_P = 2\pi n i \sin\theta \, d\theta. \tag{32}$$

To find the value of the field at P due to all the turns throughout the entire length of the solenoid, it is necessary simply to integrate this expression with respect to θ between the limits 0 and π. Setting H equal to this value of the magnetic field strength, we have

$$H = 2\,\pi n i \int_0^{\pi} \sin\theta\, d\theta$$

or[1]

$$H = 4\,\pi n i. \tag{33}$$

Remembering that n is the total number of turns divided by the length of the solenoid, equation (33) may be written in the form

$$H = 4\,\pi \frac{N}{L}\, i. \tag{34}$$

It will be seen that the limits 0 and π for the angle θ correspond to the assumed condition that L is very large compared to r.

The value of H may be expressed either in dynes per unit pole or in lines per square centimeter. In case the current is measured in amperes, the equation (33) becomes

$$H = \frac{4}{10}\,\pi n i. \tag{35}$$

Since this is the magnetic field strength or flux density at the center, the total flux ϕ through the solenoid is given by the equation

$$\phi = \frac{4}{10}\,\pi n i A, \tag{36}$$

where A is the area of cross-section of the coils. It is here assumed that the magnetic field is uniform over the entire area of the solenoid. When, for any reason, a magnetic field

[1] **This integration is as follows:**

$$\int_0^{\pi} \sin\theta\, d\theta = \Big[-\cos\theta\Big]_0^{\pi} = 1 + 1 = 2.$$

of known strength is required, it is most frequently realized by means of the long solenoid carrying a known current.

The magnetic field is uniform for a certain region near the center of the solenoid, but toward the ends it is no longer parallel to the axis, and its value is not given by equation (33). If the long solenoid is bent into a circular form with the ends joined, thus forming a toroidal coil, the end effects and the external field vanish, and the lines of force are circles with their centers lying on the axis of the tore. If the wire turns are close together, the windings may be considered as forming approximately a uniform current sheet, within which the value of the uniform magnetic field is given by equations (33)-(35).

110. The Electrodynamometer. The ***electrodynamometer*** is an instrument of great utility for the measurement of current strength, voltage, or power, in either direct-current or alternating-current circuits. For the present it will be treated as a current-measuring device.

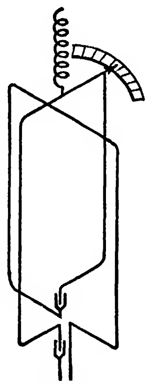

FIG. 72.

It consists essentially of two rectangular coils, connected in series, placed with their planes vertical and at right angles to one another, as shown in Fig. 72. One coil is fixed in position while the other is hung from a torsion head by a light fiber of silk, so that it is free to rotate about a vertical axis. A light spiral spring surrounds the suspending fiber, and is attached to the movable coil and to the torsion head. This spring furnishes the control for the suspended system. Electrical connection with the movable coil is provided by means of mercury cups, into which its terminals dip, and which are placed directly in a vertical line below the point of suspension. Some clamping arrangement is usually provided to prevent damage to the suspended system during transportation.

When current is passed through the two coils connected in series, their magnetic fields react to produce a torque, which rotates the movable coil about its vertical axis. This torque may be opposed by twisting the torsion head and the attached spring through a certain angle, until the torsion of the spring just compensates the torque due to the reacting fields. This angle is read by means of a pointer attached to the head, which plays over a graduated scale on the top of the frame of the instrument.

The torque due to the reacting fields is proportional to the current strength in each coil, and hence, to the square of the current strength. The compensating torque of the spring is directly proportional to the angle through which the torsion head is rotated in order to keep the suspended coil in its position of equilibrium. It will be seen that the direction of the deflecting torque is not changed, even though the current is reversed through the coils.

From these considerations, it will be seen that

$$i^2 = k'\phi,$$

where ϕ is the angle through which the torsion head is rotated in order to maintain the movable coil in its zero position, or position of equilibrium, and k' is a constant which depends on the stiffness of the spring, the dimensions of the coils, and the number of wire turns. The preceding equation may be written in the form

$$i = k\sqrt{\phi}, \tag{37}$$

where $k = \sqrt{k'}$. If the value of k is known, the value of i may be readily computed.

Thus, it appears that the square root of the observed angle through which the spring is rotated, multiplied by a constant, gives the value of the current strength. This constant may be determined by passing a current of known strength through

the instrument and observing the angle through which the torsion head must be rotated in order to keep the moving coil in its initial position of equilibrium. The current may be measured by any desired method of suitable accuracy. Since a precise determination is necessary for the calibration of the instrument, it is customary to use a silver or copper voltameter in series with it, computing the current strength from the gain in mass of the cathode in a measured interval of time. If currents and angles of twist are plotted on squared paper, the curve will be parabolic.

In measuring direct currents the reaction between the field of the movable coil and the earth's field may be considerable. To avoid error from this cause, the instrument is so placed that the plane of the movable coil is at right angles to the magnetic meridian.

The electrodynamometer may be calibrated and used also for the measurement of voltage and power in either direct or alternating-current circuits. It is sometimes equipped with a mirror and scale, and the deflection angles are read directly instead of being annulled by a torsion spring. When so used, it is called a ***reflecting electrodynamometer***.

As a ***voltmeter*** this instrument is made with many turns of fine wire in the coils, which are in series, and usually with a high non-inductive resistance also in series. This acts as a multiplier and enables the range of the instrument to be increased.

As a ***wattmeter***, the fixed coil (current coil) consists of a few turns of large wire, and its terminals are connected in series with the circuit in which the power consumption is to be measured. The movable coil (pressure coil) is made of many turns of fine wire in order to secure a high resistance, and is connected in parallel with the power circuit. A non-inductive resistance is frequently connected in series with the pressure coil.

111. The Heating Effect of the Current. The potential difference between two points is measured in terms of the work done in conveying the unit charge between these points. This relation is given by the equation

$$V = \frac{W}{Q},$$

or

$$W = VQ, \tag{38}$$

where V is the potential difference and W is the work done in conveying the charge Q. By setting $Q = it$, equation (38) becomes

$$W = Vit, \tag{39}$$

or, if the value of V is substituted from Ohm's law,

$$W = Vit = i^2Rt. \tag{40}$$

The thermal equivalent of the work is given by

$$W = JH, \tag{41}$$

where J is the mechanical equivalent of heat, or the number of work units equivalent to one heat unit. Combining (40) and (41), we find,

$$W = JH = Vit = i^2Rt. \tag{42}$$

The work will be given in ergs when the electric units are all taken in the absolute C. G. S system, and in joules when the volt, ampere, ohm, and second are used. With the absolute units J has the approximate value

$$J = 4.18 \times 10^7 \text{ ergs per calorie};$$

while, with the practical units,

$$J = 4.18 \text{ joules per calorie}.$$

If both sides of equation (42) are divided by the time, the power relations of the electric quantities are obtained in the form

$$P = \frac{W}{t} = \frac{JH}{t} = i^2R, \tag{43}$$

which expresses P in watts when practical units are used consistently.

The equations (42) and (43) might be used to find any one of the electric quantities involved, all the others being known. There are other more accurate methods available for measuring current and resistance, however, and the equations are more frequently used to determine the mechanical equivalent of heat, J.

The method here described is that of the flow calorimeter. Figure 73 shows the arrangement of the parts. A spiral coil

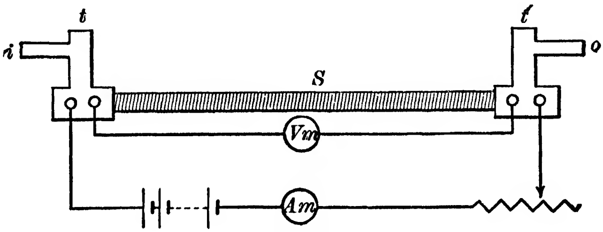

FIG. 73.

of wire S within a glass tube carries a current which is measured by the ammeter Am. A voltmeter Vm placed across the terminals of the coil gives the potential difference between its ends. From these readings the rate of energy supply to the coil is found. This energy heats the wire. If a stream of water is made to flow continuously through the tube, there will be a constant difference in the temperature of inflow and outflow, provided the rate of energy supply by the current is just equal to the rate of energy withdrawn by the water stream. From equation (42) we may write

$$Vi = \frac{JH}{t},$$

or

$$J = \frac{Vit}{H}.$$

The heat removed by the water stream is given by the product of the mass of water and the difference between the initial and final temperatures, whence we have

$$J = \frac{Vit}{m(t_1 - t_2)}. \tag{44}$$

In the continuous-flow calorimeter, heat is carried away at a uniform rate, being absorbed by the water which flows steadily through the tube. If the temperature of the water supply is constant, and if the flow is maintained at a uniform rate, the thermal condition will become fixed, as will the electric condition. That is, the temperatures of inflow and outflow will become constant, and the resistance of the wire will not change. Under these conditions there are no corrections to be made for the thermal capacity of the apparatus. By keeping the flow of water steady, and the mean temperature of inflow and outflow within five degrees of the room temperature, corrections for radiation and conduction become very small and may be neglected.

112. Laboratory Exercise XXV. *To determine the mechanical equivalent of heat with the flow calorimeter.*

Apparatus. Flow calorimeter with accessories, ammeter, voltmeter, control rheostat, two thermometers, watch, and source of steady current.

Procedure. (1) Arrange the apparatus as shown in Fig. 73. Adjust the flow of water until it is steady, with a difference in temperature between inflow and outflow of from three to five degrees. Let the water flow for a few minutes before taking readings so that the temperatures may become constant. The thermometer readings should be estimated to one hundredth of a degree, and the two instruments should be compared before beginning the experiment.

(2) See that no air bubbles are lodged on the wire turns of the coil. After the current has flowed a few minutes, set a

weighed vessel in position to receive the outflowing water, and note the exact time at which the flow into the vessel begins. Take simultaneous readings of the ammeter, voltmeter, and both thermometers at half-minute intervals, recording these values in a table previously ruled. When one or two liters of water has passed, remove the vessel, note the exact time, and record it. Weigh the water collected, and compute the number of calories of heat absorbed by the water.

(3) Repeat for five sets of observations, using different rates of flow and different values of the current. Let the final result be the mean of the five thus found. The data may be arranged as shown in the following table:

Time	i	V	Temp. In	Temp. Out	Mass of Water	J

113. The Electrolytic Effect of the Current. If a wire carrying a current of electricity is cut and its ends are submerged in a jar containing a water solution of an inorganic acid or salt, current will still continue to flow, and there will be a deposit of ions on the cathode, that is, the terminal from which current leaves the solution. The mass M of this deposit is, from Faraday's laws, proportional to the amount of charge Q passing. This may be expressed by the formula

$$M = zQ,$$

where z is the mass deposited by the unit of charge. We have also

$$Q = it;$$

hence, if the current strength is constant throughout the time t, its value may be found from the relation

$$i = \frac{M}{zt}. \tag{45}$$

The value of z, which is called the ***electrochemical equivalent,*** is characteristic of the substance deposited. It has been so accurately determined for silver and copper that voltameters containing solutions of these metals are used for precise measurements of current strength. The method is valuable for the calibration of current-measuring instruments, but it is not commonly used outside of the precision laboratory. It is slow and requires considerable equipment, and it gives results that are more accurate than are required in practice.

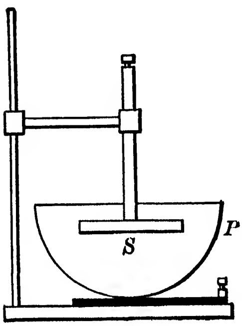

FIG. 74.

The ***silver voltameter*** is used for the most precise determinations, and its form is usually that shown in Fig. 74. The cathode S is a plate of pure silver so mounted that it can be immersed in a solution of silver nitrate contained in a platinum bowl P. The ***international ampere*** is defined in terms of the silver voltameter; the value of z for silver being ***0.0011180 gram per coulomb.*** (See § 6.)

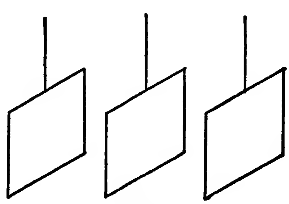
FIG. 75.

The ***copper voltameter*** is easier to use than the silver voltameter, and is nearly as precise in its results. It usually takes the form shown in Fig. 75. The middle plate is the cathode and the two outside plates are joined and constitute the anode. A twenty-five per cent solution of copper sulphate with the addition of one or two per cent of sulphuric acid is used. The electrochemical equivalent of copper is ***0.0003294 gram per***

coulomb. This value varies slightly with the current density and with the temperature and concentration of the solution.

114. Laboratory Exercise XXVI. *To determine the constant of an electrodynamometer with the copper voltameter.*

APPARATUS. Electrodynamometer, copper voltameter in duplicate with accessories, rheostat, and reversing switch.

PROCEDURE. (1) Set up the instrument so that the coils are at right angles to one another, and with the plane of the movable coil at right angles to the magnetic meridian. Adjust the leveling screws until the coil swings freely, and set the torsion head against its stop on zero. Read accurately the

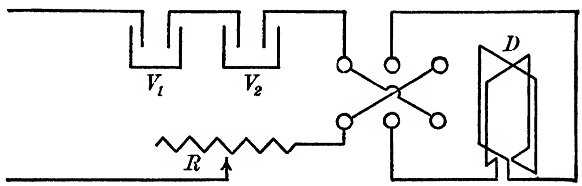

FIG. 76.

position of the coil pointer, and take this as the zero or equilibrium position. Stops are provided to limit the deflection.

(2) Connect the circuit as in Fig. 76, using a twisted pair of wires to the electrodynamometer. Pass a current of suitable strength through the circuit and note the angle of compensation. Reverse the current and note whether the compensation angle varies. Do this for two positions, with the plane of the movable coil respectively *parallel* with and *perpendicular* to the magnetic meridian. Any readings made should be with the former position, and with reversed current.

(3) Adjust the current to a suitable value and proceed with the voltameter determination as outlined in § 113. Let the current pass for at least half an hour, reversing every two minutes, and record the compensation angles.

(4) Calculate the current strength from equation (45), and find the value of the constant in equation (37).

EXERCISES

1. A circular loop of wire of radius 60 cm. is placed with its plane vertical and in the magnetic meridian. A current of 10 amperes flows north at the top of the coil. A south magnetic pole of 200 units strength is placed on the axis of the coil, at a distance of 80 cm. from its plane. Calculate the force on this pole. Show clearly in a diagram its direction. What is the force on the pole if placed at the center of the loop? What is its direction?

2. Make a diagram which will show clearly the magnetic field reactions in the electrodynamometer. In the case of the error due to the earth's field, which one of the three, (*a*) total force, (*b*) horizontal component, (*c*) vertical component, is the effective one. Make a diagram showing clearly the directions of the reacting fields and of the resulting forces and torques. Assume approximate dimensions, and calculate the possible value of the torque due to this cause.

CHAPTER V

CAPACITY AND THE CONDENSER

Part I. Definitions and Units

115. Fundamental Ideas and Definitions. Up to this time we have considered electricity as resembling in some ways an incompressible fluid, and we have assumed that all parts of the circuit carried the same current strength at the same time. In this and the following chapter new aspects of electric circuits will be presented, in which there will be considered the storage of energy in certain parts of the circuit.

If the wire connecting the poles of a battery is cut, its ends will be at a definite difference of potential, and the charges residing on the free ends will be small. If, however, these free ends of the wires are expanded into plates with large surface areas, there will be a momentary current through the circuit, and a greater charge will accumulate on the plates. As the plate area is increased, and as the distance between the plates is made less, the charge on the plates, for the same potential difference, increases. This ability of the system of conductors to hold or store a quantity of electricity is called the ***capacity***[1] of the system. Such a system of conducting plates is called a ***condenser.*** It is a device by means of which the capacity of an isolated conductor can be very greatly increased, due to the presence near it of another charged conductor. This other conductor may be connected to the earth or, more commonly, to the opposite pole of the electric generator.

[1] In order to distinguish electrostatic capacity from current-carrying capacity, etc., the term capacitance is sometimes used.

The capacity of a condenser can be shown to be directly proportional to the area of surface of its plates, and inversely to the distance between them. In order to realize the greatest possible capacity in a small space, a great many very thin sheets of metal foil are used for the conductors, and these are separated by selected sheets of thin mica in the higher grades of condensers, or by sheets of paraffined paper in the cheaper grades.

116. Classification of Condensers. Condensers may be grouped in two classes. One class is that in which the dielectric must sustain a high potential. Such a condenser consists of a few plates widely separated, and the capacity is too small to measure by the ordinary methods. Such condensers are commonly used in high-frequency, alternating-current circuits. Their properties and the methods of making measurements with them are treated in the larger works on the theory and equipment of wireless telegraphy.

Condensers of the other class have larger capacity and are intended for use with low-voltage batteries and ordinary galvanometers. They have many layers of thin foil separated by thin dielectric, closely pressed together.

117. Units of Capacity. In Fig. 77, AB represents such a system of interleaved plates. If the key K is pressed to b, a potential difference V is applied to the terminals of the condenser, and a transient deflection of the galvanometer is observed. The zero position is quickly regained, however. This sudden throw of the galvanometer signifies a rush of electricity into the condenser. The condenser is then said to be charged. If the key K is then raised to a, thus removing the charging potential difference

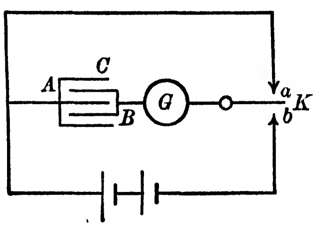

Fig. 77.

and closing the condenser circuit through the galvanometer, a transient deflection is again observed, this time in a direction opposite to the first one. The condenser is now discharged, the plates having been brought to the same potential.

The current strength [1] during charge or discharge is not constant, as will be shown in §§ 125 and 127, if a condenser is included in the system. It is necessary to take into account the total quantity of charge which passes rather than the current itself. This is given by the expression

$$Q = \int i \, dt, \tag{1}$$

where i is the ***instantaneous value*** of the current strength. The quantity of electricity stored in a perfect condenser, that is, one whose dielectric has infinite resistance and no absorption,[2] is always found to be directly proportional to the charging potential difference. This relation may be written in the form

$$Q = CV, \tag{2}$$

where C is a constant whose value is given by the ratio

$$C = \frac{Q}{V}. \tag{3}$$

This constant, which is characteristic of the particular condenser, is the measure of the ***capacity*** of the condenser. From equation (3) it is seen that the capacity is numerically equal to the charge in the condenser when unit potential difference is impressed across its terminals.

If Q and V are given in absolute C.G.S. electromagnetic units, C will be expressed in the same system. A condenser

[1] In cables and transmission lines the capacity is distributed, and the calculation of the current strength at any time, and at any point along the conductor, becomes somewhat complicated. The theory of problems of this class is given in *The Propagation of Electric Currents in Telegraph and Telephone Conductors* by FLEMING (Van Nostrand, 1911).

[2] See § 119.

will have unit capacity when unit potential difference develops in it the unit charge. In order to express capacity in practical units, Q must be given in coulombs and V in volts, whence

$$(4) \qquad \frac{[Q_{\text{C.G.S.}} \times 10]_{\text{coulombs}}}{[V_{\text{C.G.S.}} \times 10^{-8}]_{\text{volts}}} = [C_{\text{C.G.S.}} \times 10^{9}]_{\text{farads}}.$$

Any given charge of Q absolute units will be represented by a number ten times as great when expressed in coulombs; and any given number of absolute units of potential difference will be divided by 10^8 in order to give the equivalent number of volts. The number of absolute capacity units must then be multiplied by 10^9 in order to give the equivalent number of practical units. This means that the value of the absolute unit of capacity is 10^9 times as great as the practical unit which corresponds to the volt and the coulomb. This practical unit of capacity is called the ***farad***, and is the capacity of a condenser which has a potential difference of one volt at its terminals, when charged with one coulomb of electricity.

The farad itself is too large a unit to be useful, being of an order of magnitude much greater than that of the capacities commonly met in practice, hence, the millionth part of the farad, the ***microfarad***, equivalent to 10^{-15} in absolute units, is chosen as a more convenient and more practical unit. A condenser of capacity one farad would be too enormous to construct: the height of a pile of condenser plates each one meter square, which would be required for a capacity of one farad, provided that the thickness of each conducting sheet together with its mica dielectric were one millimeter, would be of the order of one hundred miles.

A submarine cable is a condenser in which the copper core constitutes one plate, with the water as the other plate, while the insulating material surrounding the core is the dielectric. Similarly, in a telephone cable, any single conductor may be regarded as one plate of a condenser, the other plate being the

adjacent wire of a pair, or the lead sheath of the cable itself. The capacity of a telephone cable should not be greater than 0.08 microfarad per mile. The earth, considered as an isolated conductor, has a capacity of about 700 microfarads. The capacity of three miles of average submarine cable is about one microfarad. A laboratory standard frequently used is one having a third of a microfarad capacity, equivalent to about one mile of cable. The capacity of a pair of number eight copper wires 1000 feet in length and twelve inches apart is about 0.0032 microfarad.

118. Standards of Capacity. Standard condensers, or capacity boxes for use in the laboratory, may be arranged with single values, commonly $\frac{1}{2}$, $\frac{1}{3}$, or 1 microfarad in each box, or they may be subdivided, with a maximum value of one or more microfarads. Subdivided condensers are so arranged in sections that different values of the capacity, from a few hundredths of a microfarad to the maximum, can be secured by the adjustment of plugs or switches. Two methods of connecting the separate sections in subdivided capacity boxes are shown in Figs. 78 and 79. The arrangement shown in Fig. 78 permits of multiple combinations only, while with that shown in Fig. 79, both series and multiple combinations are possible. In certain geometric forms, notably the sphere, cylinder, and parallel plate, it is possible to calculate the capacity from the

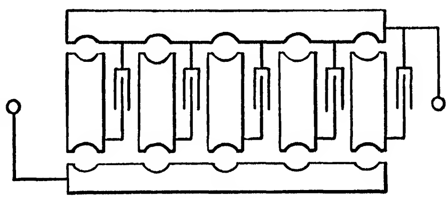

Fig. 78.

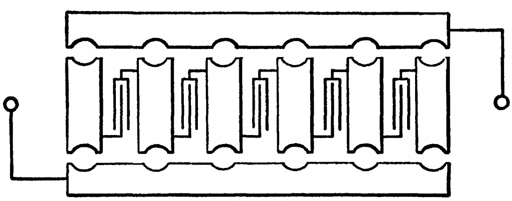

Fig. 79.

dimensions. These forms serve as reliable standards when dry air is used as the dielectric, but the capacities will be small unless inconveniently large dimensions are assumed. If dry air or vacuum constitutes the dielectric of a condenser, the value of the capacity will not be dependent upon the charging potential, nor upon the time for which it is applied. With such solid dielectrics as glass or paraffin, however, the capacity is found to depend on the mode of charging. The phenomena of leakage, absorption, and residual charge must be taken into account carefully.

119. Leakage, Absorption, and Residual Charge. When the dielectric of a condenser shows a true conductivity, it is said to possess ***leakage.*** No substance can be regarded as an absolute non-conductor, though a pair of charged plates with dry air as the dielectric will retain the charge almost indefinitely. A high-grade mica condenser will retain its charge for some hours with but slight change, while an average paraffined-paper condenser shows a marked falling off in its charge within a few seconds. This dielectric conductivity may be strictly like that in metals, or it may be electrolytic in type. The conductivity of dielectrics usually increases with rise in temperature, and with an increase in the impressed voltage. Solid substances which are not changed in chemical composition at high temperatures, such as glass or porcelain, become good conductors when raised to incandescence.

When a given potential difference applied to a condenser gives it a certain charge for a short time of application, and a greater charge for a longer time, the condenser is said to possess ***absorption.*** If such a condenser with negligible leakage is charged by a given potential difference and then is left to itself after removing the charging voltage, the potential difference across its terminals is found to diminish somewhat, at first rapidly, then slowly. Under the action of the charg-

ing potential some molecular changes probably occur in the dielectric, which require time, and this strained condition also requires time for recovery. Absorption is by some writers called *soakage*, both terms arising from the early view that the electric current was of the nature of fluid flow, and that more or less penetration into the substance of the dielectric occurred. In any event the process of charging affects the dielectric like a mechanical stress, and there are but few solid substances which recover immediately after the removal of such stress. This is shown by the intimate relation which exists between the phenomena of absorption and the elastic after-effect of the dielectric substance. With glass, absorption and elastic after-effect are both large, while with quartz they are both practically zero. An air condenser shows no absorption.

When the terminals of a charged condenser are connected by a conductor, they are brought to the same potential, and the condenser is said to be discharged. If they are again connected after the lapse of a brief time, another smaller discharge occurs, and this may be repeated several times. This so-called ***residual charge*** is closely associated with the absorption of the dielectric, and is due to the slow recovery of the dielectric from the electrostatic strain. Condensers with quartz or dry air as the dielectric do not show residual charge.

The construction and use of a condenser would be simplified if the dielectric material was free from the properties of leakage and absorption. The significance of the capacity of a condenser is not definite unless the circumstances of charging and discharging are fully specified. The precision condenser must be carefully studied in order to ascertain the influence of temperature changes.

Cables and transmission lines act as condensers. When they are subjected to high potential differences, the absorption in the dielectric may result in large residual charges. When a

high-tension circuit is opened, such condensers should always be effectively discharged by repeated or continuous grounding before they are touched, otherwise surprising discharge voltages may develop.

120. Specific Inductive Capacity. The capacity of a condenser depends not only upon the form and dimensions of the plates, but also upon the nature of the dielectric medium. Suppose the capacity of a given condenser with air as the dielectric is C_a, while the capacity of the same condenser with some other substance as the dielectric is C_0. The ***specific inductive capacity***, or the ***dielectric constant*** of the substance, is defined by the equation

$$k = \frac{C_0}{C_a}. \tag{5}$$

Strictly speaking, the reference medium for which the dielectric constant is taken as unity should be a vacuum. However, dry air differs so little from a vacuum in this respect that its dielectric constant may also be taken as unity.

Measurements of specific inductive capacity yield results which vary widely with the physical state of the substances and with the conditions of the test. Average values for a few substances are given in the following table:

Petroleum	2.0
Ebonite	2.0–3.0
Paraffin	2.3
Glass	2.0–10.0
Mica	5.0–7.0

121. Dielectric Strength. When condensers are to be used with high voltages, the property of ***dielectric strength*** is quite as important as good insulation. If the potential difference impressed exceeds a certain critical value, the dielectric will be pierced. In case the dielectric is a gas or a liquid, its continuity is restored immediately after the spark. In a solid

dielectric, however, the path of the spark is a permanent defect, and if sufficient electric energy is supplied by the gener ator, current will continue to flow along this path in the form of an electric arc.

The dielectric strength is expressed in terms of the potential difference in volts (or in kilovolts) required to pierce a given thickness of the substance. It is not a quantity that can be very definitely measured. The results vary with the character of the voltage, whether direct or alternating, and also with the distance between the plates or electrodes, the shape of the plates, and the time during which the voltage is applied. Some approximate values for average samples of common materials are:

Mica	60,000 volts per mm.
Vulcanized rubber	10,000 volts per mm.
Insulating oils	5000–10,000 volts per mm.

In any case, the presence of moisture greatly lessens the dielectric strength. Although air is an excellent insulator, its dielectric strength is lower than that of most solid or liquid substances.

122. Capacities in Series and Parallel. The capacity of a condenser in the form of two parallel plates is given by the formula

$$C = \frac{kA}{4\pi d}, \tag{6}$$

where A is the area of one plate, d is the distance between the plates, and k is the specific inductive capacity of the dielectric.[1] Figure 80 represents three con-

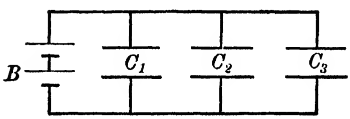

Fig. 80.

[1] The capacity will be in electrostatic units if A and d are in centimeters. If the result is to be expressed in microfarads, the factor 9×10^5 will be introduced into the denominator of equation (6).

densers connected in *parallel.* Since the capacity of a condenser is proportional to the area of its plates, it follows that the capacity C, equivalent to that of the three condensers, is given by

$$C = C_1 + C_2 + C_3. \tag{7}$$

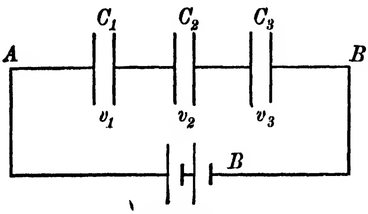

FIG. 81.

Three condensers connected in *series* are shown in Fig. 81. In this case the quantity in each condenser is the same, and is equal to the charge which enters the system from the battery B. Moreover, if a potential difference V is applied at the terminals AB, we have

$$V = v_1 + v_2 + v_3, \tag{8}$$

where v_1, v_2, and v_3 are the potential differences between the plates of the three condensers, respectively. Hence, we have

$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}. \tag{9}$$

Since $Q_1 = Q_2 = Q_3$, the equation (9) may be written in the form

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]. \tag{10}$$

But $V = Q/C$, where C is the equivalent capacity and Q is the charge in one condenser; hence,

$$\frac{Q}{C} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \tag{11}$$

and

$$\left[\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]. \tag{12}$$

PART II. CHARGE, CURRENT, AND ENERGY RELATIONS

123. The Variation of Charge with Time. Assume a constant potential difference V impressed on a circuit which contains a capacity C and non-inductive resistance R (Fig. 82). It is to be understood that the resistance R includes all the ohmic resistance throughout the entire circuit. The condenser does not instantly acquire its full charge on closing the key K, nor is the discharge an instantaneous process. It is important to investigate the time relations of charge and current during the process of charging and discharging the condenser.

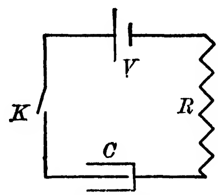

FIG. 82.

The available potential difference V, which is assumed to be constant, will be divided into two parts. One part V_1 will maintain the current strength through the ohmic resistance R, and the other part V_2 will appear at the condenser terminals and store energy in the form of charge. Neither of these values is constant, but the sum of the two is constant, and always equal to V. We may then write

$$V = V_1 + V_2. \tag{13}$$

Since V_1 and V_2 are both varying continually, their instantaneous values must be used. The potential difference which maintains current through R is always iR, and the instantaneous value of i is dQ/dt; whence we have

$$V_1 = R\frac{dQ}{dt}. \tag{14}$$

The potential difference at the condenser terminals at any instant is given by the formula

$$V_2 = \frac{Q}{C}; \tag{15}$$

whence, throughout the period of charge or discharge,

$$V = R\frac{dQ}{dt} + \frac{Q}{C}. \tag{16}$$

Assuming that the condenser is being *charged*, equation (16) may be integrated, and expressions may be found for the values of charge and current at any time t seconds after closing the key K. Separating the variables, equation (16) becomes

$$dt = RC\frac{dQ}{VC - Q}. \tag{17}$$

Integrating this expression between the limits zero and Q, and remembering that $Q = 0$ when $t = 0$, we have

$$t = RC\int\frac{dQ}{VC - Q} = -RC\log_e\frac{VC - Q}{VC}, \tag{18}$$

where e is the base of the Napierian system of logarithms. Solving this equation for Q, we have,

$$Q = VC - VCe^{-t/RC}. \tag{19}$$

From equation (19) it is seen that for $t = 0$, $Q = 0$, which was the original assumption; but if $t = \infty$, then $Q = VC$, which represents the maximum and final value of the charge in the condenser.

As an illustration of the foregoing relations, consider a condenser of 10 microfarads capacity, which has a charging potential of 1000 volts suddenly applied to its terminals, the circuit resistance being 200 ohms. The final value of the charge after an infinite time is given by equation (19),

$$Q = VC = 1000 \times 10 \times 10^{-6} = 0.01 \text{ coulomb.}$$

Choosing intervals of time of 0.001 second, and substituting these values for t in equation (19), the charge corresponding to each instant of time may be found. From these values the curve I, Fig. 83, is drawn. This curve shows the relation between coulombs and time. It is evident that *practically* the

full value of the charge is reached in a few thousandths of a second, although its full value is not reached until a much longer time has elapsed. The final value of the charge is seen to be quite independent of the value of R.

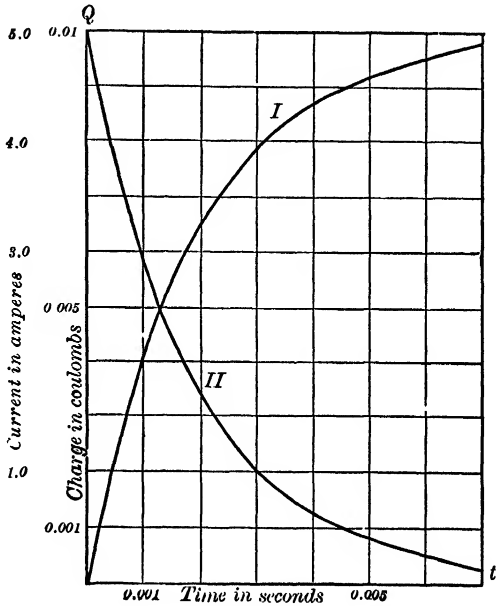

FIG. 83.

124. The Time Constant. It is frequently necessary to compare the behavior of condensers with regard to the quickness with which they acquire their charges. It is obviously impossible to use for this purpose the total time involved in the process, since this is theoretically infinite. Custom has, however, sanctioned the use of a certain time interval called the ***time constant*** of the circuit. Its value is RC seconds. The corresponding value of Q is readily derived from equation (19). If t is made equal to RC seconds, then

$$Q = VC - \frac{1}{e} VC, \tag{20}$$

and it is clear that at this time after closing the key, the charge has risen to a point which falls short of its final value by $1/e$ times that final value, that is, about 0.37 times that final value.

125. The Variation of the Charging Current with Time. The instantaneous value of the current at any time t can be

found by differentiating equation (19) with respect to the time. This gives

$$i = \frac{dQ}{dt} = \frac{d}{dt}[VC - VCe^{-t/RC}],$$

or

(21) $$i = \frac{V}{R}e^{-t/RC}.$$

At the outset, when $t = 0$, it is seen that

$$i = \frac{V}{R},$$

and the current begins to flow as if there were no capacity in the circuit. The condenser begins to show its effect, however, as time increases, and when $t = \infty$, $i = 0$. If t is made equal to RC seconds, equation (21) takes the form

(22) $$i = \frac{1}{e}\frac{V}{R},$$

which shows that the current has fallen to $1/e$ times its initial value when $t = RC$. If values of C, R, and V, as given in the numerical illustration of § 123, are substituted in equation (21), the current may be calculated for any time t. Corresponding values of current and time are plotted in curve II, Fig. 83, from which it appears that the initial current is large, but that it rapidly decreases and approaches zero as the charge approaches its final value. The value RC seconds which was substituted for t is called the ***time constant*** of the circuit, as stated in § 124.

126. The Distribution of Energy. In order to study the distribution of energy in a circuit during the process of *charging* a condenser, we shall use the fact that energy is always given by the product of the potential difference, the current, and the time. The energy supplied to the circuit for any short interval of time dt is therefore given by the equation

(23) $$dW = Vidt,$$

where V is the constant impressed potential difference and i is the current strength. This energy may be set equal to the sum of the energy dissipated as heat in the ohmic resistance, and the energy stored in the condenser. Writing this equation, we have

$$Vidt = i^2Rdt + \frac{Q}{C} i\,dt. \tag{24}$$

If the instantaneous values of i and Q are substituted in the two terms of the right-hand member of (24), these terms may be separately integrated between limits $t = 0$ and $t = \infty$, and values may be found for the energy transformed into heat, and for that stored in the condenser. Substituting the value of i from equation (21), the term i^2Rdt, which we shall denote by dW_R, takes the form

$$dW_R = \frac{V^2}{R} e^{-2t/RC} dt$$

and

$$W_R = \int_0^\infty \frac{V^2}{R} e^{-2t/RC} dt = -\frac{V^2}{R}\frac{RC}{2}\Big[e^{-2t/RC}\Big]_0^\infty = \tfrac{1}{2}CV^2. \tag{25}$$

From this equation it is evident that the total energy dissipated as heat in the ohmic resistance of the circuit is not dependent upon R, but only upon V and C.

In order to find the energy stored in the condenser, we may substitute the instantaneous values of Q and i from equations (19) and (21), in the last term of (24), and integrate between the same limits as before. The term $Qidt/C$, which we shall denote by dW_C, is, therefore, of the form

$$dW_C = \frac{V^2}{R}\Big[e^{-t/RC}(1 - e^{-t/RC})\Big]dt,$$

whence, we find

$$W_C = \frac{V^2}{R}\int_0^\infty \Big[e^{-t/RC}(1 - e^{-t/RC})\Big]dt = \tfrac{1}{2}\,CV^2. \tag{26}$$

This is seen to be exactly the same result as that of equation (25). Hence it appears that one half of the total energy given to the circuit is lost as heat in the resistance, and one half is stored in the condenser.

The charged condenser is analogous to a stressed spring, in which energy is stored while under stress; an equivalent amount of energy is returned when the constraint is released.

127. The Variation of Charge and Current during Discharge. Expressions similar to those of the preceding articles may be derived for the instantaneous values of charge and current during the *discharge* of a condenser. Returning to equation (16), and setting $V = 0$, which means that the impressed voltage is cut off and the circuit left to itself, we have

$$0 = R\frac{dQ}{dt} + \frac{Q}{C}. \tag{27}$$

Separating the variables, we find

$$dt = -\frac{RC\,dQ}{Q}. \tag{28}$$

Integrating this between the limits Q_0 and Q, which represent respectively the initial and final values of the charge, we have

$$t = -RC\int_{Q_0}^{Q}\frac{dQ}{Q} = -RC\log_e\frac{Q}{Q_0},$$

or

$$-\frac{t}{RC} = \log_e\frac{Q}{Q_0}. \tag{29}$$

Equation (29) may be put in the exponential form,

$$Q = Q_0\,e^{-t/RC}. \tag{30}$$

If the initial value of the charge is given by the formula

$$Q_0 = VC,$$

equation (30) becomes

$$Q = VC\,e^{-t/RC}. \tag{31}$$

This gives the value of the charge in the condenser at any time t seconds after discharge begins.

To find the value of the current at any instant during the discharge, equation (31) is differentiated with respect to t, which gives

$$(32) \qquad i = \frac{dQ}{dt} = -\frac{V}{R} e^{-t/RC}.$$

This is seen to be the same expression as that for the charging current given in equation (21), except that the sign is negative, which indicates a reversed direction.

The discussion in the preceding articles shows that a condenser acquires its charge according to an exponential function of the time. The charge reaches its final and maximum value theoretically only after an infinite period, although in most condensers in actual use the charge is practically complete in a fraction of a second. The final value of the charge is quite independent of the resistance of the circuit. Through its influence on the current, however, the resistance does control the *rate* at which the charge is stored or given up. Moreover, neither the energy lost as heat in the resistance, nor that stored in the condenser, depends on the actual value of the resistance. It may be shown also that the rates of storing and giving up energy are quite different in the processes of charging and discharging.

EXERCISES

1. A condenser has a capacity of 0.3 mf., and is charged with a potential difference of 1.434 volts. Calculate the value of its charge in (*a*) coulombs, (*b*) microcoulombs, (*c*) C. G. S. units.

2. Derive the dimensional formula for capacity.

3. Check the equations of §§ 123–127 by means of dimensional formulas.

CHAPTER VI

ELECTROMAGNETIC INDUCTION

PART I. MUTUAL AND SELF INDUCTANCE

128. The Linking of Circuits with Lines of Force. A straight wire carrying a current is known to have about itself a magnetic field. The general form of the lines of force is a circle, concentric about the axis of the wire. The existence of this field is an indication of current flowing along the wire, and the intensity of the field has been shown to bear a direct ratio to the current strength. (See equation (8), Chapter IV.)

The line of force must be regarded as a closed curve along which a magnetic pole will move. From whatever point the pole may start, it will return again to the same point. If the line of force is due to a bar magnet, or to an electromagnet, part of the path will be through iron and part through air.

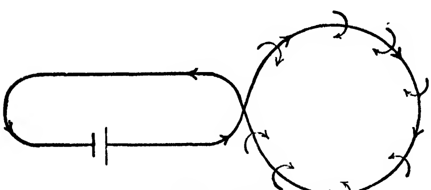

FIG. 84.

Moreover, the electric circuit is itself a closed curve along which the electric charge passes from the generator back to the generator again. The lines of force must then be considered as *linked* with the electric circuit, as represented in Fig. 84.

In considering the linking of lines of force with an electric circuit, two different cases may be distinguished: (1) *when the circuit is originally without current, and is brought into a*

magnetic field; (2) when the circuit conveys the current which produces the magnetic field.

In the first case, suppose a closed loop of a single turn of wire not carrying a current is brought into the neighborhood of a bar magnet. Some of the lines of force of the magnetic field will link with the loop. If ϕ lines of force thus link with the single turn, the number of linkings is given by $N = \phi$. If there are S turns of wire linking with ϕ lines of force, then the total number N of such linkings is given by the equation

(1) $$N = S\phi.$$

In the second case, let ϕ represent the total number of lines of force due to the flowing current which thread through the circuit (Fig. 84). If there is a single turn of wire, the number of linkings is given by $N = \phi$. If, however, there are S wire turns, each line of force is considered as linking with every turn. The total number N of such linkings is given by the product of the number of wire turns and the number of magnetic lines; that is,

(2) $$N = S\phi.$$

This product of magnetic flux lines by wire turns is frequently called ***flux turns.***

It is important to consider in what ways the number of linkings, or flux turns, may be changed. Assuming that the permeability of the medium is constant, suppose first that a loop of a conductor not carrying current is placed in a magnetic field. The number of linkings may be changed by changing (*a*) the field strength, (*b*) the position of the loop, (*c*) the dimensions or shape of the loop, (*d*) the number of turns of wire.

In the case of a closed circuit which does carry current the number of linkings may be changed by changing (*a*) the current strength, (*b*) the number of turns of wire, (*c*) the shape or dimensions of the circuit.

Faraday first showed that any change in the number o linkings between wire turns and flux lines gives rise to a induced electromotive force. Moreover, Lenz's law state that the induced current arising from this E. M. F. is alway so directed as to oppose the change. Lenz's law is merely th statement of the principle of the conservation of energy fo the electrical case. From this principle it is apparent tha the energy put into the circuit to bring about a change in th number of linkings is precisely equal to the energy of th induced current arising from the change.

129. The Faraday Equation. Assume a conductor ben into the form ABC, Fig. 85, and lying in the magnetic fiel NS, with its plan perpendicular to th lines of force. O the horizontal an parallel wires A an C lies a bar ab whic can move in eithe direction along th wires, and is alway in contact with them

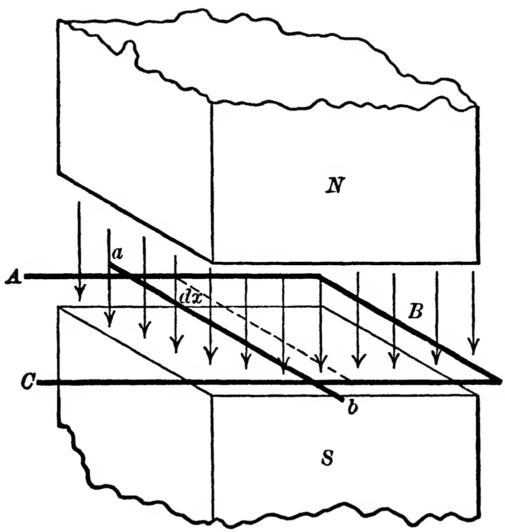

FIG. 85.

Suppose the bar i moved to the righ through a short di tance dx in a tim interval dt, thus cu ting across the line of force, and changin the number of linkings between the circuit and the fielc This motion sets up an induced current in ab, in the directio from b to a. If V is the induced electromotive forc and i the instantaneous value of the induced current, the

the energy dW_1 of the induced current is given by the equation

(3) $$dW_1 = Vi\,dt.$$

Let H be the value of the magnetic field strength and l the length of ab between A and C. Then, when a current i flows in ab, there will be a force acting on it given by the equation

(4) $$F = ilH.$$

When the bar is moved through a distance dx against this force, the work dW_2 done is given by the equation

(5) $$dW_2 = ilH\,dx.$$

Since from Lenz's law the work done in moving the conductor is equivalent to the energy associated with the current induced, the expressions (3) and (5) may be equated, and we have

(6) $$Vi\,dt = -\,ilH\,dx.$$

The negative sign shows that the current induced is directed so that its magnetic field reacts with the field H to oppose the motion, as required by the law of conservation of energy. In place of $l\,dx$, which is an area, dA may be written, and the product $H\,dA$ gives the change in the number of linkings dN, due to the motion of ab. Equation (6) may then be written in the form

$$Vi\,dt = -\,i\,dN,$$

or

(7) $$V = -\frac{dN}{dt}.$$

This expression is called ***Faraday's equation.*** It states that the induced potential difference is numerically equal to the time rate of change of the number of linkings. One C. G. S. unit is induced when one line of force is cut by one wire in one second. In order to induce an electromotive force of one volt in the circuit, 10^8 lines of force must be cut per second by a single wire.

130. Mutual Inductance of Two Circuits. Consider a circuit A (Fig. 86) carrying a current of strength i. The magnetic field strength at any point in the neighborhood of A is proportional to i (§ 104). Since the field strength at any point is measured by the number of lines of force per unit area, it follows that the number of lines through any chosen area near A is proportional to i.

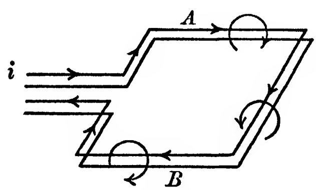

FIG. 86.

Suppose that another conductor forming a closed circuit B is near A, and that the number of linkings between wire turns of circuit B and lines of force due to circuit A is equal to N. The number N is proportional to i, and if the relative positions of A and B, the number of wire turns in each, and the permeability of the surrounding medium are not changed, then

(8) $$N = Mi,$$

or

(9) $$M = \frac{N}{i},$$

where the factor M is a geometric constant of the pair of circuits, which is quite independent of the current strength.

If the current in the circuit A changes by an amount di, the number of linkings with the B circuit will change by a corresponding amount dN, whence

(10) $$M = \frac{dN}{di}.$$

Moreover, if this change takes place in a time dt, we may write

(11) $$M = \left[\frac{dN}{dt}\right]_B \div \left[\frac{di}{dt}\right]_A.$$

It has been shown in equation (7) that the time rate of change of linkings gives the value of the induced potential difference. Whence, the equation (11) may be written in the form

$$M = \frac{V}{\left[\frac{di}{dt}\right]_A}. \tag{12}$$

This constant M is called the ***mutual inductance*** of the two circuits, and it may be defined in any one of the following ways: (*a*) from equation (9), *M is numerically equal to the total number of linkings with the B circuit, when unit current flows in the circuit A*; (*b*) from equation (10), *M is numerically equal to the change in the number of linkings with the B circuit, when the current in the A circuit is changed by unit amount*; (*c*) from equation (11), *M is the factor determined by the constant ratio between the time rate of change of linkings, and the time rate of change of current strength*; (*d*) from equation (12), *M is the value of the potential difference, or E. M. F., induced in the B circuit, when the current in the A circuit is changing at unit rate.*

131. Units and Standards of Mutual Inductance. The practical unit of mutual inductance is called the ***henry***: *a pair of circuits has a mutual inductance of one henry when an E. M. F. of one volt is induced in one of them if a change of one ampere per second occurs in the other.* The *henry* is equal to 10^9 C. G. S. units.

The mutual inductance of two circuits can be computed only when the number of lines of force arising in one circuit, and linking with the other circuit, is known. This is possible only in a few simple cases.

One arrangement for which the mutual inductance may easily be calculated is shown in cross-section in Fig. 87. A long bar of wood or hard rubber AA' is turned to a uniform diameter, except for a short distance at the middle aa', where the diameter

is made slightly less than that of the rest of the bar. In this middle channel is wound a large number of turns, S, of fine wire, usually a thousand or more, the ends being brought out

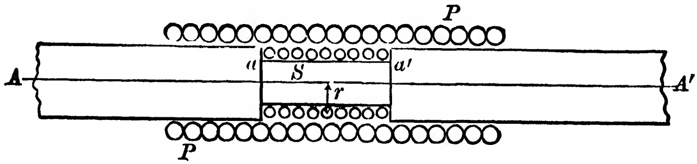

FIG. 87.

to terminal binding posts. Outside of this coil, and running from end to end of the core, is wound in a uniform layer the coil P.

For the long coil, called the primary, T represents the total number of turns, L is the length, and T/L, or n, is the number of turns per centimeter of length. For the short secondary coil, S is the total number of turns, r is the radius of one turn, and A is the area of cross-section.

Remembering that the mutual inductance of two circuits is given by the number of linkings established when unit current flows, the value of M for the present case is easily calculated. Let a current of strength i C. G. S. units flow through the primary coil. The value of the magnetic field strength at the center of the coil is, from equation (33), Chapter IV,

$$H = 4\,\pi n i. \tag{13}$$

Since this may be regarded as uniform over the entire cross-section, the total magnetic flux through the secondary coil is given by the formula

$$\phi = HA = 4\,\pi n i A = 4\,\pi^2 n i r^2. \tag{14}$$

All of these lines interlink with all the turns of the secondary; hence, by equation (1), the total number of linkings is

$$N = \phi S = 4\,\pi^2 n i r^2 S. \tag{15}$$

From equation (9) we have

$$M = \frac{N}{i};$$

substituting in this the value of N from equation (15), we have

(16) $$M = 4\,\pi^2 n r^2 S \text{ C. G. S. units,}$$

or

(17) $$M = \frac{4\,\pi^2 n r^2 S}{10^9} \text{ henrys.}$$

This form of mutual inductance, which is so carefully constructed that its value can be calculated from its dimensions, is called a standard ***current inductor***. The ratio of the length to the radius should not be less than fifty. The secondary coil is sometimes wound outside of the primary, in which case the area of the cross-section, by which the value of the flux density is multiplied in (14), is that of the primary coil. The secondary coil should lie close to the primary to avoid leakage.

The equation (16) was obtained on the assumption that the permeability of the medium is unity. If any magnetic substance of permeability μ is introduced, the value of the mutual inductance becomes

(18) $$M = 4\,\pi^2 n r^2 S \mu \text{ C. G. S. units.}$$

Since the permeability changes with the magnetic field strength, and hence with the current, M can be calculated in this way only when the corresponding values of μ and H are known. For this reason, it is customary to avoid all magnetic materials in the construction of such coils.

A more convenient laboratory standard with any desired value is made by winding the necessary number of turns on a marble spool. The winding is then thoroughly impregnated with an insulating varnish and baked hard. After mounting on an ebonite or wooden base it is carefully calibrated. Standards prepared in this way are very constant. They should be occasionally checked in a standardizing laboratory.

132. The Mutual Inductance of Symmetrical Circuits. If two circuits A and B have a mutual inductance M, it can be shown that the number of linkings with B due to unit current in A is exactly equal to the number of linkings with A due to unit current in B. The general proof of this requires more powerful methods of analysis than those introduced thus far, but in the case of two symmetrical circuits, the proof is simple and is as follows.

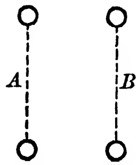

FIG. 88.

Consider two circuits A and B, shown in cross-section in Fig. 88, which are in every way symmetrical. Let M_1 be the mutual inductance when a current flows through A, and let M_2 be their mutual inductance when the same current flows through B. For a current of strength i in A, the number of linkings with B is given by the equation

$$N_1 = M_1 i. \tag{19}$$

For the same current through B, the number of linkings with A is given by the equation

$$N_2 = M_2 i. \tag{20}$$

By symmetry we have

$$N_1 = N_2;$$

whence

$$M_1 = M_2. \tag{21}$$

133. Self-inductance of a Circuit. With a single circuit carrying a current i, as shown in Fig. 84, the number of lines of force threading through the circuit, and hence, the number N of linkings, is proportional directly to the current strength. This fact may be expressed by the equation

$$N = Li, \tag{22}$$

or

$$L = \frac{N}{i}. \tag{23}$$

If the current changes by some small amount di, the number of linkings will change by some corresponding amount dN, whence

$$L = \frac{dN}{di}. \tag{24}$$

Moreover, if this change takes place in a time dt, we may write

$$L = \left[\frac{dN}{dt} \div \frac{di}{dt}\right]. \tag{25}$$

Since by equation (7) the time rate of change of the number of linkings gives the induced potential difference, the equation (25) may be written in the form

$$L = \frac{V}{\frac{di}{dt}}. \tag{26}$$

If the geometric form, the number of wire turns, and the permeability of the medium are constant, L is itself constant, and quite independent of the current strength. This quantity L is called the ***self-inductance*** of the circuit, and it is defined in any one of the following ways: (*a*) from equation (23), *L is numerically equal to the total number of linkings, when unit current is flowing;* (*b*) from equation (24), *L is numerically equal to the change in the number of linkings when the current is changed by unit amount;* (*c*) from equation (25), *L is the factor determined by the constant ratio between the time rate of change of linkings and the time rate of change of current;* (*d*) from equation (26) *L is the value of the potential difference, or E. M. F. induced in the circuit when the current changes at unit rate.*

134. Units and Standards of Self-inductance. The practical unit of self-inductance is the ***henry*** *: a circuit has an inductance of one henry when one volt is induced at its terminals by a change in current strength of one ampere per second.* The *millihenry* and *microhenry* are convenient subdivisions of the henry. (See § 12.)

Taking any one of the defining equations for self-inductance, its dimensional formula is found to be

$$L = [L]; \tag{27}$$

hence its absolute unit should be the same as the unit of length, the ***centimeter.*** The henry is equivalent to 10^9 centimeters.[1]

The secondary circuit of a $\frac{3}{4}$-inch induction coil will have an inductance of approximately 15 henrys; an ordinary telegraph sounder 20 to 30 millihenrys; and the coils of a sensitive, suspended needle galvanometer from one to two henrys. Good resistance-box coils should have an inductance of less than one microhenry. The inductance of a dynamo field magnet may exceed 1000 henrys. The primary of an induction coil 20 inches long will have an inductance of approximately 20 henrys, while the secondary of such a coil, with a resistance of 30,000 ohms, may have an inductance of 2000 henrys.

As in the case of mutual inductance, the calculation of the self-inductance of circuits from their dimensions is feasible only in a few cases.[2] One form of circuit for which this calculation is easily made is the long solenoid. Assume a long solenoid of radius r, area of cross-section A, and length l, with T total turns of wire, and n turns per centimeter. From equation (33), § 109, the value of the field strength at the center of the coil is given by

$$H = 4\pi n i, \tag{28}$$

and the total flux across the central cross-sectional plane is

$$\phi = HA = 4\pi n i A = 4\pi^2 n i r^2. \tag{29}$$

[1] Since the distance along the arc of a great circle of the earth, from the equator to the north pole, was originally taken as 10^9 centimeters, an early name for the practical unit of inductance was the *quadrant*.

[2] The formulas given here for calculating self and mutual inductances from the dimensions of the coils are only approximate. Accurate formulas, which are rather complicated, are given in the BULLETIN OF THE BUREAU OF STANDARDS, Vol. 8, 1912. Approximate formulas for coils of various shapes will be found in the various electrical handbooks.

Assuming that all of these lines of force link with all of the wire turns, which is very nearly true, the total number of linkings is given by the formula

$$N = \phi T = 4\,\pi^2 n i r^2 T. \tag{30}$$

Since $T = nl$, equation (30) becomes

$$N = 4\,\pi^2 n^2 i r^2 l. \tag{31}$$

From equation (23), we have

$$L = \frac{N}{i} = 4\,\pi^2 n^2 r^2 l \text{ C. G. S. units,} \tag{32}$$

or

$$L = \frac{4\,\pi^2 n^2 r^2 l}{10^9} \text{ henrys.} \tag{33}$$

If the permeability of the medium within the coil is μ, equation (32) becomes

$$L = 4\,\pi^2 n^2 r^2 l \mu. \tag{34}$$

However, in the construction of self-inductance coils which are to be used as standards, magnetic materials are avoided, because of the variation of the magnetic permeability with current strength.

PART II. CURRENT, ENERGY, AND CHARGE RELATIONS

135. Current and Energy Relations in Inductive Circuits. A piece of matter cannot set itself in motion, but requires energy from without to effect a change in its momentum. Similarly an electric current cannot set itself in motion, and energy must enter the circuit in order that electric charge may be transferred. It has been shown in § 129 how energy can enter a circuit by making lines of force cut that circuit, so as to change the number of linkings with the wire turns.

When a current is established, a magnetic field is created in the ether, the medium about the circuit, and when the current ceases, this magnetic field collapses and disappears. This magnetic field is of the nature of a strained condition in the medium, and work is required to establish the field. Moreover, when such a field collapses or disappears, the stored energy is returned to the circuit in the form of an induced current.

The magnetic field about a circuit carrying a current is then to be considered as the seat of a certain amount of energy whose magnitude depends upon the strength of the current and also upon its distribution, that is, upon the shape and dimensions of the circuit and the number of turns of wire. This is quite analogous to the case of a system of material particles in rotation, in which the kinetic energy of rotation depends upon the masses of the particles as well as upon their space distribution.

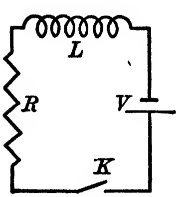

FIG. 89.

Consider a circuit (Fig. 89) in which the inductance and resistance are concentrated in L and R respectively, with an impressed potential difference V. In a non-inductive circuit the current rises to its full value instantly when the circuit is closed.

This is not the case, however, when an inductance is present, for in this case magnetic flux lines are linked with the wire turns of the circuit. We have seen that this means that a potential difference is established which depends on the rate of change of current, and which opposes the change. When the key K is closed and current begins to flow through L, its associated field sweeps out and cuts other wire turns, thus setting up a potential difference given by equation (26),

$$V = -L\frac{di}{dt}. \tag{35}$$

136. The Helmholtz Equation. The impressed potential difference V (Fig. 89) may be considered as at any instant equal to the sum of two components: one part V_1 maintains the current in the ohmic resistance of the circuit, while the other part V_2 maintains the growing current in the inductance. We may then write

$$V = V_1 + V_2$$

or

$$V = iR + L\frac{di}{dt}. \tag{36}$$

Equation (36) is called the ***Helmholtz equation.*** It gives the instantaneous value of the potential difference at the terminals of an inductive circuit. The inductance L is always essentially a positive quantity and may be equal to or greater than zero.

Suppose $L = 0$; then the second term of the right-hand member of equation (36) is zero, and the impressed potential difference is equal to the iR drop through the circuit.

Suppose that L is greater than zero; then for a rising value of i, that is, with di/dt positive, it is obvious that the iR drop through the circuit is not so large as the impressed V, and the quantity $L\,(di/dt)$ is of the nature of a reversed potential difference which retards the rise in the current.

For a decreasing current the term $L(di/dt)$ is negative; then the iR drop is greater than the impressed V. This means that the instantaneous value of the current is greater than it would be in a non-inductive circuit, and the effect of L is to retard the fall in current strength.

If the current strength is constant, that is, if $di/dt = 0$, the impressed V is equal to the iR drop.

It is then obvious that in a circuit possessing inductance, through which a varying current is flowing, the impressed voltage is not represented by the iR drop in the circuit, but includes also that part which is utilized in storing energy in the magnetic field about the circuit. The effect of the inductance is to make the changes in the current lag behind the changes in the impressed potential difference.

Self-inductance is thus seen to be that *property of the circuit* which opposes any change that is made in the current strength. It is analogous to the inertia of matter, which opposes any changes in the velocity impressed on a body.

The Helmholtz equation may be written in the form

$$i = \frac{V}{R} - \frac{L}{R}\frac{di}{dt}. \tag{37}$$

This equation shows that during the period of increase of i, the instantaneous value of the current is less than the final value which V is able to maintain by the amount

$$\frac{L}{R}\frac{di}{dt}.$$

The theory of dimensions requires that $L(di/dt)$ must itself be a potential difference, and the negative sign shows that it tends to retard the growth of i. If di/dt is negative, as it is for a decreasing current, the sign of the last term of equation (37) is positive; hence, i is greater than its steady initial value V/R.

In large distributing systems which carry heavy currents,

and in which the time constants are small, the effects of sudden breaks or short circuits are often exceedingly violent. Enormous induction currents are quickly established, which have the suddenness and disastrous effects of violent explosions.

137. The Growing Current and the Time Constant. Let us assume that a potential difference V is suddenly impressed on a circuit of resistance R and inductance L. The Helmholtz equation,

(38) $$V = iR + L\frac{di}{dt},$$

is a differential equation which expresses the general relation between the variable quantities i and t. The integration of this equation gives the instantaneous value of the current at any time t seconds after the potential difference is impressed. Separating the variables, we have

(39) $$iR - V = -L\frac{di}{dt},$$

or

(40) $$i - \frac{V}{R} = -\frac{L}{R}\frac{di}{dt},$$

whence

(41) $$\frac{di}{i - \frac{V}{R}} = -\frac{R}{L}dt.$$

The final value of the current after it has become steady is V/R; let us denote this by I. Integrating between the limits of zero and t, at which instants the currents are zero and i, respectively, we obtain

(42) $$\int_0^i \frac{di}{i - \frac{V}{R}} = \int_0^t -\frac{R}{L}dt,$$

whence

(43) $$\Big[\log_e (i - I)\Big]_0^i = \Big[-\frac{R}{L}t\Big]_0^t,$$

o

or

(44) $$\log_e(i - I) - \log_e(-I) = -\frac{R}{L}t,$$

or

(45) $$\log_e \frac{i - I}{-I} = -\frac{R}{L}t,$$

or

(46) $$i = I(1 - e^{-Rt/L}).$$

From equation (46) it is seen that when $t = 0$, $i = 0$, and when t equals infinity, $i = I$. The general form of the curve that shows the relation between values of i and t is shown in Fig. 90, curve I, which is asymptotic to the line $i = I$. Only after

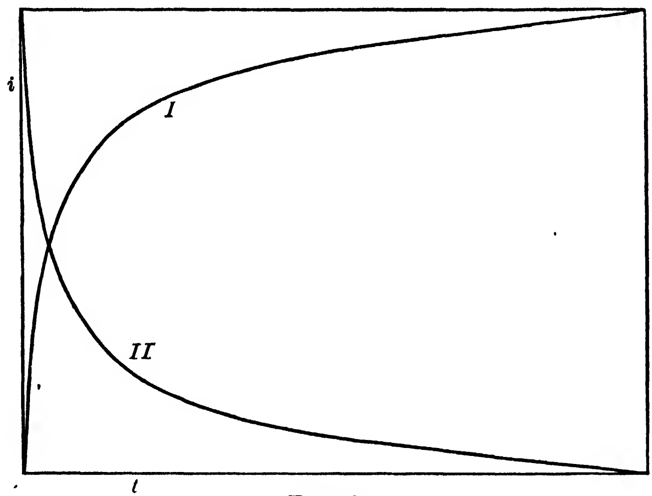

FIG. 90.

a time which is theoretically infinite, does i become strictly equal to I; but for values of L and R which are actually in use, i practically attains its final value in a few seconds, or even in a fraction of a second.

In order to compare the quickness with which currents grow in inductive circuits, it is obviously impossible to use the full time required for attaining the steady state, as these times

are in every case theoretically infinite. It is customary, however, to use the time required for the current to reach some *definite fraction* of its final value. If t is made equal to L/R seconds in equation (46), that equation takes the form

$$i = I - \frac{1}{e} I, \tag{47}$$

and it is seen that at this time the current falls short of its full or final value by an amount equal to $1/e$ times the final value, that is, about 0.37 times the final value. This factor L/R is a characteristic constant of the inductive circuit, and is called the ***time constant.*** The effect of the inductance in delaying the rise of the current to its full value is measured in terms of this time constant.

138. The Falling Current and the Time Constant. Consider a circuit of resistance R and inductance L, in which a steady current i_0 is being maintained by an electromotive force of value V. If the electromotive force is suddenly cut off by the opening of a switch, the relation between the decreasing current and the time is given by the integration of the Helmholtz equation, which for this case will be written in the form

$$0 = Ri + L\frac{di}{dt}. \tag{48}$$

Separating the variables, we have

$$\frac{di}{i} = -\frac{R}{L}\,dt. \tag{49}$$

Integrating both sides of this equation between the limits zero and t, at which instants the currents are i_0 and i respectively, we have

$$\int_{i_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt, \tag{50}$$

whence

$$\Big[\log_e i\Big]_{i_0}^{i} = -\frac{R}{L}\Big[t\Big]_0^t, \tag{51}$$

or

(52) $$\log \frac{i}{i_0} = -\frac{R}{L} t,$$

or

(53) $$i = i_0 e^{-Rt/L}.$$

From equation (53) it is seen that when $t = 0$, i has the value i_0, and when t is infinite, i becomes zero. The graphical relation between i and t is shown in curve II, Fig. 90. For a time equal to L/R seconds, it is seen that

(54) $$i = \frac{1}{e} i_0,$$

and this value of the time, in which the current falls to $1/e$ times its initial value, is called the ***time constant.*** This time constant is a characteristic of the circuit, by means of which we can express the effect of the inductance in retarding the fall of the current to zero.

If we integrate both sides of equation (53) between the limits zero and infinity, we find the total charge Q which flows through the circuit when the electromotive force is cut off:

$$Q = \int_0^\infty i\, dt = \int_0^\infty i_0 e^{-Rt/L} dt;$$

whence

(55) $$Q = i_0 \frac{L}{R}.$$

From equation (55) the time constant is given a new significance; it is seen to be that time interval during which the initial current would have to flow if it remained constant, in order to convey the charge which does actually pass.

In the design of electromagnets, where quick action is required, or where the time relations of the mechanism to other parts is specified, the time constant is important, as it is the factor which controls the rate of rise or fall of the magnetizing current.

For an average telegraph sounder the time constant will be less than 0.01 second. For a relay of 150 ohms resistance, it will be about 0.03 second. For the field magnets of a dynamo it may be as high as ten seconds. These figures signify the time interval required for a growing current to reach a value equal to 0.63 of the final value, or for a decreasing current to fall to 0.37 of its initial value.

139. The Energy in an Inductive Circuit. When a potential difference V is impressed on a circuit and a current i is established, the energy dW given to the circuit in a time dt is expressed by the equation

$$dW = Vi\,dt. \tag{56}$$

If the circuit is inductive, a part of the energy is stored in the magnetic field about the inductance, while another part is dissipated as heat in the ohmic resistance. Separating these components and remembering that $L(di/dt)$ is a potential difference, we may write equation (56) in the form

$$dW = i^2R\,dt + L\frac{di}{dt}i\,dt. \tag{57}$$

In order to find the energy W_L stored in the inductive part of the circuit, the last term of equation (57) may be integrated between the limits zero and I, where I represents the final value of the current:

$$W_L = \int_0^I L\frac{di}{dt}i\,dt = \frac{1}{2}LI^2. \tag{58}$$

This is called the ***intrinsic energy equation;*** it gives the value of the energy stored in the circuit due to the establishment of the magnetic field. If the current is reduced to zero, the magnetic field will collapse and disappear, and the energy will be returned to the circuit, appearing as a flow of charge, if the circuit is closed.

It is of interest to note the analogy between equation (58) and the expression for the kinetic energy of a rotating body:

$$K.E. = \frac{1}{2} K\omega^2,$$

where K is the moment of inertia and ω is the angular speed. It is apparent that self-inductance in a circuit bears the same relation to a changing current that inertia does to changing speed. The inductance, like the inertia, always tends to oppose the change.

When a hot spark is required for ignition purposes, it is not sufficient merely to break the circuit of a dry cell or storage battery, for the potential difference appearing at the terminals is that of the battery only, and represents but a small amount of power. If, however, a coil of large inductance is put in series with the battery, an amount of energy given by equation (58) is stored in the magnetic field at a relatively slow rate. This energy may be made as large as desired by the proper regulation of L and I. The sudden breaking of this circuit causes a rapid decrease in the current, and a correspondingly large potential difference appears at the separated terminals. The stored energy is released quickly and a hot spark is the result.

140. Inductance and Capacity in Parallel. If an inductive coil is shunted by a condenser in series with some resistance, as shown in Fig. 91, the time constants of the two portions of the circuit may be so chosen that the effects of capacity and inductance will annul each other. From equation (21), § 125, the current flowing through AA' alone is given by the equation

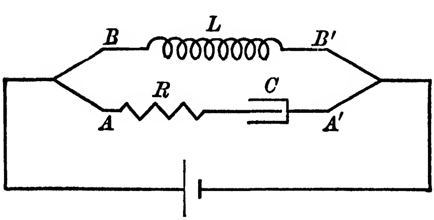

FIG. 91.

(59) $$i_c = Ie^{-t/RC}.$$

From equation (46), the current flowing through BB' alone is given by the equation

(60) $$i_L = I(1 - e^{-Rt/L}).$$

If the time constants are made equal, that is, if

(61) $$\frac{L}{R} = RC,$$

and if both branches of the circuit have the same potential difference simultaneously impressed, then the total current flowing through the circuit is given by the equation

(62) $$i_c + i_L = I.$$

This equation shows that under the assumed conditions, the current which flows is precisely that which would flow if the circuit had a resistance R but neither capacity nor inductance.

This principle is applied in an important way in modern telephone circuits, where the inevitable capacity of lines and cables distorts the wave form of the voice currents, thus greatly diminishing the clearness of the speech transmission. The effect of the capacity is to cause the various components of the voice currents to travel with different speeds and to die away at different rates, and it is desirable to overcome this effect as far as possible. This is accomplished by introducing inductance in the form of loading coils at intervals throughout the circuit, and it is only by this means that the recent advances in long distance and cable transmission have been possible.

141. A Relation between Self and Mutual Inductance. Consider two circuits, A and B, connected in helping series, with the same current i flowing in each. Let L_a and L_b represent respectively the inductance in each circuit, and let M be

their mutual inductance. By equations (8) and (22) the number of linkings is

(63) for the A circuit: $N_A = L_A i + Mi,$

(64) for the B circuit: $N_B = L_B i + Mi.$

Equation (63) gives the number of linkings between lines of force arising in the A circuit, and wire turns of the B circuit. Equation (64) gives the number of linkings between lines of force arising in the B circuit, and wire turns of the A circuit. The total number N of linkings due to lines of force common to both circuits is then given by the sum of N_A and N_B; whence

$$N = N_A + N_B = (L_A + L_B + 2\,M)i. \tag{65}$$

If L_1 represents the self-inductance of the system considered as a single coil, then

$$N = L_1 i;$$

whence

$$L_1 = (L_A + L_B + 2\,M). \tag{66}$$

If the coils A and B are now connected in opposing series, the method applied in deriving (63) and (64) will give,

(67) for the A circuit: $N_A = L_A i - Mi,$

(68) for the B circuit: $N_B = L_B i - Mi.$

The total number N of linkings due to lines of force common to both coils will be given by the sum of (67) and (68); whence

$$N = N_A + N_B = (L_A + L_B - 2\,M)i. \tag{69}$$

If L_2 represents the self-inductance of the system considered as a single coil, we have

$$N = L_2 i,$$

whence

$$L_2 = (L_A + L_B - 2\,M). \tag{70}$$

Subtracting (70) from (66), and solving for M, we have

$$M = \frac{L_1 - L_2}{4}. \tag{71}$$

If two similar circuits with equal self-inductances are imagined to be absolutely superposed in space, it may be shown that

$$L_A = L_B = M. \tag{72}$$

142. The Quantity of Electricity in an Inductive Circuit. If a potential difference is induced in a circuit by some change in the number of linkings of its wire turns with magnetic lines of force, the value of the resulting current, in general, will not be constant. Moreover, the induced current is transient, persisting only while the change in the linkings is taking place. The value of this transient and variable current is difficult to measure. In general, it is more useful to measure the total quantity Q of charge which passes. The value of this quantity is easily calculated. At any instant, the value of the current flowing through a resistance R under a potential difference V is, by Ohm's law,

$$i = \frac{V}{R}. \tag{73}$$

Remembering that $i = dQ/dt$, and that $V = dN/dt$, we may write

$$dQ = \frac{1}{R}\,dN. \tag{74}$$

Integrating this expression for dQ between the limits N_2 and N_1, which are respectively the initial and final values of the flux turns, corresponding to the values zero and Q of the charge, we have

$$Q = \frac{1}{R}\int_{N_2}^{N_1} dN = \frac{N_1 - N_2}{R}. \tag{75}$$

Representing the change in flux turns by ΔN, equation (75) may be written in the form

$$Q = \frac{\Delta N}{R}. \tag{76}$$

This equation states that the induced charge is numerically equal to the ratio between the total change in the number of linkings and the total resistance of the circuit through which the charge passes. It is seen that the amount of the induced charge is independent of the time. It must be remembered that the mere condition that lines of force are interlinked with wire turns does not induce the flow of a charge. Induction currents only arise when some change in the number of linkings occurs, that is, when the lines of force are being cut by wire turns.

As an illustration, consider the case of the current inductor (§ 131), with a steady current through the primary. Let the corresponding number of linkings be denoted by N_1. If the circuit is broken, the magnetic field collapses, and N_2 is zero. For this case, the charge induced in the secondary is

$$Q_1 = \frac{N_1}{R}.$$

If current is started through the primary, the linkings with the secondary circuit increase from zero to N_2, and

$$Q_2 = \frac{N_2}{R}.$$

If the primary current is reversed in direction, the lines of force are first withdrawn and then immediately reëstablished in the opposite direction. For this case

$$N_1 = N_2 = N$$

and

$$Q = \frac{N_1 - (-N_2)}{R} = \frac{2N}{R}. \tag{77}$$

The quantity is given in absolute units when N and R are in absolute units. If R is in ohms and Q is in coulombs, then

$$Q = \frac{2N}{R\,10^8}. \tag{78}$$

EXERCISES

1. Check the equation of § 135 by means of dimensional formulas.

2. Check the defining equations for self and mutual inductance by means of dimensional formulas.

3. A pair of circuits has a mutual inductance of 3 millihenrys. A current of 1 ampere is started through the primary. What is the value of the charge induced in the secondary, if its resistance is 0.1 ohm?

4. A circuit has a resistance of 1.0 ohm, and an inductance of 0.5 henry. An initial current of 100 amperes is flowing, when the impressed potential difference is cut off by opening a switch. Find (*a*) the initial rate of decrease of the current; (*b*) the time constant of the circuit; (*c*) the value of the current strength for $t = 0.01$, 0.1, and 1.0 second; (*d*) the rate of change of current for the times given in (*c*).

5. A potential difference of 110 volts is impressed on a circuit of resistance 3 ohms and inductance of 0.04 henry. Calculate (*a*) the initial rate of rise of current strength; (*b*) the value of the current strength for $t = 0.02$ second; (*c*) the iR drop and the inductive drop at this instant; (*d*) the rate of change of current at this instant; (*e*) make the calculations of (*c*) and (*d*) for $t = 0.5$ second.

6. In order to trace the rate of growth of a current in an inductive circuit, find the fraction of its final value for $t = L/R$, $t = 2\,L/R$, $t = 3\,L/R$, etc.

CHAPTER VII

ELECTRICAL QUANTITY AND THE BALLISTIC GALVANOMETER

143. Fundamental Relations. In the preceding chapters three important equations have been discussed, which deal with the quantity or amount of charge passing through a circuit.

In § 102, it was shown that the value of the quantity, when a constant current i flows through a circuit for t seconds, is given by the equation

$$(1) \qquad Q = it;$$

whereas, if the current is not constant, the value of the charge is, by equation (1), § 117,

$$(2) \qquad Q = \int i \, dt.$$

In § 117, it was shown that the quantity stored in a condenser is given by the equation

$$(3) \qquad Q = CV.$$

In § 142, equation (76) gives the value of the total induced charge in terms of the change in the number of linkings and the resistance of the circuit, in the form

$$(4) \qquad Q = \frac{\Delta N}{R}.$$

For the measurement of quantity passing more or less continuously through a circuit, as in (1) above, and of relatively

large value, as in the case of lighting or power circuits, a quantity meter in some form must be used. These meters are called coulomb meters or ampere-hour meters. For a detailed description of them the student is referred to the larger treatises and the journals.

For the measurement of the transient and relatively small quantities produced by the discharge of a condenser, or arising in an inductive circuit, as in (3) and (4) above, the ballistic galvanometer is used.

144. The Ballistic Galvanometer. The term "ballistic" is derived from a Greek word signifying *to throw.* Its descriptive meaning is obvious when it is considered that this galvanometer is used only to measure time integrals of transient currents. The conditions under which it is used do not admit of steady deflections, but rather of an initial *throw*, followed by a series of swings of decreasing amplitude. It is this first throw which is observed, and which is found to be proportional to (and therefore a measure of) the passing charge.

The current through the galvanometer is not constant, and it is of very short duration. The reaction of its magnetic field with the field of the instrument produces an impulsive torque on the suspended system. This is opposed by the torque due to (a) the elasticity of the suspension in the moving-coil type, (b) the controlling magnetic field in the moving-needle type, and (c) the torque due to air friction, or to the magnetic field reactions of induced currents, set up either in metallic parts of the instrument or in the wires of the circuit itself.

The moving system must have a sufficient moment of inertia so that it does not begin to move until practically all the charge has passed, and its motion must not be too fast, or the observer will have difficulty in reading the throw. Any galvanometer of either type may be used ballistically with certain restrictions.

145. The Suspended-needle Type. The moving-needle ballistic galvanometer is quite like any other sensitive galvanometer of its type with the addition of two characteristic features: (*a*) the needle is massive, or in some way loaded so as to give it a large moment of inertia; (*b*) the galvanometer is so designed that the damping is very small.

It is usually made with a small bell-shaped needle in order that air friction, the chief cause of damping in this type, may be avoided. Its sensibility can be increased by using the astatic system, and may be varied by using interchangeable coils, variable coil distance, or variable control with permanent magnets.

To set up such a galvanometer, select coils of suitable resistance, and with the control magnet removed, fix the mirror on the suspension so that it faces away from the scale. Now by means of the control magnet, or suitable bar magnets laid flat on the table near the galvanometer, bring the needle about until the mirror faces the scale. Careful manipulation of the magnets will give a very small residual control; hence the galvanometer can be made as sensitive as desired. A working period of 10 seconds is usually sufficiently sensitive, while a period of 15 seconds or more is impracticable under ordinary working conditions because of stray fields from adjacent electric circuits. On account of the susceptibility of this type of galvanometer to such disturbances, it has been largely superseded by the moving-coil instruments.

For a suspended-needle galvanometer, if damping is neglected, it may be shown that the relation between the charge passing through its coils and the deflection is given by

$$Q = G \sin \frac{d}{2}, \tag{5}$$

where G is a constant and d is the first throw.

146. The Suspended-coil Type. In the moving-coil type of ballistic galvanometer, the transient current sets up a mag-

netic field about the coil which lasts only as long as the current flows. This field, during the brief time for which it exists, reacts with the field of the permanent magnet, and this causes an impulsive torque and a sudden angular displacement of the system. The relation between the quantity of electricity passing and the resulting deflection is given by the equation

$$Q = Gd, \tag{6}$$

where G is a constant and d is the first throw.

The values of the constants in equations (5) and (6) can be determined readily from the theory, but the calculations involve an accurate knowledge either of the dimensions of the parts and the magnetic field strength, or of the figure of merit and the period; and the result must be corrected for damping. Practically such calculations are of little value. Methods for finding these constants experimentally will be discussed in §§ 149–151.

147. Damping. A simple pendulum vibrating in air will make a great many vibrations before it finally comes to rest. If the pendulum ball swings in oil or water, its motion will be retarded to a much greater degree, and the vibrations will cease much sooner than before. Indeed the medium may have so great a viscosity that the pendulum, after deflection and release, will return slowly to its equilibrium position without crossing the zero point. The same variations in the motion of the suspended system of a galvanometer may be observed if the damping is varied through wide limits.

By *damping* is meant the effect of all the retarding forces which oppose the motion of a vibrating body. By means of these forces, the energy stored in the body is gradually dissipated. It is usually assumed that the resultant of these forces is proportional at every instant to the velocity of the body.

Air friction causes damping in either type of galvanometer. In the moving-coil type a more effective cause of damping is the establishment of induced currents in the metal parts of the coil frame, or in the circuit of the coil itself when the ter-

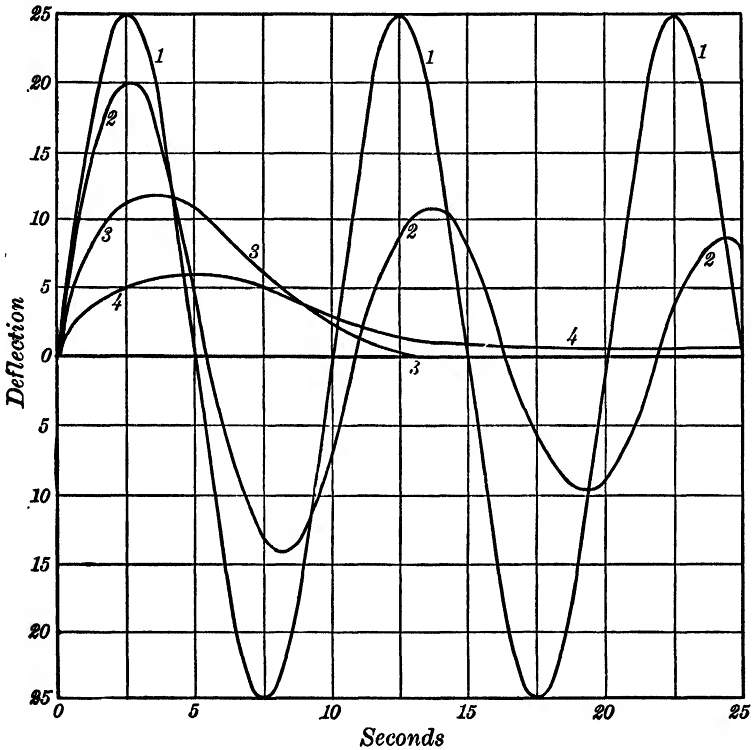

FIG. 92.

minals are connected. Frequently a small closed rectangle of copper wire is attached to the coil for this purpose.

It may be shown from the general equations for oscillatory motion, that for a strictly undamped condition, the deflection is a simple harmonic function of the time, of the form

$$d = k \sin bt, \tag{7}$$

where bt is the phase angle. The graphic representation of

this relation between deflection and time is shown in curve 1, Fig. 92. If the damping is small, the successive amplitudes slowly decrease in value, as shown in curve 2. As the damping increases, the suspended system comes to rest more quickly, as shown in curve 3. For damping greater than a certain critical value, the motion loses its periodic or oscillatory character and follows curve 4.

This latter type of non-oscillatory motion is called ***aperiodic.*** It is desirable in many forms of measuring instruments, inasmuch as the observer does not have to wait for the oscillatory motion to die away.

Moving-coil galvanometers may be so designed that the first throw is proportional to the quantity passing through the coils, even though the damping is large enough to produce the dead beat or non-oscillatory motion. The sensibility is greatly reduced, however. A Weston voltmeter, or indeed any galvanometer in which the damping is chiefly due to induced currents, will give a straight-line curve when charges are plotted against throws. If the induced currents flow through the circuit connected to the galvanometer terminals, the damping will vary with the resistance of the circuit, that is, the factor G in equation (6) will not be a constant for all values of the circuit resistance.

148. Logarithmic Decrement. On account of the damping, the alternate amplitudes of swing of a galvanometer on either side of the equilibrium position grow successively less. If d_1, d_2, etc., represent these successive amplitudes, we may write

$$\frac{d_1}{d_2}=\frac{d_2}{d_3}=\frac{d_3}{d_4}=\cdots=k. \tag{8}$$

The factor k is called the ***damping factor***, or the ***damping ratio***, and its logarithm to the base e is called the ***logarithmic decrement***, which is usually represented by λ. Any deflection

is less than it would have been if there had been no damping. If d_0 represents the throw which would have occurred if the motion had been entirely undamped, and d the observed first throw, it may be shown that we have approximately [1]

$$d_0 = d\left(1 + \frac{\lambda}{2}\right) \tag{9}$$

or

$$d_0 = d\sqrt{k}. \tag{10}$$

These equations must be regarded as giving approximate relations only, and the method loses its validity when the correction factor reaches a value larger than about 1.20.

The theory of logarithmic decrement, with methods of determining values of the correction factors, was formerly important when using suspended needle galvanometers. Present practice employs almost exclusively the suspended coil type, rather heavily damped, for which such correction factors are of no significance. Logarithmic decrement and damping are of great importance, however, when dealing with oscillatory currents, such as are constantly used in high frequency circuits and wireless telegraphy, and a full discussion of the theory will be found in the larger works upon those subjects.

149. Calibration of a Ballistic Galvanometer. The calibration of a ballistic galvanometer consists in sending known charges of electricity through the instrument, and observing the corresponding deflections. These charges may be produced in either of two ways, as follows:

I. With a standard condenser and a battery of known electromotive force, the charge stored in the condenser is given by equation (2), § 117,

[1] Theoretically, the approximate relation is $d = d_0 \cdot e^{-\lambda/2}$ or $d_0 = d \cdot e^{\lambda/2}$. Since $e^{\lambda/2} = 1 + \lambda/2 + \cdots$, we have approximately the relation (9). Since $e^{\lambda} = k$, we have $e^{\lambda/2} = \sqrt{k}$, whence $d_0 = d \cdot \sqrt{k}$, as in (10).

(11) $$Q = CV.$$

The galvanometer constant is given by equation (6) in the form

$$G = \frac{Q}{d} = \frac{CV}{d}.$$

II. By means of any arrangement of a magnetic field and of wire turns of a circuit, in which the change in the number of linkings can be computed, we can find the charge from the relation given in equation (76), § 142,

(12) $$Q = \frac{\Delta N}{R}.$$

Such an arrangement can be realized by means of a mutual inductance M, connected as shown in Fig. 93. Any change in the current through the primary coil P is accompanied by an induced charge in the secondary circuit which causes a throw on the ballistic galvanometer.

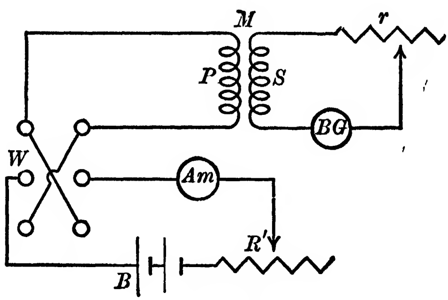

FIG. 93.

The value of the charge is derived as follows. A constant battery of suitable electromotive force is put in series with a control rheostat R', an ammeter Am, and a reversing switch W. The switch W enables us to reverse the current through the primary of the mutual inductance. The secondary coil S is in series with the galvanometer BG, and with an adjustable resistance r. If the current strength in the primary coil P is changed from some initial value i_1 to some final value i_2, a corresponding change takes place in the number of linkings with the wire turns of the secondary coil S. The charge induced is given by equation (75), § 142, in the form

(13) $$Q = \frac{N_1 - N_2}{R}$$

where R is the total resistance of the secondary circuit. By equation (8), § 130, this equation becomes

(14) $$Q = \frac{Mi_1 - Mi_2}{R}.$$

If the current is brought from zero to some value i_2 by closing the switch, we have

(15) $$Q = -\frac{Mi_2}{R}.$$

If the initial current i_1 is broken by opening the switch, $i_2 = 0$, and we have

(16) $$Q = \frac{Mi_1}{R}.$$

The changed sign shows that the induced charge is in opposite directions in these two cases. If the initial current is reversed by throwing over the switch W, then

(17) $$Q = \frac{M[i_1 - (-i_2)]}{R} = \frac{2\,Mi_1}{R}.$$

By (6), the value of the galvanometer constant is given by the equation

$$Q = Gd,$$

whence

(18) $$Q = \frac{2\,Mi_1}{R} = Gd,$$

or

(19) $$G = \frac{2\,Mi_1}{Rd}$$

In equation (18), Q will be in coulombs when M is in henrys, R in ohms, and i in amperes.

In case a standard current inductor is used, equation (19) becomes, by (16), § 131,

(20) $$Q = \frac{2\,Mi_1}{R} = \frac{8\,\pi^2 nr^2\,Si_1}{R}.$$

This gives Q in absolute units if i and R are in absolute units. To get Q in coulombs, i and R must be in amperes and ohms respectively, and the number of linkings must be divided by 10^9; this gives

(21) $$Q = \frac{8\,\pi^2 nr^2\,Si_1}{R10^9}.$$

After several pairs of corresponding values of the charge and the deflection have been found, they may be plotted on cross-section paper. If G is constant, for any particular instrument, the curve will be a straight line. Figure 94 shows several such curves for a galvanometer, when different resistances are connected in series with it. The values for Q used in plotting curve 1 were obtained from a standard condenser and standard cell. (See § 150.) In this case the galvanometer was in series with a resistance of some hundreds of megohms, the condition being practically that of open circuit, so far as induced currents in the coil are concerned.

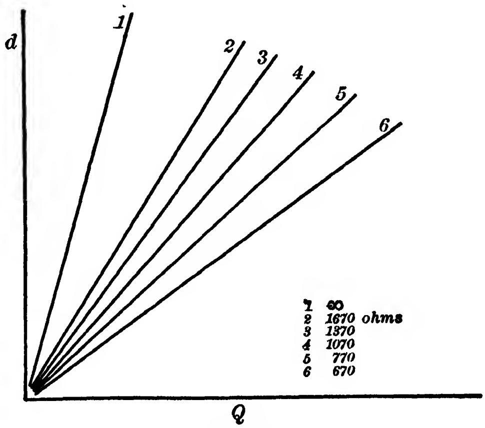

FIG. 94.

The values of Q in the other curves were obtained from a

standard of mutual inductance by the method of § 151. The values of the total secondary circuit resistances are given for each of the curves. It is obvious from an inspection of these calibration curves that the throw for any given value of Q increases as the circuit resistance increases, being greatest when the condenser method is used. In this case the galvanometer is swinging with its circuit practically open, so that the throw is not influenced by damping due to induced currents.

In the case of the slightly damped galvanometer systems, for which the motion is oscillatory, the application of the corrections for damping will reduce the calibration curves taken under different conditions of circuit resistance to a standard curve for undamped motion. Moreover, this curve will be coincident with the calibration curve by using a standard condenser after it has been corrected for damping.

In the case of the heavily damped, or dead beat moving coil galvanometers, there is no simple way of reducing any calibration curve to an undamped standard curve, and it becomes necessary *to calibrate the galvanometer for the precise condition under which it is to be used.*[1]

In using the ballistic galvanometer it is desirable that the throw shall be slow enough to be easily read, that the time of return shall be as small as possible, and that the sensibility shall be as great as possible. It has been shown that these conditions are best fulfilled when the damping is just sufficient to make the galvanometer aperiodic. This condition can usually be realized by the adjustment of the resistance in series with the instrument.

[1] Special keys have been devised which introduce a fixed resistance into the galvanometer circuit immediately after the condenser charge has passed, or which open the circuit immediately after the inductive charge has passed. Such keys are easily arranged and should always be used if it is necessary to calibrate the galvanometer with one type of circuit and use it with another type of circuit. In general, however, the galvanometer should be calibrated under the same conditions of circuit resistance as those with which it will be used.

150. Laboratory Exercise XXVII. *Calibration of a moving coil ballistic galvanometer with a standard condenser.*

APPARATUS. Standard adjustable condenser, standard cell, discharge key, and ballistic galvanometer.

The discharge key, shown in Fig. 95, is a highly insulated

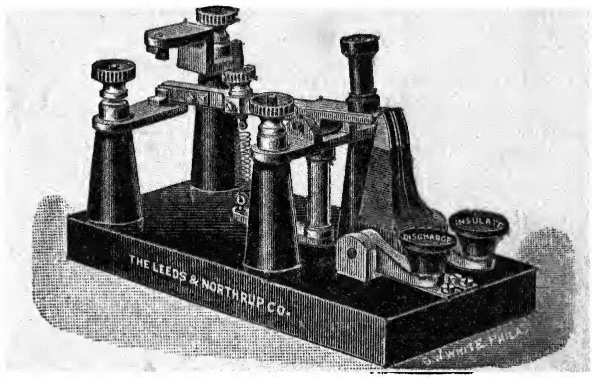

FIG. 95.

three-way key, arranged for prompt and convenient manipulation. The binding post in contact with the rocking lever is always attached to one of the condenser terminals. The battery is put in series with the condenser through the upper

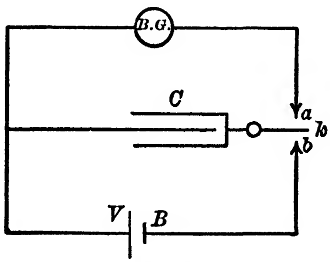

FIG. 96.

contact points, and the galvanometer is connected to the condenser through the lower contact points.

PROCEDURE. (1) Arrange the circuit as in Fig. 96. With k pressed to b, the condenser, of capacity C, is charged with

the potential difference V due to the standard cell. Then the charge Q given to the condenser is given by the equation

$$Q = CV.$$

With k raised to a, this charge is sent through the galvanometer, and the corresponding deflection is observed. The value of G is then given by the equation

$$G = \frac{Q}{d}.$$

(2) In order to determine whether G is constant for all values of the deflection d, it is necessary to vary Q and take

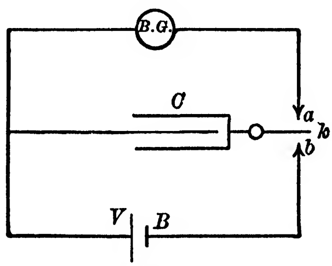

FIG. 96 (repeated).

deflections over the entire working range of the scale. This variation in Q is most easily secured by using an adjustable capacity at C. In case a condenser of fixed value only is available, a storage battery and volt box may be used at B, the values of the potential differences being determined with a potentiometer or an accurate voltmeter.

(3) Take several readings of the scale deflection for each value of the capacity, and record the E. M. F. of the battery used.

(4) Calculate the value of Q for each observed value of d, and plot a curve coördinating Q in microcoulombs and deflections. The quantity Q will be expressed in coulombs when C is in farads and V is in volts. If C is in microfarads, Q will

be in microcoulombs. The observed values of d are subject to correction for damping.

In all experiments using condensers, it is essential that the insulation of the different parts of the circuit should be high. The crossing of wires and contact between the wires and the table should be avoided. Considerable practice is necessary in order to observe the first throw accurately, owing to the rapid motion of the galvanometer coil.

151. Laboratory Exercise XXVIII. *Calibration of a ballistic galvanometer with a standard mutual inductance.*

Apparatus. Standard mutual inductance, ballistic galvanometer, reversing switch, ammeter, storage battery, and control rheostat.

Procedure. (1) Arrange the circuit as in Fig. 93, making r equal to zero.

(2) Start with some small value of the current, reverse W, and note the deflection. Reduce R' until the deflection on reversal is nearly full scale. Read and record several values of d for successive reversals, together with corresponding values of the current. Reduce the current and repeat the readings with deflections about half as large as before.

Report. (1) Tabulate all the readings, together with the means of the galvanometer deflections. Record also the total resistance of the secondary circuit, and the probable precision of the ammeter and galvanometer readings.

(2) Plot a curve between microcoulombs as abscissas and deflections as ordinates. Account for the form of the curve if it departs from a straight line.

(3) In order to show the variations in these curves plot the calibration curve for the galvanometer as taken by the method of § 150. On the same sheet plot the curve as found above.

Increase r from zero to some value such that the total resistance of the secondary circuit is made successively two, three, and four times the original value. In the manner described in (2) take data for calibration curves for these three condi-

tions, and plot them also on the same sheet with the other two. Account for the variation in the slope of the curves and state what inferences may be drawn from the experiment.

152. The Magneto-Inductor. The magneto-inductor is a convenient and self-contained device for producing a change in the number of linkings between the lines of force of a fixed bar magnet and the wire turns of a movable coil. With this change in the number of linkings known, the value of the induced charge in a circuit of given resistance may be found. One form of the apparatus represented in Fig. 97 consists of a bowl-shaped casting *ABC*, in the center of which is fixed a strong and permanent bar magnet *NS*. Attached to the top of the bar magnet is a circular plate which almost completes the magnetic circuit, leaving a narrow circular air-gap at *bb*. Through this air-gap passes freely a brass tube *DD*, which carries at *cc* a coil of fine wire. The ends of this wire are attached to two binding posts at the top of the tube.

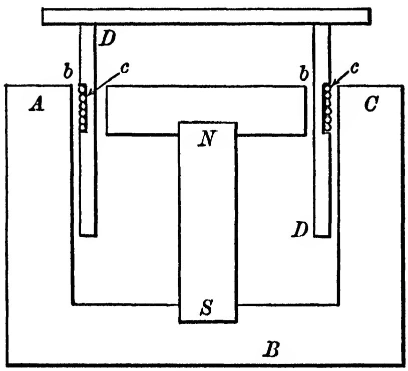

Fig. 97.

Let the magnetic flux across the air-gap be represented by ϕ. Then with S turns in the coil, the change in the number of linkings as the coil drops freely through the air-gap is given by the equation

$$(22) \qquad N = S\phi.$$

If R is the total resistance of the circuit which includes the galvanometer and the coil, then the charge induced is

$$(23) \qquad Q = \frac{N}{R} = Gd.$$

In order to eliminate G and determine N we may write, by equation (18),

$$Q = \frac{2\,Mi}{R} = GD, \tag{24}$$

where M is a mutual inductance used as in § 151. Dividing (23) by (24) and solving for N, we have

$$N = 2\,Mi\frac{d}{D}. \tag{25}$$

The total flux across the air-gap is found from the equation

$$\frac{N}{S} = \phi = \frac{2\,Mid}{SD}. \tag{26}$$

If A is the area of cross-section of the bar magnet NS, the flux density through it is

$$B = \frac{\phi}{A} = \frac{2\,Mid}{ASD}. \tag{27}$$

If M and i are measured in henrys and amperes, respectively, the equation (25) becomes

$$N = 2\,Mi\frac{d}{D}10^8, \tag{28}$$

and the same factor, 10^8, must be introduced in equation (26) and (27).

If the magnet has been properly made, and if the air-gap is small, the flux lines will be constant in number for a long time. The device then affords a rapid method of calibrating a ballistic galvanometer with a minimum of apparatus. If used in circuits like those of §§ 222 and 236, the drop coil should be left connected in the galvanometer circuit throughout the experiment in order to keep the circuit resistance constant.

153. Laboratory Exercise XXIX. *To determine the constants of a magneto-inductor.*

Apparatus. Magneto-inductor and apparatus as in Laboratory Experiment XXVIII, § 151.

Procedure. (1) Arrange the circuit as in Fig. 93. Connect the coil of the magneto-inductor in series with the ballistic galvanometer and the secondary of the mutual inductance. Drop the coil and observe the throw d. Repeat several times, using different values of S if possible.

(2) With the coil fixed in position, reverse a suitable current through P, and observe the corresponding throw D. Repeat with several different values of the current strength.

(3) From these data calculate the value of N, using equation (28). Also find values of ϕ and B, and state the units in which all the quantities are expressed. Make a study of the apparatus in order to determine whether the drop coil is cutting all the lines of force due to the magnet.

EXERCISES

1. Given a magneto-inductor together with its constants, and a ballistic galvanometer. Show how the ballistic constant of the galvanometer is determined.

2. Show fully from Lenz's law that the charge induced in the secondary circuit of a mutual inductance on reversing the current in the primary, is twice as great as for either make or break.

CHAPTER VIII

THE MEASUREMENT OF CAPACITY

154. General Considerations. The measurement of capacity is an important part of the work of an electrical laboratory. It involves the calibration of standards, the comparison of laboratory or secondary standards, and measurements on cables, aërial or submarine, and telegraph, telephone, and power lines.

Capacity is of special importance when present in alternating current circuits. For an alternate charge and discharge at low frequencies, such as those used in commercial light and power circuits, capacity measurements agree with those made by single charge and discharge.

Under the action of high frequencies where the charges and discharges alternate very rapidly, from a few thousand per second upward, a given capacity will measure less than for a single prolonged charge, or for a comparatively slow charge. For the theory and methods of capacity measurement at high frequency the student should consult the textbooks and treatises on wireless telegraphy and high frequency circuits. Only methods which are applicable to low frequency or to static charge and discharge will be considered here.

A capacity may be measured by comparison with a standard capacity, by comparison of its stored charge with that discharged by a self or mutual inductance, and by absolute methods which are independent of previously measured standards.

Suppose two condensers of capacities C_1 and C_2 are charged respectively with potential differences V_1 and V_2, developing

charges Q_1 and Q_2. These relations may be expressed by the equations

(1) $$Q_1 = C_1 V_1$$

and

(2) $$Q_2 = C_2 V_2.$$

If the same charge is given to both condensers, which is always the case if they are connected in series, we have

(3) $$C_1 V_1 = C_2 V_2,$$

or

(4) $$\frac{C_1}{C_2} = \frac{V_2}{V_1}.$$

If the two condensers are charged with the same potential difference, however, we have

(5) $$V_1 = V_2;$$

dividing (1) by (2), we obtain the relation

(6) $$\frac{C_1}{C_2} = \frac{Q_1}{Q_2} = \frac{d_1}{d_2}.$$

The equations (5) and (6) give two fundamental relations by which capacities are generally compared. In equation (4) potential differences may be measured, or better, their ratio may be found. In equation (6) a ballistic galvanometer is required to measure the quantities.

155. Comparison of Capacities by Direct Deflection. Suppose C_1 (Fig. 98) represents the capacity of a standard condenser and C_2 is that of a condenser whose capacity is to be measured. Either condenser may be introduced separately into the circuit by means of the three-way key k. A discharge key, like that shown in Fig. 95, is placed at K, and serves to charge either condenser, and to discharge it through the ballistic galvanometer. With k across 1 and 3, and with K

pressed to point b, the charge Q_1 in condenser C_1 is given by the equation

$$Q_1 = C_1 V$$

where V is the potential difference of the battery. When k is raised to point a, this charge passes through the galvanometer and we have

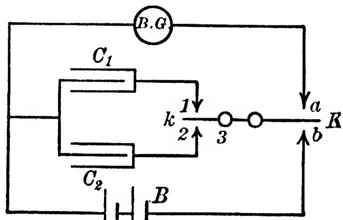

FIG. 98.

$$Q_1 = C_1 V = G d_1, \tag{7}$$

where G is the ballistic constant of the galvanometer. If k is now made to connect the points 2 and 3, and K is thrown in succession to b and a as before, the charge in C is given by the equation

$$Q_2 = C_2 V = G d_2. \tag{8}$$

Dividing (7) by (8), we obtain the relation

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2},$$

whence

$$C_2 = C_1 \frac{d_2}{d_1}. \tag{9}$$

In the method as just described, the galvanometer is in circuit with the condenser throughout the entire time of the deflection. During this time some part of the absorbed charge is certain to appear. The effect of this absorption will be to produce a current in the galvanometer coil during the entire time of the deflection. This tends to increase the deflection, and the longer the period of the galvanometer, the greater becomes the effect of the absorbed charge. This leads to a value of the capacity which increases with the period of the galvanometer, and hence it is a variable magnitude, which depends upon the experimental conditions.

Figure 99 shows the relation between the quantity discharged from a given condenser and the time. Assume that during the first 0.01 second practically all of the charge has passed, and that the absorbed charge does not all appear until some seconds later. If now, the condenser can be disconnected from the galvanometer immediately after the point n is reached, then the quantity sent through the coils will be independent of the absorbed charge, and hence of the period. The charge which passes before the point n is reached may be called the free charge, and the capacity computed therefrom may be called the free capacity. Standard condensers thus rated may be measured with a high degree of precision, provided the temperature coefficient of the condenser is known and the corresponding correction is applied. To compare the free capacities of two condensers, a discharge key is required which will open the galvanometer circuit within a few hundredths of a second after the battery is applied.

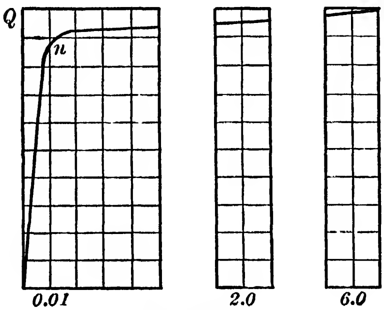

FIG. 99.

156. Laboratory Exercise XXX. *To compare capacities by the direct deflection method.*

APPARATUS. Standard condenser, the condenser to be measured, ballistic galvanometer, three-way key, discharge key, and one or more battery cells.

PROCEDURE. (1) Arrange the circuit as in Fig. 98. Set the three-way key on the standard condenser side, and charge the condenser. Then quickly discharge through the galvanometer and read the corresponding throw. This should be repeated several times. The mean of these readings may be called d_1.

(2) With the three-way switch on the unknown condenser

side, a similar set of readings will be taken. The mean of these may be called d_2.

(3) Calculate from equation (9) the value of the unknown capacity.

(4) The readings may be repeated with different values of V, thus bringing the deflection to different parts of the scale.

The chief objection to the method is that deflections must be observed. Zero methods are always preferable when possible.

In case widely different values of capacity have to be compared, it is well to vary the values of V so that the deflections may be kept approximately the same. This can be done by using a volt box in series with the battery, the charging potentials being taken from traveling contacts AB (Fig. 63).

Since the voltage between A and B is proportional to the resistance between these points, equation (9) may be written in the form

$$C_2 = C_1 \frac{r_1 d_2}{r_2 d_1}. \tag{10}$$

It is important that the insulation should be high. For this reason, all crossing of wires or contact of the insulated covering with the table or other pieces of apparatus should be avoided. The connecting wires should be run as air lines throughout. This precaution will be necessary in all work on capacity.

157. Laboratory Exercise XXXI. *To study leakage, absorption and residual charge.*

Apparatus. Condenser, ballistic galvanometer, discharge key, and one or more cells of battery.

Procedure. I. ***Leakage.*** (1) Arrange the circuit as in Fig. 96. Charge C by pressing k to b for an instant, and immediately discharge through the galvanometer, reading the deflection d_1. Again charge C, insulate by holding k on the middle position, and after 10 minutes discharge, read the throw d_2.

(2) Continue in this way, making the time for which the key is on the middle position successively longer until the discharge throw has reached about one tenth of its initial value.

(3) Plot a curve between values of time and throw. Since the throws are proportional to the charges, the curve may be used to calculate the percentage of the original charge which leaks away per minute. For a given interval, say the first one of ten minutes, the leakage is proportional to $d_1 - d_2$, while the percentage that leaks away per minute is given by the formula

$$\frac{d_1 - d_2}{d_1} \times \frac{100}{10}. \tag{11}$$

(4) If the leakage is small, as it is in a mica condenser, the time interval for which k is held on the middle point is taken longer. If the leakage is rapid, as it is in a paraffined-paper condenser, the interval may be a few seconds only.

II. ***Absorption.*** (1) With the circuit as in Fig. 96, and a mica condenser at C, charge and immediately discharge through the galvanometer, and read the throw.

(2) Charge again for ten seconds and discharge.

(3) Repeat with increased time of charging up to two or three minutes, recording the discharge throw after each charge.

(4) Repeat the same program with a paper condenser at C.

(5) Tabulate the results.

III. ***Residual Charge.*** (1) With the circuit as in Fig. 96, charge a mica condenser for three minutes, then discharge and note the throw. Bring the key promptly to the insulating position, being careful not to make contact with the battery terminal. To avoid doing so, the battery may be disconnected.

(2) After successive periods of one minute insulation, discharge the condenser and note the throws.

(3) Repeat the same program with a paper condenser at C.
Include in the report a discussion of the information which

hese experiments have yielded as to the qualities of the con-lensers used.

158. The Comparison of Capacity with the Wheatstone 3ridge. FIRST METHOD. Two resistances and the two con-lensers to be compared are ar-anged in series. Across the alter-iate junction points a galvanometer ınd a battery are connected, as shown n Fig. 100. When the three-way ːey K is raised to a, the condensers ıre charged, and when pressed to b hey are discharged. The resist-ınces R_1 and R_2 are so adjusted hat no deflection of the galvanom-ıter occurs when the key is ma-ıipulated. Let i_1 and i_2 be the cur-ents through R_1 and R_2 at any nstant, and let V_1 and V_2 be the corresponding potential lifferences across R_1 and R_2 respectively. Assuming that he condensers are perfect, the charges in each are given by he equations

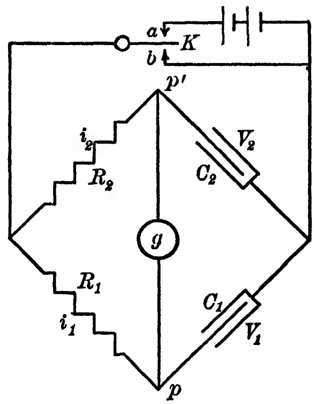

FIG. 100.

$$Q_1 = C_1 V_1, \qquad Q_2 = C_2 V_2.$$

Che condition for no deflection is that the potential is the ame at p and p'; hence we have

12) $$\frac{Q_1}{Q_2} = \frac{C_1}{C_2}.$$

f we let dt represent a short time interval, we may write

$$Q_1 = i_1 dt = \frac{V_1}{R_1} dt,$$

$$Q_2 = i_2 dt = \frac{V_2}{R_2} dt,$$

whence

$$\frac{Q_1}{Q_2} = \frac{R_2}{R_1}. \tag{13}$$

Combining equations (12) and (13) we have

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}. \tag{14}$$

It will be observed that the sequence in which the sides of the bridge enter in the formula differs from that in the corresponding formula for resistance comparisons.

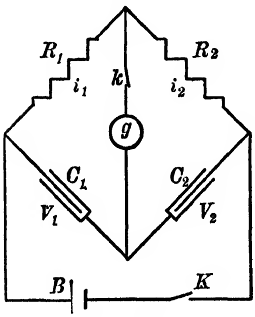

FIG. 101.

SECOND METHOD. With the condensers and resistances arranged in a closed circuit as before, the galvanometer and battery are connected as in Fig. 101, with tap keys at k and K. When K is closed the two condensers are charged in series, and the quantity given to each is the same. If V_1 and V_2 represent the charging potential differences, we may write

$$Q_1 = C_1 V_1, \qquad Q_2 = C_2 V_2,$$

whence

$$\frac{C_1}{C_2} = \frac{V_2}{V}. \tag{15}$$

Moreover, we have

$$V_2 = i_2 R_2, \qquad V_1 = i_1 R_1,$$

when no current flows through the galvanometer $i_1 = i_2$; hence we have

$$\frac{V_2}{V_1} = \frac{R_2}{{}_1R}. \tag{16}$$

Combining equations (15) and (16) we obtain the relation

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}. \tag{17}$$

Absorption and leakage tend to give values other than the true capacity, and this difficulty is exaggerated if the time of charge is prolonged. A uniform and quick tapping of the keys will tend to eliminate this effect. For long cables of large capacity the time of charging may be several seconds, but for ordinary laboratory condensers the keys should be tapped in quick succession. An advantage of the method is that any sensitive galvanometer, not necessarily a ballistic one, may be used.

For cable testing the circuit is shown in Fig. 102. In this case C_1 is a standard capacity and C_2 is the cable whose capacity is to be found. The cable will usually be coiled in a tank and earth connections will be made as at E. For rapid determinations in commercial testing a low frequency alternating current and a telephone receiver may replace the battery and galvanometer. The time constants of the arms R_1 and R_2 must be small compared with the period of the alternating current. The vibration galvanometer may also be used in place of the telephone receiver.

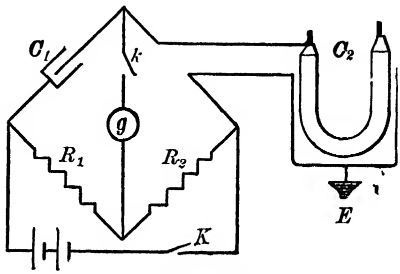

FIG. 102.

159. Laboratory Exercise XXXII. *To compare capacities with the Wheatstone bridge. First method.*

APPARATUS. Standard condenser, condenser to be measured, two resistance boxes of at least 2000 ohms range, discharge key, several dry cells, and a sensitive galvanometer.

PROCEDURE. (1) Arrange the circuit as in Fig. 100. Make R_1 small, say 10 ohms, and make R_2 as large as the range of the box will permit. Raise the key to a, then discharge and note the throw.

(2) Proceed further as in (2), (3) and (4) of Laboratory Exercise XXXIII, § 160.

The first throw of the galvanometer is often followed by a deflection in the opposite direction. This is due to absorption and may be disregarded, attention being given to so balancing the resistances that the first throw is made equal to zero.

160. Laboratory Exercise XXXIII. *To compare capacities with the Wheatstone bridge. Second method.*

APPARATUS. Standard condenser, condenser to be tested, two resistance boxes of at least 2000 ohms range, two tap keys, several battery cells, and a sensitive galvanometer.

PROCEDURE. (1) Arrange the circuit as in Fig. 101. Make R_1 small and make R_2 as large as the range of the box will permit. Tap k several times to insure the complete discharge of the condensers, then tap K with k down, and note the deflection.

(2) Make R_2 small and R_1 large, and repeat the procedure. Note the deflection, which will probably be in the direction opposite to the first one. If it is not, the range of the resistance must be increased or a standard condenser of different value must be selected. In case the two deflections are in opposite directions, proceed to adjust R_1 and R_2 until no deflection is observed. Estimate and record the least deflection which could be observed on the scale. Increase R_2 until this least observable deflection appears, also decrease R_2 until the deflection in the opposite direction is just perceptible. Record these values of R_2, also their mean, which gives the most probable value.

(3) Repeat the readings with different values of R_1 and R_2, keeping them both as large as possible throughout. The precision of the method is greater when C_1 and C_2 are nearly equal, and when R_1 and R_2 are high.

(4) Calculate the value of the unknown from equation (17).

This method, being a zero method, is superior to the direct deflection method. High insulation of all parts is necessary. Good results can be obtained only when the condensers compared have dielectrics of the same kind, for if there is a difference in the absorption of the two condensers it is difficult to secure a balance. The effects of absorption may be rendered less troublesome by making the time of charge very short and by discharging promptly.

161. Laboratory Exercise XXXIV. *To compare capacities with the Wheatstone bridge and vibration galvanometer.*

APPARATUS. Condensers to be compared, Kohlrausch bridge, and vibration galvanometer.

The Kohlrausch bridge was described in § 73. The vibration galvanometer is described in § 171.

PROCEDURE. (1) Arrange the circuit as in Fig. 103, with the vibration galvanometer and source of alternating current replacing the galvanometer and battery of the preceding method. The resistance R_1 and R_2 will be the two segments of the long wire of the Kohlrausch bridge. One terminal of the vibration galvanometer is connected to the movable contact of the bridge, and the other is connected at the junction of the two condensers.

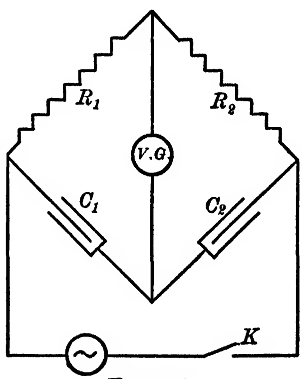

FIG. 103.

(2) Adjust R_1 and R_2 until the band of light is sharply defined and of minimum width. Vary the resistances slightly until the least observable broadening appears, and estimate the precision of the settings.

(3) Calculate the results by means of equation (17).

Uncompensated inductance in the bridge arms will prevent a balance, and the wire turns of the Kohlrausch bridge are not non-inductive. However, as the lengths of the segments change, the resistances change in equal ratio, and the time constants remain essentially equal. Hence, sharp settings are easily made.

162. The Comparison of Capacities by the Method of Mixtures. This method consists essentially in (*a*) charging two condensers in series by a potentiometer method, (*b*) so connecting them that the charges will mix, tending to neutralize one another, and (*c*) testing for the resultant or outstanding charge, if there is any.

In Fig. 104, C_1 is the condenser to be tested, C_2 is the condenser of known capacity, R_1 and R_2 are resistance boxes, B is a battery of several cells, g is a ballistic or other sensitive galvanometer, and S is a highly insulated double-pole, double-throw switch for the purpose of making the connections in the proper order. The switch is so arranged that when the handle is thrown to the right the contact at 5 is made an instant before that at 6. The wires that cross at D are connected. The point D is sometimes connected to ground.

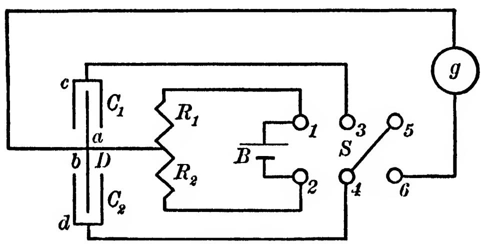

FIG. 104.

Whatever the position of the switch, a current i is always flowing through both resistances, and if the switch is thrown so as to connect 1 with 3 and 2 with 4, both condensers will be charged. The condenser C_1 is charged with the potential difference V_1 that exists across the terminals of R_1, and C_2 is charged with the potential difference V_2 that exists across the terminals of R_2. If the switch is thrown to the right, as soon as 3 is connected to 5 the charges in the two condensers will mix, for the plates a and b are already connected and the switch now makes connection between c and d.

If the charges in C_1 and C_2 are equal, the positive charge from c will exactly neutralize the negative charge from d, and the negative charge from a will neutralize the positive charge

from b. For this condition no charge will remain in the condensers.

But if the charges in C_1 and C_2 are not equal, both condensers will still be charged to some extent. An instant after the switch makes contact between 3 and 5 it connects 4 and 6, and when this occurs the condensers are both discharged through the galvanometer.

After the switch has been thrown to the left, if no deflection of the galvanometer occurs on rocking the switch to the right, we may conclude that when the switch was thrown to the left, the condensers were charged with equal quantities. In this case we have

$$Q_1 = Q_2, \quad C_1 V_1 = C_2 V_2.$$

Moreover, we know that $V_1 = iR_1$, and $V_2 = iR_2$, where i is the steady current flowing from the battery. We may then write

$$C_1 R_1 i = C_2 R_2 i,$$

whence

$$C_1 = \frac{R_2}{R_1} C_2. \tag{18}$$

If the capacity to be determined is that of a cable, the necessary time for charging may be a minute or even longer. For ordinary condensers a second or two is more than enough.

The particular advantages of the method are that it is less affected by absorption than the methods of the preceding articles, and that it is applicable over a wide range of values.

If absorption is troublesome, interchange the battery terminals and again adjust for a zero deflection. Use the mean of the two resistance settings. This effect will be reduced by a rapid rocking of the switch.

The method of mixtures is probably the most widely used of all comparison methods for the calibration of standards, precision comparisons, and practical cable testing.

163. Laboratory Exercise XXXV. *To compare capacities by the method of mixtures.*

Apparatus. Two resistance boxes, standard condenser and condenser to be measured, several dry cells, sensitive galvanometer, and special switch.

Procedure. (1) Arrange the circuit as in Fig. 104, being careful that all wires are air lines, not resting on the apparatus nor touching one another.

(2) In seeking the correct values of R_1 and R_2 for securing equal charges in the condensers, operate systematically as follows. Make R_1 small and R_2 as large as possible. Throw the switch to the left and then to the right and note the direction of the galvanometer deflection. Then make R_2 small and R_1 as large as possible and repeat the procedure. The deflections will probably be in opposite directions. If not, the capacities are probably too widely divergent for the resistance ranges used. This range must then be extended, or a standard capacity nearer to the unknown must be chosen.

If the deflections are opposite in direction it is obvious that the values of the resistances for zero deflections lie between the extreme limits, and these values are readily found.

Since the charge deflecting the galvanometer is a differential one it is obvious that the sensibility of the method increases with the E. M. F. of the battery used. The values of R_1 and R_2 should not be less than one or two thousand ohms.

(3) Estimate and record the least observable deflection that can be read on the galvanometer scale. When values of the resistances are found for the condition of zero deflection, keep R_1 constant and find what changes in R_2 will give the least perceptible deflection to the right and left, respectively. Substitute the mean of these values of R_2 in equation (18).

(4) Repeat with several different sets of values of R_1 and R_2. Tabulate all data and results, the probable precision of the readings, and the percentage accuracy of the results.

164. The Discharge of a Condenser through a High Resistance. If a condenser of capacity C is charged with an initial potential difference V_0, and then is allowed to discharge through a very high resistance R, the charge will gradually disappear and the potential difference between the plates will slowly sink to zero.

Let V be the instantaneous value of the potential difference at some time t seconds after the initial charge. The quantity in the condenser at this instant is given by $Q = CV$, and the rate at which the charge is decreasing is given by the equation

$$-\frac{dQ}{dt} = -C\frac{dV}{dt}. \tag{19}$$

The instantaneous value of the decreasing current may be written in the form

$$-\frac{dQ}{dt} = \frac{V}{R}; \tag{20}$$

hence, combining (19) and (20), we have

$$\frac{V}{R} = -C\frac{dV}{dt}. \tag{21}$$

Separating the variables, we have

$$\frac{dV}{V} = -\frac{dt}{CR}. \tag{22}$$

Integrating both sides of this equation, and remembering that when $t = 0$, $V = V_0$, we find

$$\log_e V = -\frac{t}{CR} + \log_e V_0.$$

Solving this equation for R, we find

$$R = \frac{t}{C \log_e \frac{V_0}{V}}. \tag{23}$$

Since galvanometer throws are proportional to charges, and since these are proportional to potential differences, the ratio d_0/d may replace V_0/V in equation (23). Again, the potential differences V_0 and V may be measured directly by means of an electrostatic voltmeter or an electrometer, and the ratio V_0/V may be calculated from these measurements.

This method is used where the resistance to be measured is so high that the direct deflection method of § 69 cannot be used. At best, the method is subject to many errors, and the results of different tests on the same sample are frequently not in good agreement.

It is known that for most insulators the leakage current does not follow Ohm's law. Moreover, this variation is a function of the impressed voltage. In a cable, the resistance of the dielectric substance changes with the temperature, decreasing as the temperature rises.

The impressed potential difference seems to produce certain molecular changes in the material of the nature of polarization, so that the resistance of the insulation increases steadily after the potential difference is applied. This phenomenon is called *electrification*. It is customary to specify a certain time, usually one minute, as the time of electrification.

Again, if the cable is immersed in water, the time for which it has been immersed must be specified. To say that a cable has an insulation resistance of 100 megohms per mile is only significant when the conditions of the test are carefully stated.

The value of an insulation resistance must then be accompanied by a statement of

(*a*) the temperature,
(*b*) the time of electrification,
(*c*) the time of immersion,
(*d*) the impressed potential difference.

The dielectric strength also is usually specified.

165. Laboratory Exercise XXXVI. *To measure a high resistance by the loss of charge method.*

APPARATUS. Sample of cable in tank of water, ballistic galvanometer, high potential battery, discharge key, and standard condenser.

PROCEDURE. (1) Arrange the circuit as in Fig. 105. The sample cable of capacity C is represented by AB.

If the cable is lead covered the contact c may be made directly to the sheath, B being connected to the core. Otherwise, a tank of water with connection shown at c is required. Using a suitable voltage, calibrate the galvanometer with the standard condenser, then charge and discharge C and compute its capacity from equation (9).

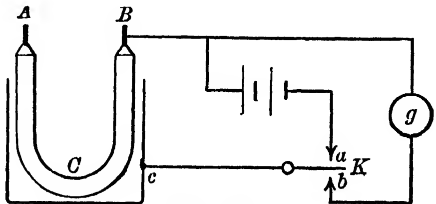

FIG. 105.

(2) Press K to a for one minute, then quickly press to b, and read the throw d_0 on the galvanometer. Keep K on b for a minute or two until C is fully discharged, then charge for one minute and set the key on the insulating position for a few seconds. Then press K to b, and read the throw d. Select the time for insulating such that d is approximately half d_0.

(3) Calculate the value of R from equation (23). The length of the sample will be measured and the result reduced to megohms per mile.

In case only a short sample is available, the method should be modified as follows. Across the terminals of the sample AB connect in parallel a condenser of known capacity C'. Assume the capacity of the small piece of cable to be negligible and use C' as the capacity of the system. It will be necessary first to determine the insulation resistance of C', which may be called R'. Then, by the method described above, find R'', the joint resistance of C and C' in parallel. From the formula for resistances in parallel, calculate the insulation resistance R of C alone.

CHAPTER IX

THE MEASUREMENT OF SELF AND MUTUAL INDUCTANCE

PART I. SELF-INDUCTANCE

166. General Methods. Three methods used for the measurement of self-inductance are:

I. ***Absolute methods***, in which the value is found independently of any previously measured inductance.

II. ***Capacity-comparison methods***, in which the unknown inductance is measured in terms of a standard capacity.

III. ***Inductance-comparison methods***, in which the unknown inductance is measured by direct comparison with a standard.

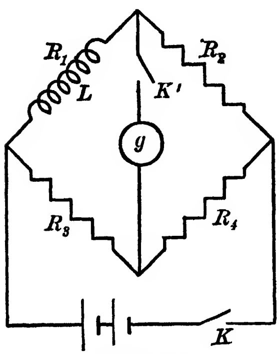

FIG. 106.

Two representative methods under I and II will be described, the formulas being given without proof.

In the ***Maxwell-Rayleigh method***, the inductance is measured in terms of a resistance and a time. The inductance L which is to be measured is put in one arm of a Wheatstone bridge circuit (Fig. 106), the other arms being non-inductive. The bridge is balanced for a steady current in the usual way. The key K is first closed, then K' is tapped, and the resistances are adjusted until there is no deflection. When a balance has been obtained, K' is closed, and the deflection d_1, which occurs when K is tapped, is read.

The steady current balance is then disturbed by altering R_1 by some small amount r, and the steady deflection d_2, which occurs when both keys are closed, is read. If the steady currents through R_1 and R_3 are denoted by i_1 and i_3, respectively, if T is the period of the moving needle of the galvanometer, and if it is assumed that there is no damping, the value of L is given by the formula

$$(1) \qquad L = rT \frac{i_1 \sin \frac{d_1}{2}}{i_3 \sin \frac{d_2}{2}}.$$

The ratio of the currents may be computed from the known resistances in the circuit and the E. M. F. of the battery used.

The value of L is thus seen to be given in terms of pure numbers together with a resistance and a time. Since resistance has dimensions $[LT^{-1}]$, the dimensional formula for inductance becomes $[L]$, whence the absolute unit is called the centimeter. The practical unit, which is 10^9 centimeters, is called the ***henry***. Formerly it was called the *secohm* because it could be expressed in terms of time and resistance, as in equation (1).

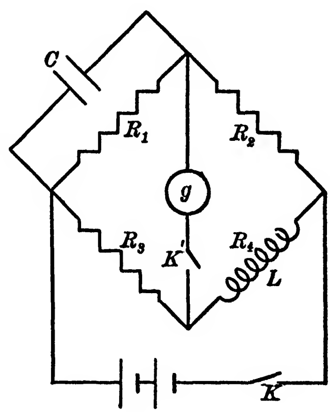

Fig. 107.

In ***Maxwell's method*** of determining an inductance in terms of a capacity, a Wheatstone bridge circuit is arranged with three non-inductive arms, and with the inductance L in the fourth arm, as shown in Fig. 107. The bridge is balanced in the usual way for steady currents, pressing K' only after K has been closed. The presence of the condenser across the arm R_1 will have no effect on this balance. If K is now tapped after K' has been closed, there will be a

deflection on the galvanometer, and this may be made zero by a new adjustment of R_1, which changes the charging potential across C. When the galvanometer does not deflect when either key is tapped first, it can be shown that L is given by the formula

(2) $$L = CR_4R_1.$$

This formula is very simple, but the double adjustment is extremely tedious, inasmuch as the change in R_1 upsets the previous balance obtained with a steady current. With a suitable resistance box provided with traveling plugs, the contact a may be moved to any position on R_1, thus varying the charging potential without annulling the bridge balance.

Another way of avoiding the necessity for the double adjustment is to use at C a condenser whose capacity may be varied either by steps or continuously. This form of condenser is not usually available. The best modification of the method is one that will be described fully in §§ 169, 170.

167. Inductance in Terms of a Capacity. Consider a circuit arranged as in Fig. 108, in which *aa* is an electromagnet of large inductance in series with a source of current D. In parallel with the inductance is a lamp p.

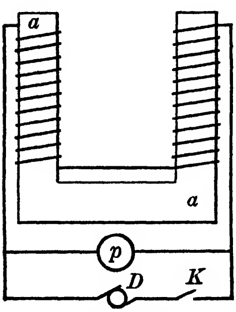

Fig. 108.

Suppose the current to be adjusted so that when K is closed the lamp glows a dull red. Then on opening the switch K the brightness of the lamp filament is first suddenly and very greatly increased for an instant, and then quickly diminished. The sudden rise of current through the lamp is due to the energy stored in the magnetic field, which is returned to the circuit when K is opened. The inductance develops a potential difference at the terminals which is greater than that originally impressed, as shown in § 136.

If a galvanometer or other measuring instrument could be inserted at p, the flow of charge could be measured and the value of the inductance could be calculated. However, any such instrument at p would be given a steady deflection when K was closed; hence it would not be in a condition to receive the charge when K was opened.

It is necessary, therefore, to devise some method whereby the galvanometer can be in its zero position until the instant that K is opened. This is accomplished in a simple manner by making the inductance one arm of a Wheatstone bridge circuit (Fig. 109). With the bridge balanced for a steady current the galvanometer terminals ab will be at the same potential, hence there will be no deflection. When K is opened the energy stored in the magnetic field about R is given back to the circuit, and the quantity of electricity thereby induced is discharged, partly through the galvanometer and partly through the rest of the circuit.

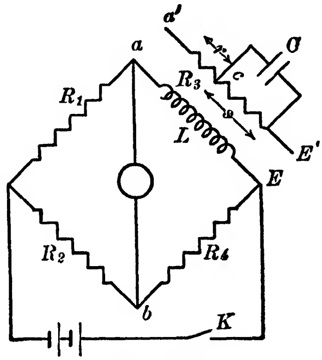

FIG. 109.

The value for L may be found as follows. With the bridge balanced for steady current, the galvanometer shows no deflection when K is kept closed. Assume a current of strength I flowing in the arm R_3. When K is opened this current drops to zero and the magnetic field about L collapses. The total change N in the number of linkings due to a current change from I to zero is, by equation (22), § 133,

$$N = LI. \tag{3}$$

Figure 110 represents exactly the same circuit as that shown in Fig. 109. A study of this diagram will show that the total

quantity of electricity discharged through the entire circuit is, by equation (76), § 142,

$$Q_1 = \frac{LI}{R_3 + R} \tag{4}$$

where R is the equivalent resistance around $aAbE$.

Suppose that the arm R_3 is now removed from the bridge and replaced by $a'E'$ (Fig. 110), in which the resistance $S + r$ is exactly equal to R_3, and around some part of which, S, is

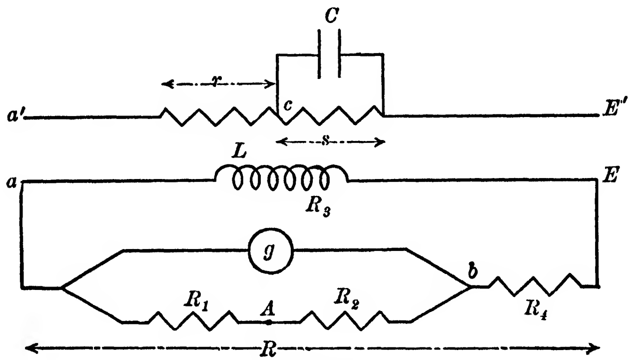

FIG. 110.

shunted a condenser of capacity C. When K is kept closed the same current as before flows through r and S, since the bridge balance has not been disturbed. The charging potential across C is then IS, and the quantity in the condenser is given by the equation

$$Q'_2 = CV = CIS. \tag{5}$$

If K is opened this quantity is discharged, part passing through S and part through all the rest of the circuit, or $R + r$. That part which passes through R is then given by the equation

$$Q_2 = CIS \frac{S}{(R + r) + S}. \tag{6}$$

When the quantities Q_1 and Q_2 are discharged through the circuit, the corresponding galvanometer deflections may be called

d_1 and d_2 respectively. However, these deflections are not caused by the entire charge, because the galvanometer is in parallel with other resistances, as shown in Fig. 110. Since these resistances are constant, the fraction of the total charge is the same in each case and the charges Q_1 and Q_2 are proportional, respectively, to the galvanometer throws. Dividing (4) by (6), we have

$$(7) \qquad \frac{Q_1}{Q_2} = \frac{\dfrac{LI}{R_3 + R}}{\dfrac{CIS^2}{R + r + S}} = \frac{d_1}{d_2};$$

and since

$$R_3 + R = R + r + S,$$

we may write (7) in the form

$$\frac{L}{CS^2} = \frac{d_1}{d_2},$$

or

$$(8) \qquad L = CS^2 \frac{d_1}{d_2}.$$

The shunted condenser may be put in series with the inductance L, and a balance secured by varying the charging potential across the condenser by means of the traveling contact at c (Fig. 109). This procedure has the advantage of being a zero method, and equation (8) then takes the form

$$(9) \qquad L = CS^2.$$

168. Laboratory Exercise XXXVII. *To determine an inductance in terms of a capacity.*

Apparatus. Box bridge and portable galvanometer, ballistic galvanometer, a few dry cells, special double-break key, standard condenser, resistance box with side plugs, and the inductance to be measured.

Procedure. (1) Arrange the circuit as in Fig. 109. Connect the inductance across the line terminals of the bridge box.

With the battery and portable galvanometer properly connected, balance the bridge to the nearest ohm, using a ratio of 100 to 100. Replace the portable galvanometer by the ballistic one, and connect the double-break key K' so that a short-circuit across the galvanometer terminals is broken an instant before the battery circuit is opened. With the bridge keys closed and the galvanometer quiet at zero, quickly press K' and read the deflection d_1.

(2) Replace the inductance by a non-inductive resistance of equal value, and shunt around all or part of it the capacity C. Let S be the value of this shunting resistance. Again close the bridge keys and quickly press K', reading the throw d_2, which should be in the opposite direction to d_1, and of approximately the same magnitude.

In order to insure a constant torsion of the suspension it is well to reverse the galvanometer terminals before taking d_2, so that both throws are read on the same side of the zero. If d_1 is much too large, inserting a non-inductive resistance in series with L will reduce it, in which case the bridge must be balanced again.

If d_2 is too small, it may be increased by increasing the value of either S or C. The total resistance of that arm of the bridge must, however, be kept the same when both d_1 and d_2 are observed.

(3) After a few trials to find the best working conditions, repeat both d_1 and d_2 several times and take the respective means. Calculate L from equation (8). Express the value in henrys and also in millihenrys.

Read again § 167, and note why the Wheatstone bridge circuit is used here. It is a difficult matter to maintain an accurate bridge balance, thus keeping the galvanometer precisely on zero without drifting. By the use of the double-break key, however, this precise balance is not necessary. The short-circuit keeps the potential at a and b the same, and the double-break key is so adjusted that the short-circuit is broken a very short time before the battery circuit is opened. This time interval should be made

so short that the galvanometer is not appreciably deflected meanwhile by any slight lack of balance which may exist. Obviously, after the battery circuit is broken, this lack of balance can have no further influence upon the galvanometer, which is then deflected solely by the quantity discharged through its coils.

169. Inductance in Terms of a Capacity. A Modification of Anderson's Method. In § 166 it was pointed out that the method due to Maxwell is tedious because of the annulling of the bridge balance when the charging potential across the condenser is changed. The following modification of ***Anderson's method*** is free from this objection.

Arrange the circuit as in Fig. 111. The inductance L is made one arm of the Wheatstone bridge and its resistance is found in the usual way, the bridge being balanced with the greatest care. The presence of S and C will not in any way affect the bridge balance for steady current.

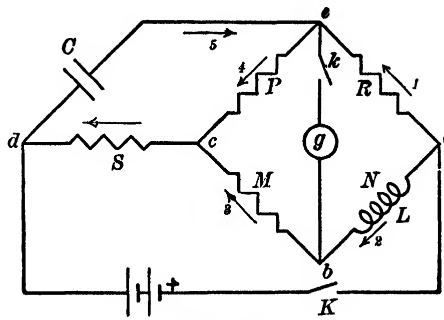

FIG. 111.

If, after the bridge is balanced, the key K is closed and k is tapped, there will be no deflection of the galvanometer. However, if k is closed and K is tapped, there will be a deflection. This deflection can be made to vanish by properly adjusting S and C. When these have been set so that the galvanometer shows no deflection for either order of tapping the keys, L may be calculated from equation (18), below.

The condition for the steady current balance is

$$(10) \qquad PN = RM.$$

The currents through ae, ab, bc, and ce may be represented

respectively by I_1, I_2, I_2, and I_3. The potential difference across bc equals MI_2; hence, for the balanced condition, we have

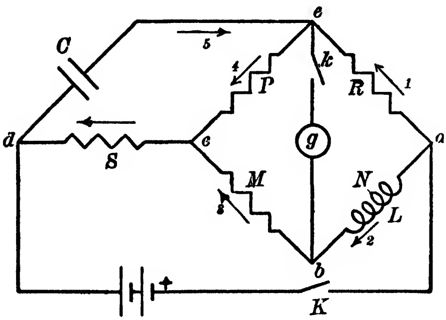

FIG. 111 (repeated).

(11) $PI_3 = MI_2$.

Also, for a balanced condition with variable current the potential difference across ae, which is equal to that across ab, is given by the Helmholtz equation,

(12) $RI_1 = NI_2 + L\dfrac{dI_2}{dt}$.

The potential difference V across the condenser terminals de is given by the equation

(13) $$V = S(I_2 + I_3) + PI_3 = SI_2 + SI_3 + PI_3.$$

The charge in the condenser is

(14) $$Q = CV = C[SI_2 + SI_3 + PI_3].$$

The instantaneous current through the condenser is given by the equation

$$i = I_1 - I_3 = \frac{dQ}{dt},$$

whence

(15) $$i = C\left[S\frac{dI_2}{dt} + S\frac{dI_3}{dt} + P\frac{dI_3}{dt}\right].$$

From (11) we have

$$P\frac{dI_3}{dt} = M\frac{dI_2}{dt};$$

whence (15) may be written in the form

(16) $$I_1 = I_3 + C\left[S\frac{dI_2}{dt} + \frac{SMdI_2}{Pdt} + M\frac{dI_2}{dt}\right].$$

If both sides of (16) are multiplied by R, the right-hand members of (12) and (16) may be equated, and we have

$$(17)\qquad NI_2 + L\frac{dI_2}{dt} = \frac{M}{P}RI_2 + CR\left[S + \frac{SM}{P} + M\right]\frac{dI_2}{dt}.$$

But, by (10), $N = MR/P$; hence, equating coefficients of dI_2/dt, we have

$$L = CR\left[S + \frac{SM}{P} + M\right],$$

or

$$(18)\qquad L = C[RM + RS + NS].$$

If S is made zero the equation reduces to $L = CRM$, which is the same as equation (2).

Referring again to Fig. 111, and assuming that the polarity of the battery is as marked, the steady currents through the bridge will be in the directions of arrows 1, 2, 3, and 4. When the key K is opened, the effect of the self-inductance L is to raise the potential at b. However, at the same time the condenser is discharging in the direction of arrow 5, which tends to raise the potential at e. Increasing S increases the charging potential across C, but at the same time decreases the current through L. Decreasing the resistance of S reverses these conditions. If S is small compared to P and M, and if the condition $L > CRM$ is fulfilled, then the potentials at e and b may be made equal and the galvanometer does not deflect.

170. Laboratory Exercise XXXVIII. *To determine an inductance by a modification of Anderson's method.*

Apparatus. Four resistance boxes, standard condenser, several dry cells, two tap keys, sensitive galvanometer, and inductance to be measured.

Procedure. (1) Arrange the circuit as in Fig. 111. With S equal to zero balance the bridge in the usual way, always tapping key k after K has been closed.

(2) Reverse the order of tapping the keys and adjust S and C until no deflection is observed. Some time will be required to find the correct value of S. The bridge balance may not be preserved due to temperature changes; hence, it is necessary to readjust the steady current balance. Several different values of the capacity should be used as well as different values of the ratio arms of the bridge.

(3) Calculate the value of L from equation (18) and indicate the probable precision of the various quantities, as well as of the value of L. The data and the calculated results should be tabulated.

It can be shown that the method is most sensitive when P and M are large and when S is small. If equation (18) is solved for S, we may write

$$S = \frac{\frac{L}{C} - RM}{R + N},$$

whence it is seen that L/C must be greater than RM in order to make S a positive quantity. Hence, the values of the various quantities must be chosen subject to the condition that L exceeds CRM.

The vibration galvanometer, § 171, and a source of low frequency alternating current, may replace the galvanometer and the battery with a considerable gain in precision.

171. The Vibration Galvanometer. In the ordinary moving coil galvanometer, the deflection is caused by the reaction of the magnetic field due to a steady current in the coil with the permanent field of the fixed magnet, and the deflection is proportional to the strength of the current.

If an alternating current is sent through the movable coil, the torque will reverse with each change in direction of the current and the resultant motion is essentially zero because of the large inertia of the suspended system.

In case the suspended coil is made very narrow and extremely light, with both the inertia and the damping small, it will follow the reversed torque due to the alternating current,

and a beam of light from a linear source reflected from a small mirror attached to the coil will be spread out into a light band on the scale.

This is the arrangement in the so-called ***vibration galvanometer.*** Its construction is quite like that of the ordinary d'Arsonval type, but differs from it in having a very narrow and light coil of fine wire hung from a bifilar suspension, which is capable of adjustment in both tension and length. These adjustments can vary the natural frequency of the suspended system through wide limits, usually from 50 to 1000 per second, and its sensibility will be high when the natural frequency is tuned to agree with that of the alternating current through its coil. In this condition of resonance a feeble current will cause a broad band of light on the scale, and for frequencies differing but little from this value the sensibility falls off rapidly.

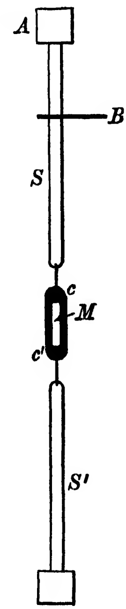

FIG. 112.

The vibration galvanometer is valuable also for zero methods with alternating currents, since harmonics in the current wave have little effect so that the conditions of a pure sine wave may be assumed.

Figure 112 shows the arrangement of the parts. The current is conducted to and from the coil cc' by the bifilar suspension S and S'. The attached concave mirror M forms an image of a linear source of light on the scale, and the width of the bright band on the scale is a measure of the amplitude of vibration of the coil.

The method of tuning the vibration galvanometer is similar to that of tuning a violin. The supporting head A can be raised or lowered by means of an adjusting screw, thus altering the tension of the suspension. A second adjusting screw slides the ivory fret B up or down along the wires, thus altering the effective length of the suspension. In this manner,

the natural frequency of the system is brought into resonance with the impressed alternating current. The sensibility of the vibration galvanometer is specified in terms of the current strength which causes one millimeter of broadening of the image of a linear bright source, with the scale at a distance of one meter from the mirror. A current strength of 10^{-6} ampere can be measured, and a current of 10^{-7} ampere can be detected.

172. Comparison of Self-Inductance. Vibration Galvanometer Method. The variable standard of self-inductance consists of two coils wound on spherical shells and connected in series, the same current passing through both coils. A common form is shown in Fig. 113. One of these coils lies within the other with the least possible clearance, and is capable of rotation about a vertical axis through an angle of 180 degrees, its position being indicated by a pointer which plays over a horizontal scale on top of the frame.

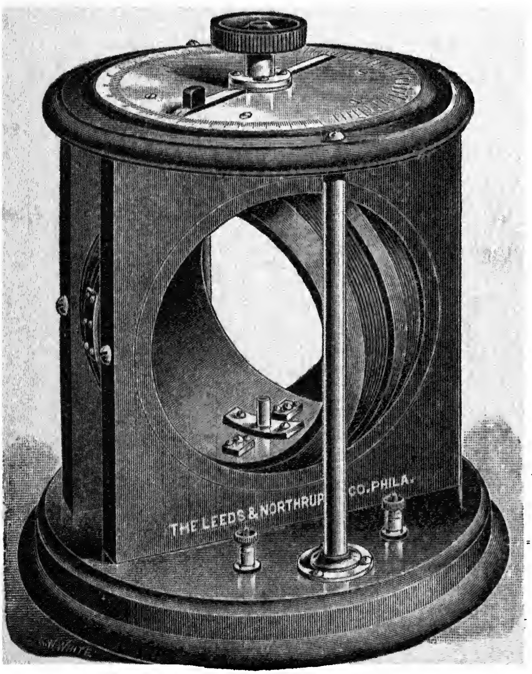

Fig. 113.

If the movable coil is so placed that its magnetic field reinforces the field of the fixed coil, the number of linkings, and

hence the self-inductance of the system, is a maximum. If, however, the movable coil is rotated 180 degrees from this position, its field will nearly or quite annul that of the fixed coil and the number of linkings and hence the inductance will be correspondingly reduced. For intermediate positions the pair of coils will have inductance values varying from nearly zero to the maximum, and these values can be read from the graduated scale. In the instrument here used this range is from 0.005 to 0.040 henry.

In the circuit shown in Fig. 114, L_1 is the standard variable inductance of resistance R_1, L_2 is the inductance to be meas-

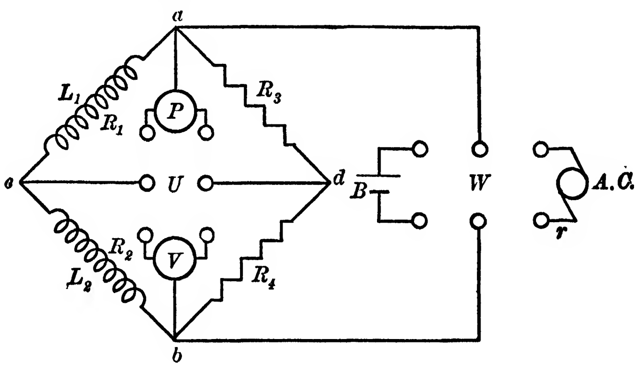

FIG. 114.

ured of resistance R_2, and R_3 and R_4 are non-inductive resistances. A double-pole double-throw switch W is arranged so as to connect either a dry cell B or a source of alternating current A. C. to the bridge at a and b.

Another double-pole double-throw switch U connects the other terminals of the bridge cd to an ordinary galvanometer P, or to the vibration galvanometer V.

With the switches set so that the dry cell and ordinary galvanometer are connected to the bridge a balance may be found in the usual way. Then

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \tag{19}$$

If both switches are thrown over, connecting the alternating current and the vibration galvanometer to the circuit, it will be found that the bridge is no longer in balance, as shown by the broad band of light on the scale. The balance can be re-

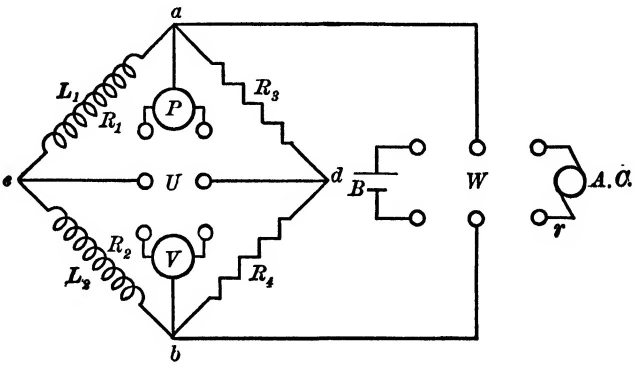

FIG. 114 (repeated).

stored, however, by rotating the movable coil of the variable inductance L_1 until the time constants of the arms *ac* and *bc* are equal. When this has been done we shall have

(20) $$\frac{L_1}{R_1} = \frac{L_2}{R_2},$$

or

(21) $$\frac{L_1}{L_2} = \frac{R_1}{R_2}.$$

Since the bridge balance was not disturbed by the rotation of the movable coil of L_1, equations (19) and (21) may be combined, whence

(22) $$\frac{L_1}{L_2} = \frac{R_3}{R_4},$$

and

(23) $$L_2 = L_1 \frac{R_4}{R_3}.$$

173. Laboratory Exercise XXXIX. *To measure a self-inductance by comparison with a variable standard. Vibration galvanometer method.*

Apparatus. Variable inductance standard, inductance to be measured, vibration galvanometer, portable galvanometer, two resistance boxes, two tap keys, two double-pole double-throw switches, and one or two dry cells.

Procedure. (1) Arrange the circuit as in Fig. 114. Balance the bridge in the usual way, using a direct current. Change to alternating current, and watch for the broad band of light on the scale to reduce to a sharp line as L_1 is rotated from one end of its range to the other. Set accurately on the position for which the vibration of the galvanometer coil disappears and read both ends of the pointer.

Record this position both in degrees and in henrys, and also note and record the range through which L_1 can be turned without causing an appreciable broadening of the line of light. Repeat several times, taking precaution to avoid being prejudiced by previous settings.

(2) Calculate the value of L_2 from equation (23) and express the result in henrys and in millihenrys. Indicate also the probable precision of the value found.

The vibration galvanometer will be adjusted by an instructor to resonance with the available alternating current circuit, and these adjustments should not be changed by the student. A fixed resistance will be permanently introduced between the alternating current terminals and the switch W. An adjustable resistance which is under the control of the student will be introduced at r. This should be set at a large value at first and only reduced as may be required.

In order to extend the range of the method, it is sometimes necessary to include in series with L_1 a box of standard inductance coils, which can be adjusted to several different values by means of plugs or switches. The range of the variable standard should be somewhat greater than the smallest value of this set of coils, thus providing a fine adjustment for the fractional parts of the lowest box value. These boxes of standard inductance coils are constructed so that different values may be obtained, and at the same time compensating non-inductive coils are automatically introduced so that the resistance remains constant.

174. Comparison of Self-Inductances. Secohmmeter Method. When a source of alternating current is not at hand, a direct current from any battery cell may be made available by means of the ***secohmmeter.*** This consists of two two-piece commutators mounted side by side upon the same shaft, each provided with four brushes set 90° apart, the shaft being rotated by an electric motor or by some other suitable mechanism.

Figure 115 shows the arrangement of the circuit. As the commutator B. C. to which the battery is connected is rotated, a pulsating reversed current is impressed at the bridge terminals *ab*. Since an ordinary galvanometer does not respond to these reversed pulses, the commutator G. C. is connected to the galvanometer and bridge terminals as shown. This is rotating with the same speed as B. C., the reversed current

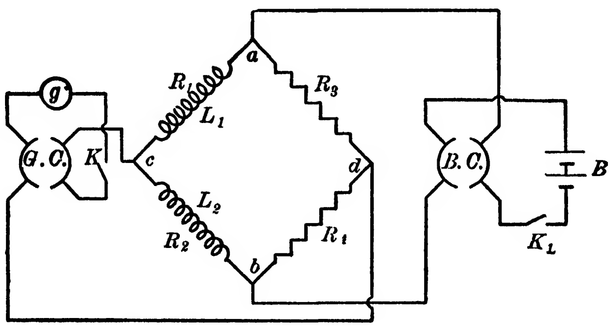

FIG. 115.

pulses are rendered unidirectional through the galvanometer, and any lack of balance of the bridge is at once indicated by a steady deflection. The experimental procedure is similar to that of Laboratory Exercise XXXIX, § 173, and the value of L is calculated from equation (23).

Part II. Mutual Inductance

175. Laboratory Exercise XL. *A study of mutual inductance.*

Apparatus. A rheostat, resistance box, one or two constant battery cells, milliammeter, two tap keys, ballistic galvanometer, and a pair of coils for testing.

Procedure. (1) Arrange the circuit as in Fig. 116, in which R is a control rheostat, R' is an adjustable resistance readable to tenths of an ohm, A is a milliammeter, and P and S are the two coils of which the mutual inductance is to be studied.

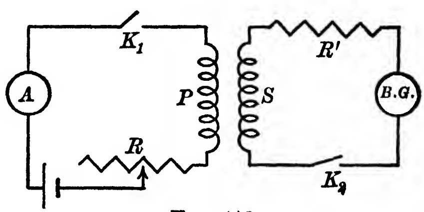

Fig. 116.

(2) Close K_2, keep R^1 constant, and vary R, observing for several values of the current strength the corresponding galvanometer deflections when K_1 is opened or closed. Tabulate the readings and plot a curve with current strengths as abscissas and deflections as ordinates. This curve will be a straight line through the origin, showing that the quantities induced in the secondary are directly proportional to the primary currents.

(3) Keep a constant value of the current through P and vary R', observing several deflections as K_1 is closed.

It is obvious that the damping in the secondary circuit will change as the resistance of the circuit changes. If a moving needle type of galvanometer is used, for which the damping is small, the correction for damping may be found and applied to the deflections.

If the moving coil galvanometer is used, the damping, which may be large, is nearly all due to induced currents, since the secondary circuit is kept closed. Changes in the resistance of the secondary circuit will then produce changes in the strength of these induced currents, and hence, also in the damping. It follows that the galvanometer throws will be affected by a variable damping, for which corrections are not easily made. (See § 147.)

In order to avoid this variable damping, it is necessary to replace the two tap keys by a double key which is so arranged that the secondary circuit is broken a very short time after the primary is closed, thus allowing the galvanometer to swing on open circuit with a constant damping. The time interval between the make and break should be very short as compared with the time of swing of the coil.

Using the double key just described, take several throws for different values of R'. Plot a curve with reciprocals of total secondary resistances as abscissas, and throws as ordinates. This should be a straight line passing through the origin, showing that the quantities induced are inversely proportional to the secondary circuit resistance.

(4) Keep the primary current constant and vary the distance between the coils, reading the galvanometer throws for several positions. Turn S through varying angles about a vertical axis, keeping the distances between centers of P and S constant, and for a constant current in P read throws on the galvanometer when K_1 is opened or closed.

(5) If possible, change the number of turns on P and on S and read the galvanometer throws as in (2). Compare these with the throws as in (2).

(6) Lay a piece of iron lengthwise through both coils, take current values and throws as in (2), and compare the throws.

(7) Set an iron plate between the coils and repeat (6).

It will be clear from the foregoing that the quantity induced in S is proportional directly to the primary current strength, inversely to the secondary resistance, and dependent also in some way upon the number of primary and secondary turns, the size, shape, and relative positions of the two coils and the permeability of the medium. For the particular pair of coils given suppose that all these factors except the first two are kept constant; then we have

$$Q = \frac{Mi}{R_s}, \tag{24}$$

where M is a general constant which contains these factors, and R_s is the total resistance of the secondary circuit. We may then write

$$M = \frac{QR_s}{i}. \tag{25}$$

We have learned, however, that

(26) $$Q = \frac{N}{R_s}$$

whence

(27) $$M = \frac{N}{i},$$

which is the definition for mutual inductance given in § 130.

It must be clearly understood that M is independent of the value of the current flowing, unless iron is present in the coils, and it is dependent solely upon the geometry of the circuit, that is, upon the dimensions, turns, and space relations of the coils. In the report state fully all the inferences to be drawn from each step of the experimental work.

176. Mutual Inductance in Terms of a Capacity. The Carey Foster Deflection Method. With a circuit arranged as in Fig. 117, the closing of the key K establishes a current of strength i through the primary coil P, and the quantity Q_1 induced in the secondary circuit is given by the equation

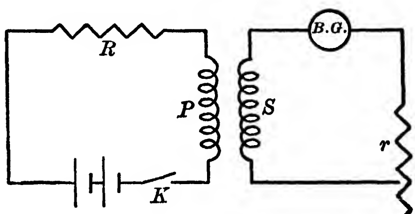

FIG. 117.

(28) $$Q_1 = \frac{Mi}{S + r + g} = Gd_1.$$

In this equation d_1 is the throw of the ballistic galvanometer, G is its constant, M is the mutual inductance of P and S, and s, r, and g are the respective resistances of the parts of the secondary circuit.

Let the circuit be changed to that represented in Fig. 118, where a condenser of capacity C is placed in series with the same galvanometer, and charged across the resistance R. When K is closed, the current i flows through R and the charging potential applied to the condenser is iR. Then the quantity given to the condenser is given by

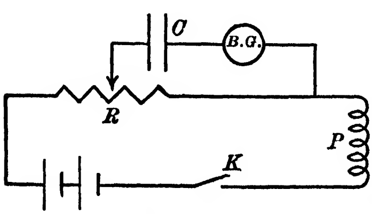

FIG. 118.

(29) $$Q_2 = CRi = Gd_2,$$

where d_2 is the galvanometer throw. Dividing (28) by (29), we have

$$\frac{Q_1}{Q_2} = \frac{M}{CR(s+r+g)} = \frac{d_1}{d_2}, \tag{30}$$

whence

$$M = CR(s+r+g)\frac{d_1}{d_2}, \tag{31}$$

It is seen from the figure that in the first case the galvanometer swings on a closed circuit, of which the resistance is, in general, not high. In the second case, however, the galvanometer swings with practically an infinite resistance in series with it. For a moving needle galvanometer the deflections d_1 and d_2 are replaced by the sines of the half-angles, respectively, and the corrections for damping must be applied. The value of M is then calculated from equation (31). For a moving coil galvanometer there is so great a variation in the damping in the two cases that it is necessary to use the zero method of § 177.

177. Mutual Inductance in Terms of a Capacity. The Carey Foster Zero Method. The circuits of Figs. 117 and 118 may be combined as shown in Fig. 119. This circuit may be considered as made up of three parts, the primary inductive circuit a, the capacity circuit b, and the secondary circuit c. When K is closed, the quantity given to the condenser is

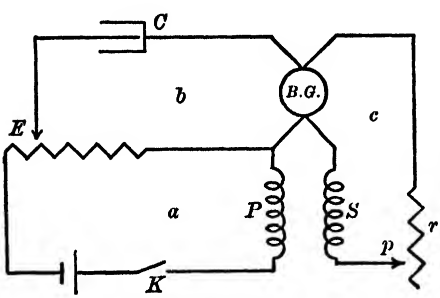

FIG. 119.

$$Q = CiR. \tag{32}$$

Since the galvanometer may be regarded as shunted by the resistance $(s+r)$, that part of Q which goes through the galvanometer is

$$Q_1 = CiR\frac{s+r}{s+r+g}. \tag{33}$$

Moreover, when K is closed, the mutual inductance sends through the galvanometer a charge whose amount is

$$Q_2 = \frac{Mi}{s+r+g}. \tag{34}$$

If adjustments of R, C, and r are made such that Q_1 and Q_2 are equal, it follows that we may write

$$CiR\frac{s+r}{s+r+g} = \frac{Mi}{s+r+g}, \tag{35}$$

which gives for the value of M,

$$M = Cr(s+r). \tag{36}$$

By the use of this equation and the method described above, the galvanometer deflections are always zero, the galvanometer resistance does not enter, and all error due to variable damping is eliminated.

178. Laboratory Exercise XLI. *To determine a mutual inductance in terms of a capacity. The Carey Foster zero method.*

Apparatus. Standard adjustable condenser, two resistance boxes, sensitive moving coil ballistic galvanometer, one or two constant battery cells, tap key and mutual inductance.

Procedure. (1) Set R (Fig. 119) at some random value, open the secondary circuit at some point say p, and tap K, noting the throw which occurs. This throw is due solely to the charge passing into the condenser. Close the circuit at p, break the condenser circuit at some point E, and note the throw when K is closed. This is due solely to the charge induced in S from P. These two throws must be in opposite directions. If they are not, interchange the terminals of P.

(2) With all connections as shown in Fig. 119, make r equal to zero, tap K, and note the throw. Make r infinite, that is,

open the circuit at p; again tap K and note the throw. If the two throws are in opposite directions, a value can be found for r for which no deflection will occur.

If the two deflections are not in opposite directions, then the condenser is sending more quantity through the circuit than is induced in S, and no value of r whatever can be found for which there will be no deflection. The quantity from the condenser must then be reduced, either by making C smaller or by diminishing the charging potential across its terminals. It is well not to change the value of R because that would alter the current through P. The charging potential is most conveniently varied by means of a traveling contact at E, which enables C to be charged across any desired fraction of R.

(3) Having found a combination of C, R and r for which the deflection is zero, record these values together with the least amount by which r can be increased or diminished before the smallest perceptible deflection occurs. Repeat the adjustment of r several times, with the attention fixed on the galvanometer and with no prejudice from previous settings. Approach the final value of r both from values that are too high and those that are too low. Find one or two other combinations of R, C, and r, and repeat the readings as outlined above.

(4) Calculate the value of M from equation (36), and state the probable precision of the result.

(5) State in the report the reasons for all the steps.

If the values of C, R, and r are given in absolute units, M will be in centimeters. If the capacity is in farads, and the resistances are in ohms, then M will be in henrys. If C is in microfarads, then

$$M = CR(s + r)10^{-6} \text{ henrys.} \tag{37}$$

179. Comparison of Two Mutual Inductances. Maxwell's Method. Two mutual inductances to be compared are arranged in a circuit as shown in Fig. 120. A constant current flows through P and P' when K is closed. In series with the

secondary coils are resistance boxes r and r'. When K is tapped, considering each side of the circuit separately, the charges induced are

(38) $$Q = \frac{M_2 i}{s + r + g}.$$

(39) $$Q' = \frac{M_1 i}{s' + r' + g}.$$

If we consider charges induced in both sides of the circuit at the same time, however, and if values of r and r' are so adjusted that $Q = Q'$, there will be no resultant charge through the galvanometer and hence no deflection.

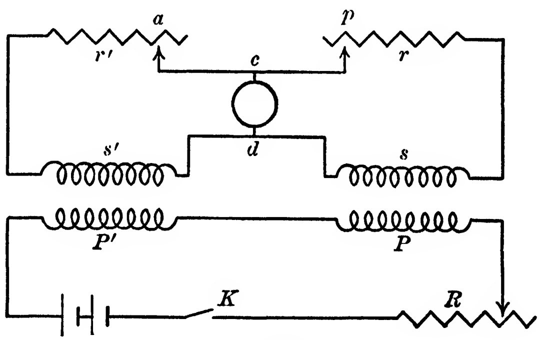

FIG. 120.

In general, there are two charges passing through the galvanometer, in opposite directions, and of different values. If the above condition is realized, so that $Q = Q'$, these charges exactly annul each other between c and d. This means that charge flows away from c on one side as fast as it is supplied on the other side, and the potentials at c and d remain constant. The galvanometer resistance then has no effect on the induced charges and g may be dropped from (38) and (39). Solving these equations for M_2, which may be taken as the unknown, we have

(40) $$M_2 = M_1 \frac{s + r}{s' + r'}.$$

The time constants of the two parts of the secondary circuit should be nearly or quite alike, and a long-period ballistic

galvanometer should be used, so that the time of throw is large as compared with the time constant of either side. Increased sensibility may be had by using an alternating current and a vibration galvanometer.

180. Laboratory Exercise XLII. *To compare two mutual inductances. Maxwell's method.*

APPARATUS. A standard mutual inductance and one to be measured, sensitive ballistic galvanometer, two resistance boxes, one or two constant battery cells, tap key, and box bridge.

PROCEDURE. (1) Arrange the circuit as in Fig. 120. Open the circuit at p and tap K, noting the throw, which is due to the charge induced in s'. Close p, break the circuit at a, tap K, and note the throw which is due to the charge induced in s only. These throws must be in opposite directions.

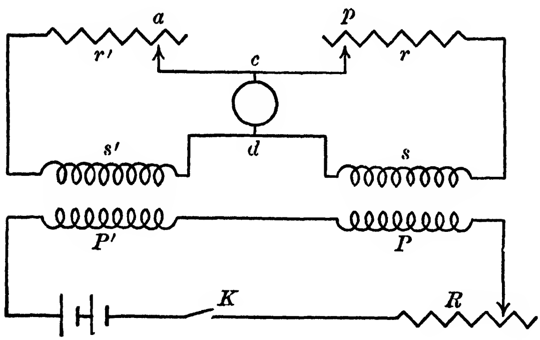

FIG. 120 (repeated).

(2) With connections as shown, adjust r and r' until no deflection occurs. Find the resistances of the coils s and s' with the box bridge.

(3) Take different values of the current through R and different settings of r and r'. Record in the data how much r or r' must be changed in order to produce the least perceptible deflection.

(4) Calculate the value of M from equation (40) and indicate the probable precision of the value found.

181. Laboratory Exercise XLIII. *Comparison of the values of M for a current inductor as determined by the Carey Foster method and by direct measurement.*

APPARATUS. As in § 178, together with a current inductor.

PROCEDURE. (1) Determine the value of M by the method of § 178. Take a large number of readings and use great care throughout.

(2) From the data furnished, calculate the value of M, using equation (17), § 131.

(3) Compare the results obtained in (1) and (2) and account for their variation.

182. Laboratory Exercise XLIV. *To determine a mutual inductance with the vibration galvanometer and a variable standard of self-inductance.*

APPARATUS. As in § 173.

PROCEDURE. (1) Arrange the circuit as in Fig. 114. Connect the two coils whose mutual inductance is to be found in helping series and call the self-inductance of this arrangement L_1. Determine its value by the method of § 173.

(2) Reverse the terminals of one coil, thus connecting them in opposing series, and call the self-inductance of this arrangement L_1'. Determine its value as before.

(3) Calculate the value of M from the relation

$$M = \frac{L_1 - L_1'}{4}$$

as given in equation (71), § 141.

CHAPTER X

MAGNETISM AND THE MAGNETIC CIRCUIT

183. Magnetism Produced by Electric Currents. That condition of matter called *magnetic* can be produced in certain subtances by the influence of a natural magnetic iron ore Fe_3O_4, or by the action of electric currents. It is highly probable that the magnetic bodies found in nature owe their peculiar and characteristic properties in some way to electric discharges. Hence, the conclusion may be drawn that the electric current is the primary source of magnetism.

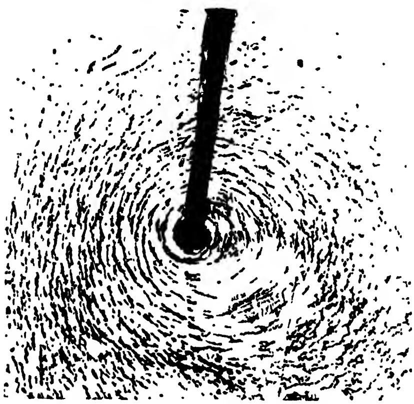

FIG. 121.

It was stated in § 128 that lines of force are to be thought of as closed curves, which may be established by the passage of a current through a circuit. The lines of force are interlinked with the circuit. Both the lines of force and the electric circuit are closed curves.

Figure 121 shows the appearance of the magnetic field about a straight wire carrying a current, the lines of force forming concentric circles about the wire. The field strength at any distance r from the wire is given by equation (8), § 105, and is

$$F = \frac{2i}{r}.$$

Figure 122 shows the field in a plane at right angles to the plane of a single circular coil, through the center of the coil. The strength of the field at the center is given by equation (14). § 106,

$$F = \frac{2\pi ni}{r}.$$

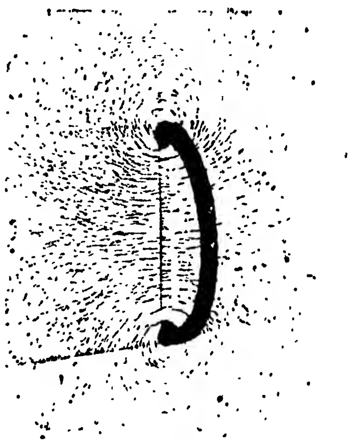

Fig. 122.

Figure 123 shows the magnetic field about a solenoid. The strength of field at the center of a long solenoid is given by equation (33), § 109.

$$F = 4\pi ni.$$

Not all substances are susceptible in the same degree to this magnetic influence. Indeed, most substances are less susceptible than dry air or a vacuum; to this class of substances has been given the name ***diamagnetic.***

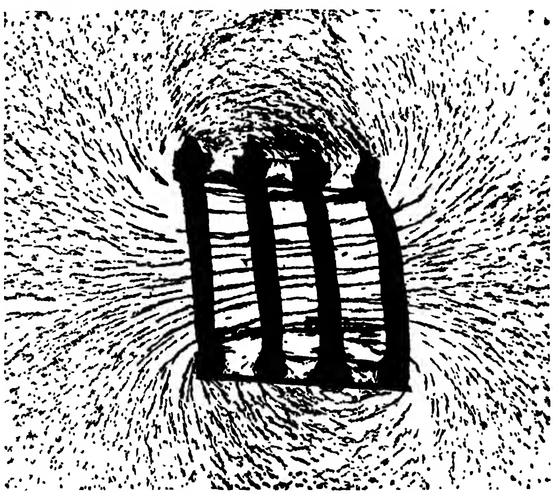

Fig. 123.

On the other hand, those substances which are *more* susceptible to magnetism than air are called ***paramagnetic.*** Three

substances, iron, nickel, and cobalt, which are chemically related, show this phenomenon in a degree much greater than any other. Since iron greatly exceeds the other elements in this respect, the name ***ferromagnetic*** is applied to the group, and this descriptive word is commonly understood even though shortened to ***magnetic.*** With the exception of certain alloys, briefly discussed later, the various forms of iron and steel are the only magnetic materials considered in the following pages.

184. The Magnetic Circuit. It has been shown in § 109 that for a short distance at the center of a long solenoid the magnetic field is very nearly uniform. If the solenoid is bent into the form of a circle, with the ends joined, the effect of the ends will vanish and a nearly uniform field will exist everywhere within the windings.

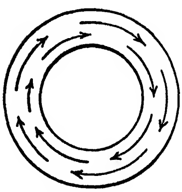

FIG. 124.

Consider a ring of iron of circular cross-section, in the form of the tore or anchor ring, uniformly overwound with wire turns through which a current flows. This constitutes an endless solenoid, containing an iron instead of an air core. Current through this toroidal winding will produce the uniform magnetic field described above, and will establish a magnetic state within the iron which is wholly confined to the iron, there being no external field.

FIG. 125.

Figure 124 represents such a ring with the magnetic lines all within the iron. If this ring, with such a

magnetic state existing within it, is placed on a horizontal surface and covered with a sheet of paper over which iron filings are sprinkled, there will appear no regularity in the arrangement of the particles, which shows that there is no external effect.

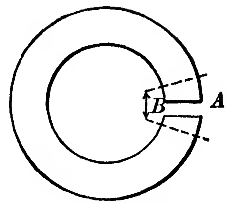

FIG. 126.

Figure 125 is a picture of the iron filings in such a case. Although there was within the ring a strong magnetic flux, no definite arrangement of the particles can be seen.

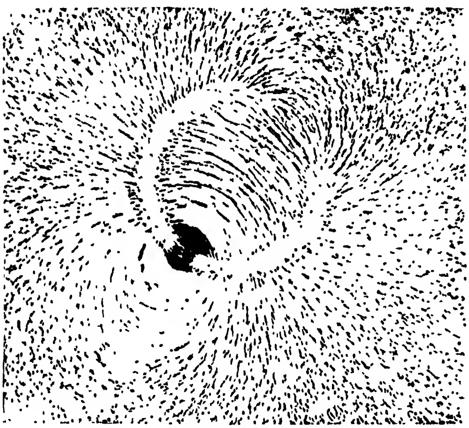

FIG. 127.

A marked change takes place, however, as soon as the magnetic circuit is interrupted by some non-magnetic substance, such as air. The same ring used for Fig. 125 was cut through on one side with a metal saw, making a gap one tenth of a millimeter wide, as shown at A, Fig. 126. A distinct regularity of arrangement of the particles is now evident, as shown in Fig. 127. If the air gap is increased to one centimeter, as shown in Fig. 126, B, a still greater effect is observed in the space surrounding the ring, as shown in Fig. 128.

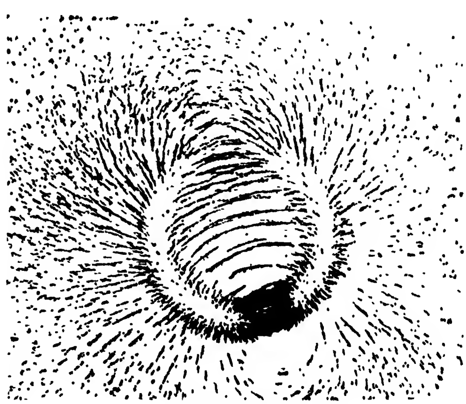

FIG. 128.

If the air gap is still further enlarged the piece takes the form of the ordinary horse-shoe magnet, and in this case the

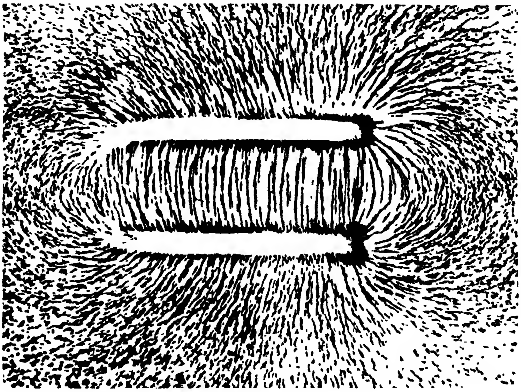

FIG. 129.

arrangement of the iron filings is shown in Fig. 129. This magnet is one taken from an ordinary telephone magneto. If

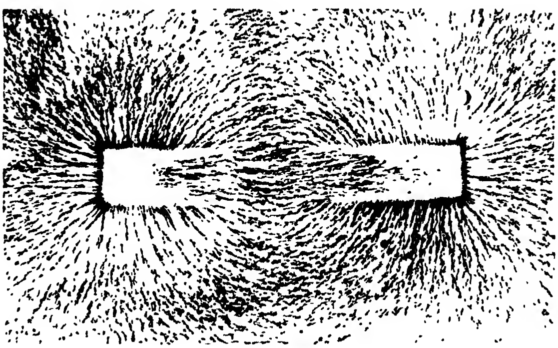

FIG. 130.

the ends are bent still further back, the iron particles group themselves as in Fig. 130.

Two conclusions may be drawn from a careful study of the preceding cases : (1) lines of force appear to be closed curves ; (2) magnetic circuits may be divided into two classes, *perfect* and *imperfect.*

185. Perfect and Imperfect Magnetic Circuits. The ***perfect magnetic circuit*** may be defined as one in which the magnetic material is continuous and homogeneous, and about which no external magnetic field exists, the magnetic flux being wholly confined to the material of which the circuit is composed. Such perfect magnetic circuits are rare. It is only approximately realized in dynamo and motor frames, Figs. 131 and 132.

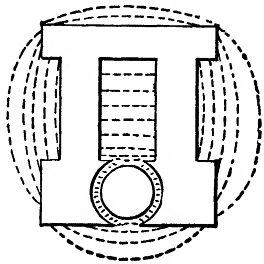

FIG. 131.

The ***imperfect magnetic circuit*** is one in which the body of the magnetic material is interrupted for a greater or less portion of its length by some non-magnetic material. It is characterized by the external field about it, in which a force is observed to act upon a magnetic pole placed in it. Most magnetic circuits are of this type.

This straying of the magnetic field outside of the material of the circuit is called ***magnetic leakage.*** This leakage varies from zero in the case of the ring to a maximum in the case of the short bar magnet. Leakage varies over the surface of a magnet, being greatest opposite two points near the ends of the bar, and least at the middle of the bar, as shown in Fig. 130.

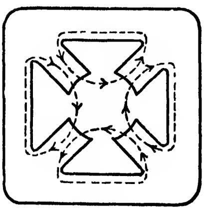

FIG. 132.

When a straight bar magnet is pivoted at its center, so that it is free to swing in a horizontal plane, it always sets its magnetic axis parallel to the direction of the field in which it is placed. This magnetic axis is usually its longest axis.

The behavior of the bar is now precisely that which it would be if its magnetism were concentrated in two points very near the ends of the bar. These two hypothetical points are called the ***poles*** of the magnet. However, it can be shown experimentally that this magnetism is not concen-

trated at certain points, but is distributed along the surface of the bar.

Magnetic polarity is developed wherever lines of force enter or leave the surface. The number of lines of force per square centimeter, that is, the surface density, is a measure of the distributed pole strength.

For a magnet floated on water or otherwise freely suspended, there is no motion of translation, simply rotation. Since the earth's field is uniform for the limited region about the magnet, this proves that the resultant turning moments acting on the two ends of the bar are equal and opposite.

A survey of the foregoing facts and phenomena leads to the fundamental principle that magnetism is a *circuital* phenomenon. Like electricity in motion, or an incompressible fluid flowing through a pipe, it cannot flow in one portion of the circuit only. Magnetic polarity is developed only when the material of the magnetic circuit abruptly changes its permeability.

186. Specification of Magnetic Quantities. There are two methods of expressing the values of magnetic quantities.

In the first method an imperfect magnetic circuit is assumed which has free poles and an external field, and the magnetic condition is specified in terms of the force action on a magnetic pole when placed in this field.

In the second method the magnetic state is specified in terms of analogies with the flow of fluids, or of electric currents.

Definitions of certain magnetic units by the first method are given in §§ 187–189. Definitions of other units by the second method are given in §§ 190–191. These units are compared in § 192.

187. Magnetic Field Strength. A magnetic field is completely specified by its action on the unit pole placed in it.

The ***direction*** of the field is given by the direction in which a free north-seeking pole will move. The ***intensity*** or ***strength*** of the field is given by the force in dynes which acts on a unit pole. Magnetic field strength is therefore expressed in *dynes per unit pole.* It is usually represented by the symbol H.

188. Magnetic Moment. It is convenient to think of the magnetic poles as being located at definite points within the magnetic substance. In reality, however, the entire body of the magnet possesses this polarity in a greater or less degree. If a bar magnet is hung in a horizontal position at right angles to a horizontal magnetic field of unit strength, it will tend to set itself parallel to the field, and the ***moment of the force couple*** which tends to rotate it is

(1) $$M = ml,$$

where m is the pole strength and l is the distance between the poles. The quantity M, which is called the ***magnetic moment*** of the bar, is really the sum of all the moments acting, considering that the distributed polarity has a different lever arm for every point along the bar.

189. Intensity of Magnetization and Susceptibility. If the pole strength of the magnet is divided by the area of the cross-section, the quotient gives the ***intensity of magnetization.*** It is represented by the symbol I, and its value is given by the equation

(2) $$\mathsf{I} = \frac{m}{a},$$

where m denotes the pole strength and a denotes the area of the cross-section.

If both numerator and denominator are multiplied by the length of the specimen, we have

(3) $$\mathsf{I} = \frac{ml}{al} = \frac{M}{V},$$

hence I may be defined also as the magnetic moment per unit of volume.

When a piece of iron is placed in a magnetic field of strength H, the inductive action of the field develops poles in the bar, and gives it an intensity of magnetization I.

The ratio of I to H is called the ***susceptibility***, and it is represented by the symbol k. We may then write

$$k = \frac{\mathsf{I}}{\mathsf{H}}. \tag{4}$$

190. Magnetic Flux. The arrangement of iron filings in a magnetic field suggests that the lines of force are continuous closed curves. It is often convenient to regard them as analogous to stream lines in a moving fluid. From this view point the expression *magnetic flux* has a definite meaning, and magnetic phenomena may be treated as *circuital* in type, the magnetic circuit having laws and properties analogous to those of the electric circuit. (See § 193.)

The magnetic flux is to be considered as distributed more or less uniformly throughout the entire magnetic circuit. This distribution is represented by lines drawn closer together where the field is strong, and farther apart where it is weak. Any definite portion of this field may be thought of as marked off from neighboring portions by the walls of an imaginary tube. By properly selecting the diameter of this tube we may arrive at a definition of the unit flux.

We will select a cross-section for this imaginary bundle of stream lines such that a conductor moving across it in one second will develop a potential difference of one absolute unit. This definite amount of magnetic flux is called the ***line of force***,[1] or the ***maxwell***. The maxwell is then defined as the

[1] The expression *line of force* has two quite distinct meanings. It is sometimes used to signify the direction of the field at a given point, and sometimes it is used, as in the case here cited, to mean the unit amount of magnetic flux,

magnetic flux which a single conductor must cut in one second in order to develop one absolute unit of potential difference.[1]

If the area over which this magnetic flux is distributed is enlarged, the conductor must move faster in order to cut the required amount in one second. This shows the necessity for another quantity, the ***flux density.*** The unit of flux density is called the ***gauss:*** a magnetic field is said to have a strength of one gauss when there is a uniform distribution of one maxwell of flux over an area of one square centimeter taken at right angles to the direction of the flux.

The total flux is usually represented by the symbol ϕ, and the flux density in air by the symbol H. For a uniform distribution over an area A we may then write

$$\phi = \mathsf{H}A. \tag{5}$$

191. Induction Density and Permeability. When a bar of iron is placed in a magnetic field the magnetic flux through it is very greatly increased, and is then expressed in terms of ***lines of induction***, the expression lines of force being restricted solely to the inducing field. The total flux of lines of induction in the bar is represented by ϕ, and the flux density of the induction by B, then

$$\phi = \mathsf{B}A, \tag{6}$$

where A is the cross-section of the iron bar. As in the case of air, the unit of induced magnetic flux is called the ***maxwell;*** and the unit of induction flux density, or one line of induction per square centimeter, is for practical purposes called the ***gauss.***

If B represents the induction density in the iron after being placed in a magnetizing field of strength H, the ratio of B to

[1] If 10^8 maxwells are cut by a single conductor in one second, the induced potential difference is one volt. There is no generally accepted name for this larger flux unit, although it is sometimes called the *practical line*, in contradistinction to the C. G. S. line or maxwell.

H is called the ***permeability***, and it is represented by the symbol μ. The permeability is given by the equation

$$\mu = \frac{B}{H}. \tag{7}$$

The general equation for magnetic flux may be written

$$\phi = \mu H A. \tag{8}$$

This reduces to equation (5) when the flux is through air, for which $\mu = 1$. When the flux is through iron it is identical with equation (6).

192. Comparison of Magnetic Quantities. When a sphere of unit radius is drawn about the unit pole as a center, there is at every point of the surface a unit field strength of one dyne per unit pole. The flux which exists through each square centimeter of surface is one maxwell. Since the area of the unit sphere is 4π square centimeters, 4π maxwells is the total flux from the unit pole.

If the unit pole is replaced by one of strength m units, the total flux is given by

$$\phi = 4\pi m. \tag{9}$$

In comparing the two methods of specifying magnetic quantities just given, it will be seen that the intensity of magnetization I is a measure of a condition which can be produced in magnetic materials alone. Magnetic flux ϕ is more general and is descriptive of a condition which can exist in any substance, whether magnetic or not. Every known substance can have magnetic flux established in it by a magnetizing field; hence an insulator for magnetism is unknown. The field strength H may be regarded as the cause of both I and ϕ.

193. The Law of the Magnetic Circuit. Magnetomotive Force. The analogy between the flow of a fluid through a pipe and electric current through a circuit was extended to

the magnetic circuit as early as 1871. The electromotive force has been defined in terms of the work done in moving a unit charge once around a complete circuit, and a similar expression for the work done in moving a unit pole once around a magnetic circuit along a line of force was called by Maxwell the *line integral* of the magnetic force. Bosanquet later called this the ***magnetomotive force***, and expressed the relations of the magnetic circuit in a formula similar to Ohm's law.

$$\text{(10)} \qquad \textit{magnetic flux} = \frac{\textit{magnetomotive force}}{\textit{reluctance}}.$$

The work done dw in moving a unit pole along any line of force is measured by the product of the length dL of the path by the component of the magnetic force along this path, or

$$\text{(11)} \qquad dw = \mathsf{H} \cos \theta \, dL$$

and the total work is found by integrating this expression around this entire path, whatever its form may be. This principle is applicable to any path whatever in the magnetic field. If the path coincides at every point with a line of force or a line of induction, however, θ is everywhere zero, and the line integral becomes simply

$$\text{(12)} \qquad W = \int \mathsf{H} dL = \mathsf{H} L.$$

We have already seen (§ 105) that the work done in moving the unit pole once around a single turn of wire carrying a current of strength i is $W = 4\pi i$. If there are N effective turns, the line integral or magnetomotive force M. M. F., becomes

$$\text{(13)} \qquad \text{M. M. F.} = 4\pi N i.$$

Moreover, by equation (34), § 109,

$$\mathsf{H} = 4\pi \frac{N}{L} i,$$

whence

(14) $$\text{M. M. F.} = \mathsf{H}L = 4\,\pi Ni.$$

If i is measured in amperes, we have

(15) $$\text{M. M. F.} = \tfrac{4}{10}\,\pi Ni.$$

The absolute unit of M. M. F. is called the ***gilbert***, and the practical unit is the ***ampere-turn***. A magnetomotive force expressed in gilberts is reduced to the equivalent number of ampere-turns by dividing by $4\,\pi/10$. Magnetizing field strength may be expressed, in terms of gilberts per centimeter or per inch, or in terms of ampere-turns per centimeter or per inch.

Suppose that a bar of iron of length L, placed within a solenoid, has an induction density of B established in it by the field of strength H. Neglecting the effect of the field outside the solenoid, we may write

$$\phi = \mathsf{B}A = \mu \mathsf{H} A = \frac{4\,\pi NiA\mu}{L},$$

or

(16) $$\phi = \frac{4\,\pi Ni}{\frac{L}{A}\cdot\frac{1}{\mu}} = \frac{\text{M. M. F.}}{R}.$$

The quantity R is called the ***reluctance*** of the circuit. (See § 194.)

The established flux is seen to be directly proportional to the M. M. F.; hence the magnetomotive force may be regarded as a measure of the effectiveness of the field in magnetizing the piece of iron. The general equation (16) presumes an initial state devoid of magnetism, but the flux established by any magnetomotive force is somewhat dependent upon any previous magnetic history of the circuit.

194. Reluctance. Reluctance is a property of the magnetic circuit which resists magnetization. It is inversely pro-

portional to the cross-section A, and directly proportional to the length L of the circuit. The factor ρ by which L/A must be multiplied to obtain the reluctance is called the ***specific reluctance*** or ***reluctivity*** of the substance. We may then write

$$R = \rho \frac{L}{A}. \tag{17}$$

The reciprocal of the reluctivity is called the ***permeability***, and is denoted by μ; hence we have

$$\rho = \frac{1}{\mu},$$

whence

$$R = \frac{L}{A} \cdot \frac{1}{\mu}. \tag{18}$$

This expression is seen to be the denominator of the middle term in equation (16).

The unit of reluctance is the ***oersted***, which is the reluctance of a circuit in which one gilbert establishes a flux of one maxwell. The oersted may also be defined as the reluctance of an air gap one centimeter long and one square centimeter in cross-section.

Reluctance and its reciprocal *permeance* are characteristics of the circuit. *Reluctivity* and *permeability* are characteristic of the given material.

Ohm's law shows that resistance is independent of current strength. The reluctance, however, varies with the magnetic flux, which in turn varies with the permeability, as shown in Fig. 147, so that the analogy between Ohm's law and the law for the magnetic circuit in equation (10) is not complete. Moreover, although energy is required to establish or reduce the magnetic flux through iron, no energy is required to maintain a continuous flux. There is, then, no analogy to Joule's law in the magnetic circuit.

The magnetic circuit may not be homogeneous, but may comprise various portions of different lengths, cross-sections,

and permeabilities, including one or more air gaps. Practical problems dealing with such circuits are solved by finding the magnetomotive force necessary to establish the desired flux in each part of the circuit. The total flux is then given by the equation

$$\phi = \frac{\Sigma \mathrm{M.M.F.}}{\dfrac{L_1}{A_1\mu_1} + \dfrac{L_2}{A_2\mu_2} + \cdots}. \tag{19}$$

Except in special apparatus for testing, it is seldom that a uniform winding can be placed over the entire circuit. More often the wire turns constitute what is called a local or bunched winding. The external field due to a bunched winding on an anchor ring is shown in Fig. 133.

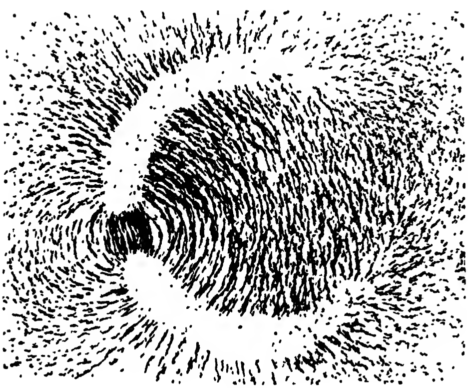

Fig. 133.

For a circuit such as that shown in Fig. 131, the M.M.F. of the windings is calculated for the required flux by equation (19), and corrections are applied for the leakage at the air gap. (See § 198.)

195. The Relation Between B and I. When a long, unmagnetized bar of iron of cross-section A is placed in a uniform field of strength H, for example at the center of a long solenoid carrying current, the total magnetic flux in the bar is made up of the sum of two distinct components (*a*) and (*b*): (*a*) through the space occupied by the bar there will be a magnetic flux due to the original magnetizing field of value HA; (*b*) when the bar of iron is placed in the field, magnetic

poles of strength m are induced, and by equation (9) the flux due to its own poles is $4\,\pi m$.

The total flux through the iron, or the induction, is therefore

$$\phi = \mathsf{H}A + 4\,\pi m,$$

and the induction density is

$$\mathsf{B} = \frac{\phi}{A} = \mathsf{H} + 4\,\pi \mathsf{I}. \tag{20}$$

The quantity B includes the effect of both I and H, while I includes only the effect of H, and not H itself.

196. The Relation Between k and μ. Substituting in equation (20) the values of B and I from equations (4) and (7), we may write

$$\mu \mathsf{H} = \mathsf{H} + 4\,\pi k \mathsf{H},$$

whence

$$\mu = 1 + 4\,\pi k. \tag{21}$$

For vacuum, dry air, and non-magnetic substances generally, $k = 0$ and $\mu = 1$. For paramagnetic substances k is positive and μ is greater than one. For diamagnetic substances k is negative and μ is less than one. However, there is no known substance for which μ is as much less than one as it is greater than one for ferromagnetic materials. For bismuth, which shows the largest susceptibility of any diamagnetic substance, $k = -14 \times 10^{-6}$, which corresponds to a permeability of 0.9998.

197. The Demagnetizing Effect of Poles. Imperfect magnetic circuits have a tendency to demagnetize themselves. The flux density within the bar is given by equation (20). The factor H is not constant, however, since the bar develops poles as soon as the magnetic state is induced in it. These poles act throughout the space occupied by the bar, and, hence, on the bar itself, and this *polar field* will have a direction opposite to that of the original field. Representing the original

field due to the solenoid by H_i, and the reversed polar field by H_p, the effective magnetizing field operative on the bar is

$$\mathsf{H} = \mathsf{H}_i - \mathsf{H}_p.$$

Hence, the induction density in the bar[1] is given by the equation

$$\mathsf{B} = (\mathsf{H}_i - \mathsf{H}_p) + 4\,\pi\mathsf{I}. \tag{22}$$

The resultant value of the magnetizing field strength will then be less than H_i; indeed it will vary from point to point along the bar. A numerical example will assist in making this clear.

Consider a bar 20 centimeters long and 2 square centimeters in cross-section placed in a magnetizing coil which is capable of producing a field of 50 gausses (Fig. 134). Let us assume

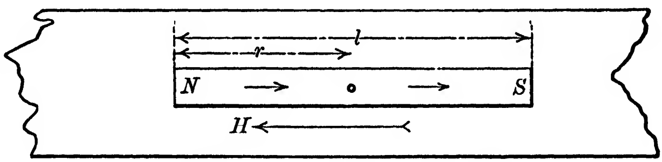

FIG. 134.

further that the intensity of magnetization of the iron, due to the field of the coil, is 1000 units. The pole strength is, from equation (2),

$$m = \mathsf{I}a = 2000.$$

If we imagine a minute longitudinal crevasse at the center of the bar 0, a unit north pole placed there will be acted on by two forces, equal in magnitude and having the same direction, which is opposite to that of H_i. One force is due to the repulsion of N and the other to the attraction of S. The sum of these magnetic forces at 0 is given by Coulomb's law,

$$F = \frac{m}{r^2} + \frac{m}{r^2} = 2\frac{2000}{100} = 40 \text{ dynes.}$$

[1] For a permanent bar magnet $\mathsf{H}_i = 0$, and $\mathsf{B} = 4\,\pi\mathsf{I} - \mathsf{H}_p$.

This force is acting in opposition to the field due to the solenoid, and the net field effective on the bar at its center is 10 units, which is 20 % of the original field. If the bar is taken twice as long, that is, 40 centimeters long, we shall have

$$F = 2\frac{2000}{400} = 10 \text{ dynes},$$

so that the net field strength at the middle of the bar is 40 units, which is 80 % of the original field. Again, if we consider a bar 200 centimeters long, we shall have

$$F = 2\frac{2000}{10000} = 0.4 \text{ unit},$$

so that the net field strength at 0 is 49.6 units, which is 99.2 % of the original value.

It is clear, therefore, that, in general, as the bar increases in length, the effect of the poles in decreasing the magnetizing field decreases; but the effect becomes negligible only for a very long bar. For a bar whose length is large as compared to its cross-section, it is clear that the magnetic induction is greater than for a short bar, if the permeability and the external field remain the same. The influence of the ends becomes negligible only when the ratio of length to diameter is large, say from 200 to 400. For such long rods the effective magnetizing force is practically the same as that of the original field.

Polar demagnetization is aided by vibration and resisted by *coercivity*. (See § 211.) It is strongest in the case of short bars. These will almost completely demagnetize themselves on withdrawal from the magnetizing field.

Short bars of iron used by themselves are not useful as test pieces. For long cylindrical or square bars, the effect of the ends can be determined[1] approximately and applied as a cor-

[1] Tables of demagnetizing factors for bars of various shapes and lengths are found in the larger works on magnetism, and in the journals.

rection factor. For ellipsoids of revolution such corrections can be precisely calculated.

Cylindrical bars sufficiently long to render the end effects negligible are neither conveniently made nor tested, and mechanical difficulties preclude the preparation of ellipsoidal specimens except in well equipped standardizing laboratories. Hence, it is necessary to secure the condition of *endlessness* either by using the specimen in the form of a ring, in which case there are no poles, or to approximate this condition by clamping the bar in massive yokes.

198. Magnetic Leakage. Iron offers an easier path to the flux lines than air, but in an imperfect magnetic circuit the flux lines do not all follow the iron. There is no insulator for magnetic flux and therefore it cannot be confined to the conductor, but the total flux is practically constant through every chosen cross-section of the conductor.

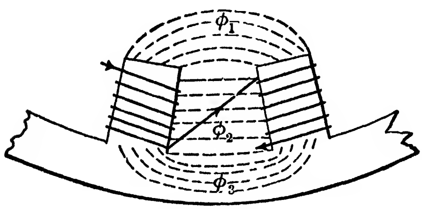

FIG. 135.

Consider two poles of a dynamo (Fig. 135), with a total flux ϕ_3. A part of this, ϕ_1, is in a position to be linked with the wire turns of the armature, and a part ϕ_2 is not useful. The ***leakage coefficient***, or leakage factor, is the ratio of the total to the useful flux, that is

$$k = \frac{\phi_3}{\phi_1}. \tag{23}$$

199. Laboratory Exercise XLV. *To study the magnetic leakage about a magnetic circuit.*

APPARATUS. Electromagnet with armature, or the field coils of a dynamo. Reversing switch, ammeter and source of current, search coils, and ballistic galvanometer.

PROCEDURE. (1) Pass current through the field coils of such value that the desired magnetic flux is assured. Place a search coil at *A* (Fig. 136), and connect its terminals to the ballistic galvanometer. Reverse the current and read the galvanometer throw. Record current, coil position, and deflection. Take several readings.

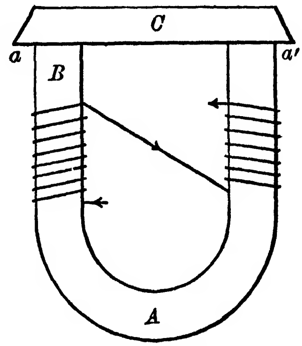

FIG. 136.

(2) Move the search coil to *B* and repeat (1).

(3) Again repeat (1) with the coil at *C*.

(4) Calculate values for the leakage coefficient for position *C*.

(5) If actual values of the flux are required, the ballistic galvanometer may be calibrated by the method given in § 151.

(6) Separate the armature *C* from the pole pieces by a single thickness of paper at *aa'*, and repeat (1). Insert at *aa'* calipered pieces of thin brass, and repeat (1), studying the change in *k* with length of air gap. For comparison of deflections the same value of the exciting current should be used throughout.

CHAPTER XI

THE EARTH'S MAGNETISM

200. The Magnetic Elements. Near the close of the sixteenth century the investigations of Gilbert established the magnetic nature of the earth. It is a weak magnet for which the value of the intensity of magnetization is about 0.08. The phenomena observed on the earth's surface are practically those which would be present if a bar magnet, short as compared to the earth's radius, were located within the earth, as shown in Fig. 137. The *geographic axis* is represented by $N'S'$, the equator by EE', and the *magnetic axis* by NS. The dotted lines show the direction of the field, which is the direction in which a north-seeking pole will point.

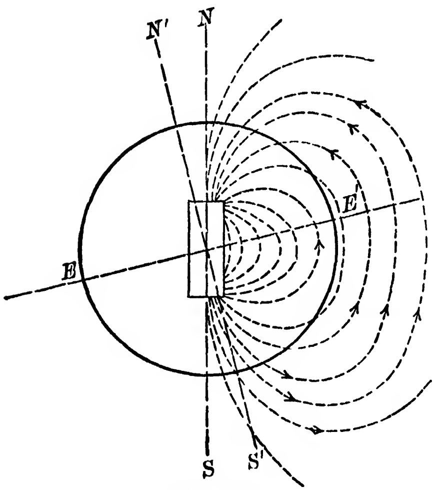

FIG. 137.

From the law of magnetic pole attraction we learn that unlike poles attract; hence, the magnetic pole of the earth which is geographically north is opposite in kind from the pole of the compass needle which seeks that direction. Confusion on this point will be avoided if it is recognized

that the north magnetic pole takes its name from being near the geographic north pole, while the north-seeking pole owes its name to the fact that it *points towards* the north. The lines of force of the earth's field must be considered as having a direction towards the north.

The *magnetic axis* departs about 15° from the direction of the axis of rotation of the earth. Its points of intersection with the earth's surface are called the *magnetic poles.* These positions will be found marked on any of the larger maps of the world. The north magnetic pole is approximately on the meridian through Omaha, and about 500 miles north of Hudson Bay, just above latitude 70° N. The south magnetic pole is on the meridian of Eastern Australia, about latitude 73° S.

A *magnetic meridian* is a vertical plane which passes through the magnetic axis of a freely suspended magnetic needle which is in equilibrium.

In order to specify precisely the earth's magnetic field at any point, three elements are usually given as follows:

(1) The ***inclination*** or ***angle of dip***, which is the angle between the magnetic axis of a freely suspended magnetic needle and a horizontal line through its center.

(2) The ***declination***, which is the angle between the magnetic and geographic meridians. It is the angle of departure of the needle from true north.

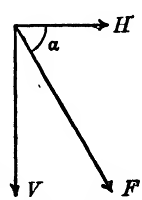

FIG. 138.

(3) The ***intensity*** of the field, which is the force in dynes which acts on a unit pole placed in the field.

In measuring the intensity it is usually most convenient to determine the horizontal component of the earth's field, represented by the symbol H, Fig. 138, and also the angle of dip α, whence the total force F is found from the relation

$$(1) \qquad H = F \cos \alpha.$$

The vertical component may be found from

(2) $$V = F \sin \alpha.$$

201. Magnetic Surveys. For many years accurate magnetic surveys have been made over all parts of the earth's surface by the scientific bureaus of various countries, and detailed records and charts have heen prepared which show the values of the magnetic elements and their variations for any given position. These studies show daily, annual, and eleven-year period variations, all of which are due to the sun; a possible variation due to the moon; secular or long-period variations of unknown cause, which extend over centuries; and certain irregular and occasional variations connected with magnetic storms and auroral displays.

At any given point, however, the earth's field is, for short periods, practically constant. Accurate determinations of the magnetic elements are of primary importance for the navigator and for the surveyor. Formerly they were also essential in the electric laboratory in connection with absolute measurements. The horizontal component of the earth's field can be determined easily in absolute measure, and it is a valuable secondary standard in such problems as current measurements with the tangent galvanometer, or ballistic galvanometer calibrations with the earth inductor.

At present, however, such methods are obsolete for the purpose of electric measurement on account of the general use of iron and steel in buildings, and on account of the stray fields from electric machinery and direct current distributing circuits. Nevertheless, a brief treatment of a few magnetic measurements will be given because of their intrinsic interest.

The student should read the chapters on the earth's magnetism in the larger textbooks of physics, and study carefully the significance of the lines drawn on the magnetic charts.

202. Laboratory Exercise XLVI. *To find the inclination with the dip circle.*

APPARATUS. Dip circle and accessories.

The ***dip circle*** (Fig. 139) consists essentially of a magnetic needle symmetrically mounted on a steel staff whose polished

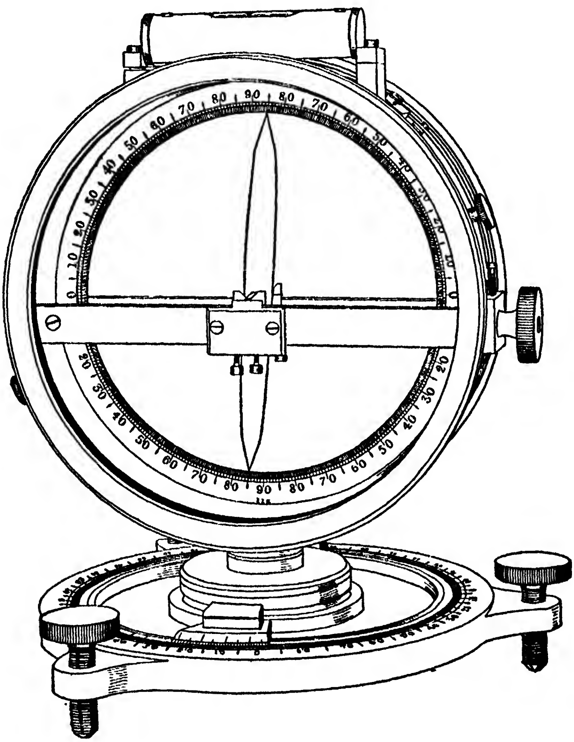

FIG. 139.

cylindrical ends roll on agate plates. If (*a*) the needle swings freely in a vertical plane coincident with the magnetic meridian, (*b*) its center of gravity lies exactly in the axis of rotation, and (*c*) its geometric axis coincides with its magnetic axis, then, if there is no friction, the needle will take a position with its axis strictly parallel to the lines of force of the

earth's field, and the angle between the axis and the horizontal as read on the vertical circle gives the dip.

Three sources of error must be taken into account when using the instrument.

1. The polished cylindrical ends of the supporting staff roll on the agate plates, which prevents perfect coincidence of the axis of rotation with the center of the vertical circle. The mean of readings taken at both ends of the needle will be free from this error.

2. The center of gravity of the needle may not coincide with the axis of rotation, in which case the observed angle will be increased or diminished according to the relative position of the center of gravity and the axis. The mean of readings taken with the polarity of the needle reversed will be free from this error.

3. The geometric and magnetic axes may not coincide. The mean of readings taken with the opposite ends of the supporting staff toward the observer will be free from this error.

Procedure. (1) Level the instrument carefully, so that for any position of the horizontal circle the bubble seeks the middle of the tube. Release the arrestment, thus lowering the needle on to the agate plates, and see that it swings freely.

(2) Rotate the instrument about a vertical axis until the needle assumes a vertical position. The needle now lies in a plane at right angles to the magnetic meridian. Rotate the instrument again in azimuth through 90° and clamp it. The needle now lies in the magnetic meridian.

(3) Read both ends of the needle. Just before taking the readings it is well to tap the base of the instrument gently with the finger in order to overcome any static friction between the staff and the agate plates.

(4) Rotate the instrument in azimuth through 180° and read both ends as before.

(5) Tabulate all data and compute the mean value of the angle of dip.

203. The Horizontal Component of the Earth's Field in Absolute Measure. This method involves the use of a mag-

netometer, which consists essentially of a magnetic needle suspended by a light fiber and capable of rotating freely in a horizontal plane. A small mirror is attached to the needle, and the deflections can be observed by the use of a telescope and scale. A bar magnet of moment M is selected. By means of magnetometer deflections and the equations derived below, values will be found for the product MH and for the quotient M/H, where H represents the horizontal component of the earth's field. The quantity M may be eliminated from these expressions, and the value of H may be found in terms of deflections and distances.

I. *To find the value of MH.*

We shall first develop a simple relation between the quantity MH and the periodic time of vibration of the suspended magnet. The magnet used will be a cylindrical bar about ten centimeters long and a few millimeters in diameter. This bar, represented by ns in Fig. 140, is hung in a suitable stirrup by

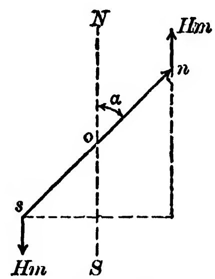

FIG. 140.

a nearly torsionless fiber attached at o. When it has been deflected through a small angle α and then released, it will vibrate in a horizontal plane until its energy is dissipated by friction against the air. The magnetic meridian is represented by NS, the pole strength of the suspended magnet by m, and the distance between the poles by l. When a pole of strength m is placed in a field of strength H, it will be acted on by

a force of Hm dynes. The magnet when deflected through some angle α, tends to regain its equilibrium position due to

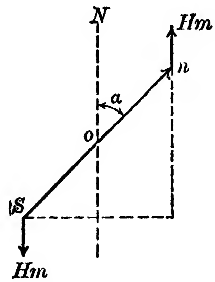

FIG. 140 (repeated).

the two forces acting on its two poles. The moment of this force couple, the so-called *restoring couple*, is given by

$$L = Hml \sin \alpha. \tag{3}$$

If it is assumed that α is so small that the sine does not differ sensibly from the angle expressed in radians,[1] then we have

$$L = Hml\alpha, \tag{4}$$

which may be written in the form

$$L = HM\alpha. \tag{5}$$

From (5) it is seen that the restoring moment, or torque, is directly proportional to the angular displacement; hence,

[1] For the sake of definite comparison, the following table is given:

ANGLE IN DEGREES	ANGLE IN RADIANS	SINE
1°	0.01745	0.017453
2	0.03490	0.034906
5	0.08720	0.087165
10	0.17360	0.17453

For an angle of 10°, the error due to the substitution of the angle for the sine is slightly more than one half of one per cent. The student should verify this and he should construct other examples from a trigonometric table.

the suspended magnet is vibrating with simple harmonic motion. The periodic time of such motion is given by the formula

(6) $$T = 2\pi\sqrt{\frac{K\alpha}{L}},$$

where K is the moment of inertia of the bar. Substituting in (6) the value of the torque from (5), we have

$$T = 2\pi\sqrt{\frac{K\alpha}{MH\alpha}},$$

whence

(7) $$T = 2\pi\sqrt{\frac{K}{MH}},$$

or

(8) $$MH = \frac{4\pi^2 K}{T^2}.$$

II. *To find the value of M/H.*

Two different arrangements of the deflecting magnet NS, with respect to the needle of the magnetometer O, will each give a value of M/H in terms of observed deflections. These positions are shown in Figs. 141 and 142, respectively. They

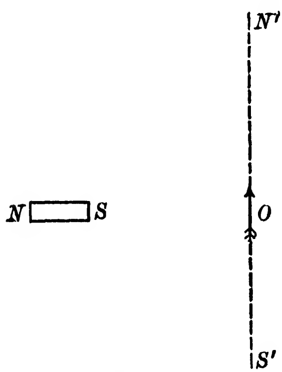

FIG. 141.

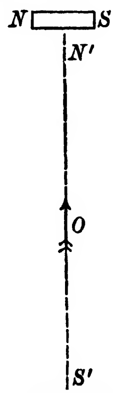

FIG. 142.

are known as the tangent position and the broadside position. The latter is the more sensitive and the theory will be developed for that case.

In Fig. 143, NS represents the deflecting magnet which has a pole strength m and an interpolar distance $2\,l$. The magnetometer needle is represented by ns, and it has a pole strength m' and a length l'. The deflecting magnet is placed with its axis at right angles to the magnetic meridian $N'S'$.

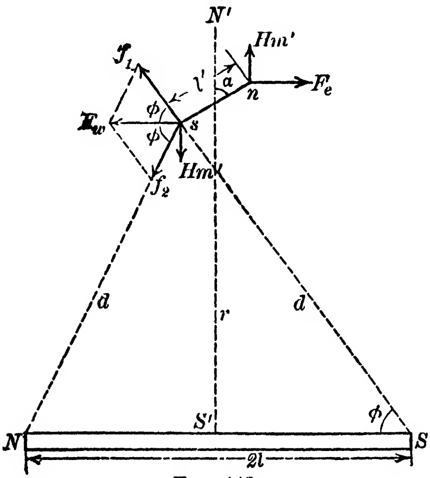

FIG. 143.

Let us assume that the length of ns is small as compared with the distance r. Then we may, without appreciable error, consider the triangle NsS as an isosceles triangle with an altitude r.

The needle ns, originally in the magnetic meridian, is deflected through an angle α by the presence of the deflecting magnet. The forces acting on s are given by the equations

$$f_1 = \frac{mm'}{d^2}, \quad f_2 = \frac{mm'}{d^2}.$$

The resultant of these two forces tends to deflect the south pole to the west, and its value is given by the equation

$$(9) \qquad F_\omega = 2\frac{mm'}{d^2}\cos\phi = \frac{2\,mm'l}{d^3},$$

since $\cos\phi = l/d$. The magnetic moment of the deflecting magnet is, by definition, $M = 2\,ml$; substituting this in (9) gives

$$(10) \qquad F_\omega = \frac{Mm'}{d^3}.$$

The moment of the deflecting couple acting on the needle is

$$F_\omega l' \cos\alpha = \frac{Mm'l'}{d^3}\cos\alpha.$$

The moment of the restoring couple due to the earth's field is $Hm'l' \sin \alpha$. Equating these moments, we have

$$\frac{Mm'l'}{d^3} \cos \alpha = Hm'l' \sin \alpha,$$

whence

$$H \tan \alpha = \frac{M}{d^3} = Md^{-3}. \tag{11}$$

The desired value of M/H could be found from equation (11) provided the magnetic length of NS were known. Since this is not easily determined, it is necessary to transform (11), and to arrange the experiment so that this length may be eliminated from the equations.

Substituting for d in equation (11) its value $(r^2 + l^2)^{\frac{1}{2}}$, and expanding, we have

$$H \tan \alpha = M[r^{-3} - \tfrac{3}{2} r^{-5}l^2 + \cdots]. \tag{12}$$

Since r is large, negative powers above the fifth may be neglected. Multiplying both sides of (12) by r^5, we have

$$Hr^5 \tan \alpha = M[r^2 - \tfrac{3}{2} l^2]. \tag{13}$$

Repeating the procedure just described, but with a different distance r_1 and a corresponding angle α_1, we may write

$$Hr_1^5 \tan \alpha_1 = M[r_1^2 - \tfrac{3}{2} l^2]. \tag{14}$$

Eliminating $3\,Ml^2/2$ between equations (13) and (14), we obtain the equation

$$H[r^5 \tan \alpha - r_1^5 \tan \alpha_1] = M[r^2 - r_1^2], \tag{15}$$

whence

$$\frac{M}{H} = \frac{r^5 \tan \alpha - r_1^5 \tan \alpha_1}{r^2 - r_1^2} \tag{16}$$

If M remains constant throughout the experiment, the value of H may be calculated from the values of MH and M/H obtained above, by means of the identity

$$H = \sqrt{\frac{MH}{\frac{M}{H}}}. \tag{17}$$

204. Laboratory Exercise XLVII. *To determine H by the magnetometer method.*

APPARATUS. Magnetometer and accessories, stop watch, meter scale, micrometer calipers, and compass.

PROCEDURE. (1) Hang the deflecting magnet in a stirrup, cover it with a bell jar, and observe the time of vibration at the position where H is to be determined. The angle of swing should not exceed 5°, and at least five determinations should be made of the time for 25 swings, the mean time of one vibration being then calculated.

Throughout the experiment no magnetic material other than the magnet in use should be about the table, or on the person of the observer.

(2) Measure the length and mean diameter of the magnet, and find its mass. From these data compute its moment of inertia by means of the formula

$$K = m\left[\frac{d^2}{16} + \frac{L^2}{12}\right],$$

where d and L are the diameter and length of the magnet, respectively.

Calculate from equation (8) the value of MH.

(3) Set the table of the magnetometer with its axis in the magnetic meridian as determined by the compass, and level the instrument so that the needle swings freely. Set the telescope and scale at a distance of one meter in front of the magnetometer, being careful to have the scale parallel to the magnetometer table.

With the deflecting magnet removed to a distant part of the room, take the zero reading on the scale. Place the magnet symmetrically in its support at a distance r from the needle and read the deflection α'. Turn the magnet end for end and again read the deflection α''. The mean of these readings, α,

will be free from error due to lack of symmetry in the distribution of the magnetism along the deflecting magnet.

Place the magnet on the other end of the table at the corresponding distance and take deflections with the north pole toward the east, and again toward the west, as before. From these four readings the mean deflection for the distance r will be found.

(4) Repeat (3) for another position at a distance r_1 from the needle, for which the deflection α_1 will be observed. It is well to take readings also for two other values of r, thus insuring two independent determinations of M/H.

(5) Substitute these data in equation (16).

(6) With the values found for MH and M/H, calculate from the identity (17) the value of H. Express the result in appropriate units.

Carelessness in handling the deflecting magnet, such as striking, jarring, or dropping it, may change the value of its magnetic moment. Only in case M remains constant throughout the progress of the experiment, is its elimination in equation (17) legitimate.

Tabulate all data, arranging the table to show the number of the deflecting magnet used, its mass, dimensions and moment of inertia, the observations for and the final value of its time of vibration, the separate and mean values of r and α, and the computed values of MH, M/H, and H.

Explain why a suspended magnetic needle experiences a rotation only and not a motion of translation.

205. Laboratory Exercise XLVIII. *To determine H with the standard tangent galvanometer and copper voltameter.*

Apparatus. Standard tangent galvanometer, one or two storage cells, control rheostat, reversing switch, and two copper voltameters with accessories.

From equations (17), § 107, and (24), § 108, it is seen that H may be expressed in terms of a current strength, the constants of a tangent galvanometer, and the tangent of the deflection angle. The current can be accurately measured with a copper voltameter.

PROCEDURE. (1) Arrange the circuit as in Fig. 144. The current from the storage cells may be adjusted to a suitable value by means of the rheostat *R*, and it may be controlled during the progress of the experiment if there is any tendency toward fluctuation.

The reversing switch *S* reverses the current through the galvanometer but not through the voltameters *V V*. Two

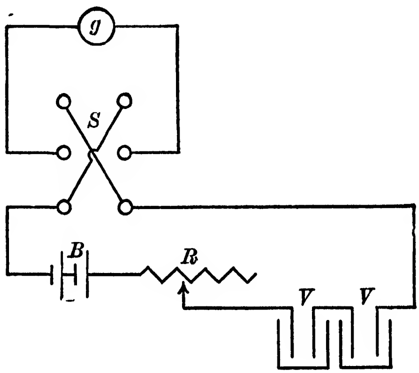

FIG. 144.

voltameters may be connected in series, one serving as a check on the other. The deposits on the cathodes of each should be the same. In each cell the outside plates will be anodes, the cathode being held midway between them.

(2) Clean the surfaces of the anodes with sandpaper, rinse them in clean water, and place them in the voltameters. The cathode plates will be carefully cleaned with sandpaper, laying them meanwhile on a piece of clean paper and avoiding touching them with the fingers. When clean and bright, rinse them in clean water, then in alcohol, and dry them by twirling them for a few seconds in the air. They should then be weighed with great care and folded in a sheet of clean paper until ready to use.

(3) Insert a pair of trial cathodes and adjust the resistance *R* until the galvanometer shows a deflection of about 45°.

Note whether the galvanometer needle deflects equally on both sides of zero, and whether the readings of opposite ends of the needle are nearly the same. If not, adjustments of the leveling screws and torsion head are necessary.

When the galvanometer and current are properly adjusted, open the switch *S*, replace the trial plates by the weighed cathodes and close the switch, noting the exact time. The cathode plates should be fully immersed.

(4) Let the current pass for 30 or 60 minutes. Observe the galvanometer deflection, reading both ends of the needle every two minutes. Immediately after each reading, throw over the switch *S*. Record in a table the readings of time and deflections.

The current should be kept constant by adjusting *R* if necessary.

(5) At the end of the period break the circuit, reading the exact time. Quickly remove the cathodes and plunge them into a bath of water made slightly acid with H_2SO_4. Rinse them in water, then in alcohol, and dry them as before. Again weigh them carefully.

(6) Calculate the current strength from equation (45), § 113. The electro-chemical equivalent of copper varies slightly with the temperature and current density, but the value 0.0003294 grams per coulomb may be used here with safety.

(7) Calculate the value of *H* from equation (24), § 108.

The watch used should be compared before and after the experiment with the standard clock, and a correction for its rate should be applied if necessary.

The electrolyte should be made from pure $CuSO_4$ and water with a density of about 1.18. One per cent of H_2SO_4 should be added.

206. Laboratory Exercise XLIX. *To compare values of H.*

APPARATUS. Small magnet with support and bell jar, and watch.

When the value of H is known at one station the value at any other station is readily found by the use of equation (8). Let H and H' be the values at two stations, and let T and T' be the corresponding periods of vibration of the suspended magnet. For the same magnet M and K will be constant, and we shall have

$$(18) \qquad T = 2\pi\sqrt{\frac{K}{MH}}, \qquad T' = 2\pi\sqrt{\frac{K}{MH'}}.$$

Squaring both of these equations and dividing the first of them by the second, we have

$$(19) \qquad \frac{H}{H'} = \frac{T'^2}{T^2}.$$

Procedure. (1) Support the vibrating magnetic needle under the bell jar at the station where H is known, and take the time of 25 or 50 vibrations, keeping the angle small. Repeat several times and calculate the mean value of T.

(2) Remove the apparatus to the station where H' is to be found, repeat the observations for the time of vibration, and find the mean value for T'.

(3) Calculate from equation (19) the value for H' at the second position.

Note for each position the location of the needle with respect to any structural iron work, or other fixed masses of magnetic material, and explain their probable effect on the period of the needle.

In this way a magnetic survey of any locality is readily carried out.

The method is analogous to that used for comparing values of the acceleration due to gravity, which involves a similar treatment of the formula for the simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

207. The Earth Inductor. A device frequently used for producing a known charge, and for measuring indirectly the horizontal or vertical components of the earth's field, is

the so-called ***earth inductor*** shown in section in Fig. 145. It consists of a non-magnetic frame of round or square section, on which is wound a coil of wire of a known number of turns S, and of effective radius r. This coil is so mounted that it may be rotated about either a horizontal or a vertical axis, and it is provided with a spring release and a stop, so that it may be rapidly turned through an angle of 180°. If the horizontal component of the earth's field is accurately known, and if the coil is rotated about a vertical axis, the *quantity* Q induced in the circuit of the coil in one half a rotation may be computed by the formula

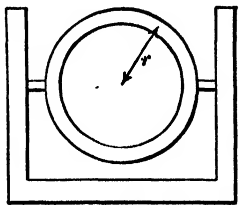

FIG. 145.

$$Q = \frac{\Delta N}{R} = \frac{2\,SHA}{R}, \tag{20}$$

where A is the area of the coil and R is the resistance of the circuit. This quantity may be used for the calibration of a ballistic galvanometer. On the other hand, equation (20) may be used for the determination of H itself.

If deflections are read on a ballistic galvanometer for both vertical and horizontal positions of the plane of the coil, the horizontal and vertical components of the earth's field, respectively, are linked with the wire turns, and the angle of dip may be determined even though values of V and H are not known. If d_1 denotes the observed deflection when the coil is horizontal, and d_2 the deflection when the coil is vertical, the corresponding charges Q_1 and Q_2 are given by the equations

$$Q_1 = \frac{2\,SVA}{R} = Gd_1, \tag{21}$$

$$Q_2 = \frac{2\,SHA}{R} = Gd_2. \tag{22}$$

Dividing (21) by (22), we obtain the relation

$$\frac{V}{H} = \frac{d_1}{d_2}.$$

From equations (1) and (2), we have

$$\frac{V}{H} = \tan \alpha,$$

whence

$$\alpha = \tan^{-1} \frac{d_1}{d_2}. \tag{23}$$

The number of turns and the dimensions of the coil must be accurately known, and the horizontal and vertical positions must be carefully determined with a spirit level. The values of V and H are subject to large variations due to local conditions, but the conditions may be regarded as remaining constant during the time required for these observations.

CHAPTER XII

MAGNETIC TESTING

PART I. MAGNETIZATION CURVES — HYSTERESIS

208. The Importance and the Nature of Magnetic Tests.[1] The industrial importance of iron and steel from the magnetic viewpoint is very great. According to the purpose for which it is to be used, it must possess high permeability, or low dissipation of energy in the process of magnetization, or the capacity of retaining a large percentage of its induced magnetism.

Hence, continued and systematic testing of the magnetic qualities of iron and steel is necessary for the producer of the material, the designer, and the manufacturer of electromagnetic machinery.

The producer must maintain a close control over his output in order to take advantage of the great variation in magnetic quality arising from slight variations in the composition and treatment of the materials.

The manufacturer must compare the predetermined efficiency of the design with the actual performance of the completed apparatus. In recent years the quality of the iron and steel produced for magnetic purposes has been greatly improved, and to-day more than ever before, the results of careful tests are being studied.

There are, in general, three kinds of tests to which iron and steel are subjected.

[1] A comprehensive treatment of the various magnetic tests, together with the modern methods, typical data, and results will be found in *Circular of* U. S. BUREAU OF STANDARDS, *No.* 17, *Magnetic Testing.*

I. **B–H** ***curves.*** A test to determine values of the flux density in the material for a given set of values of the magnetizing field. From this the ***permeability*** may be computed.

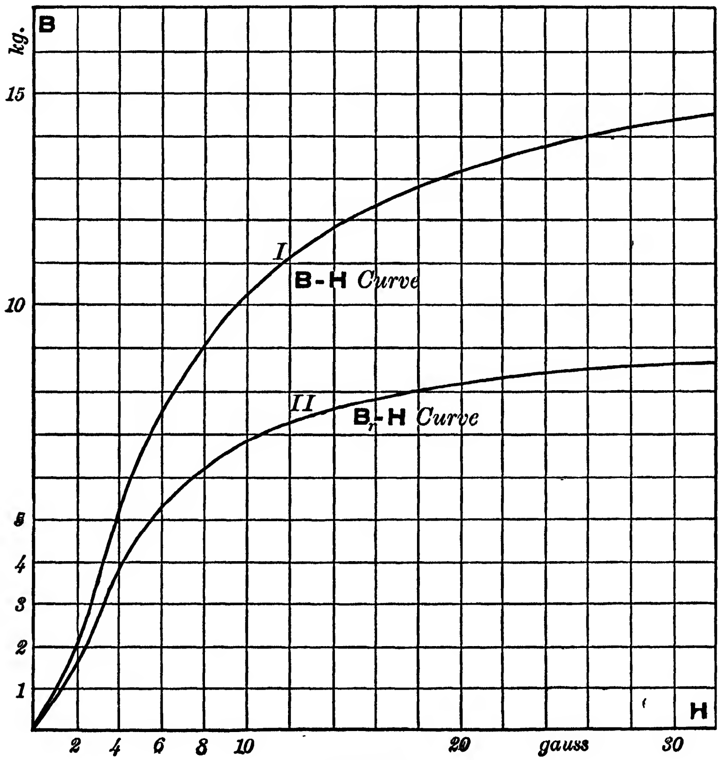

FIG 146.

II. ***Hysteresis and core-loss.*** A test to determine the energy expended in carrying the sample through a complete magnetic cycle. Work is done also in setting up eddy currents within the metal. The energy consumed per second due to both of these causes is called the ***core-loss.***

III. ***Residual Magnetism.*** A test to determine the amount of magnetism retained after the magnetizing field has been withdrawn. This may also include the determination of the tenacity with which the magnetism is held.

209. Units of H and B. The designer of electrical machinery must know the flux density which a given magnetizing field will establish in the material used. He must also know the value of the permeability, and how it varies with B and H. A

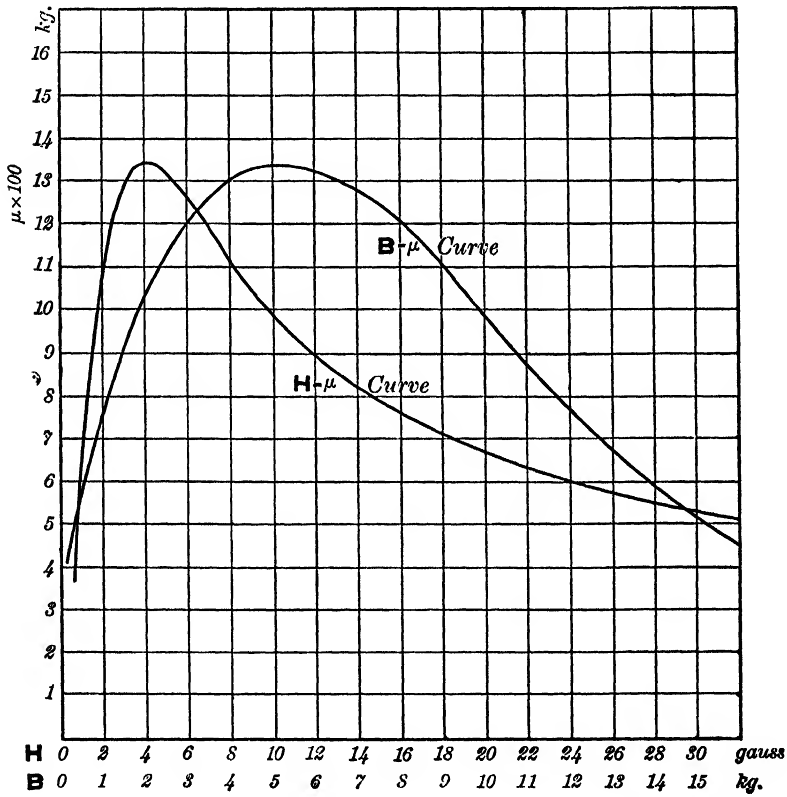

FIG. 147.

typical B–H curve for cast steel is shown in I, Fig. 146. In Fig. 147 the B–μ and H–μ curves are shown for the same material.

The magnetizing field strength H is commonly expressed in C. G. S. lines (or maxwells) per square centimeter, that is in gausses; or in gilberts per centimeter; or in ampere-turns per centimeter or per inch.

The induction density B is expressed in gausses, or in kilo-

gausses, or in lines per square inch. Corresponding values of the important magnetic quantities (§§ 187–193) for the sample of cast steel mentioned above, are shown in the following table, with H and B in gausses.

H	B	I	k	μ
1.0	650	51.7	51.7	650
4.0	5300	421.7	105.4	1325
10.0	10300	819.3	81.9	1030
20.0	13100	1041.4	52.1	655

210. B–H Curves. The upper curve in Fig. 146 and the upper ones in Fig. 148 show three distinct stages of the magnetizing process. At first, for low values of H, the induction increases slowly. Next, the induction rises rapidly with large changes for small increments of H, and finally, after reaching the knee of the curve, further increase is slow even for large increments of the magnetizing force.

The value of B can always be increased by increasing H, but a limit is soon reached above which it is not practicable to go. At this stage the iron is said to be approaching saturation. For wrought iron and cast steel this limit is approximately reached at 15,000 to 17,000 gausses. For cast iron the saturation point is at about 10,000 gausses. Curves for different sorts of iron and steel are shown in Fig. 148.

The induction density depends somewhat upon the initial state of the sample, and upon its previous magnetic history, as well as upon the rapidity and mode of change from one value of the magnetizing field to another. The discussion of these matters will be resumed in the following articles. The permeability is diminished by mechanical treatment, such as rolling or hammering.

The testing of iron at low values of the induction density has become important in recent years because of the increas-

ing use of relays for feeble currents. In fields where H is less than one gauss, the induction density curve is essentially a straight line starting with a finite inclination to the H axis.

With alternating currents of high frequencies, that is, upwards of 100,000 cycles per second, the permeability probably

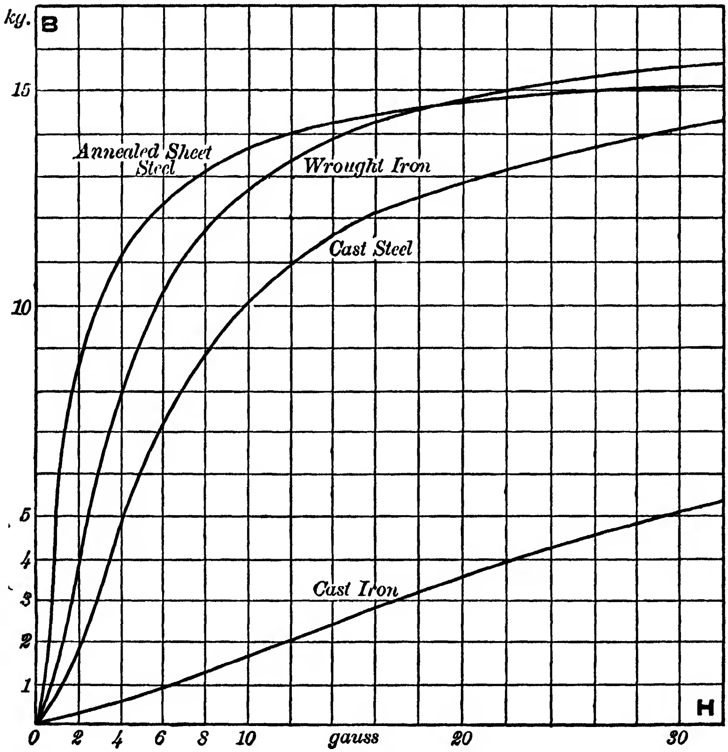

FIG. 148.

does not differ greatly from that given by the normal induction curve, but it is not significant, and it is difficult to measure because of the skin-effect due to the rapid alternations. The induction near the center of the sample is exceedingly small, because before the effect has penetrated appreciably, the field is withdrawn and reëstablished in the opposite sense.

Due to this effect, iron has little influence in increasing the inductance of coils in high frequency circuits.

211. Residual Magnetism and Coercive Force. It was stated in § 194 that no energy is required to maintain the

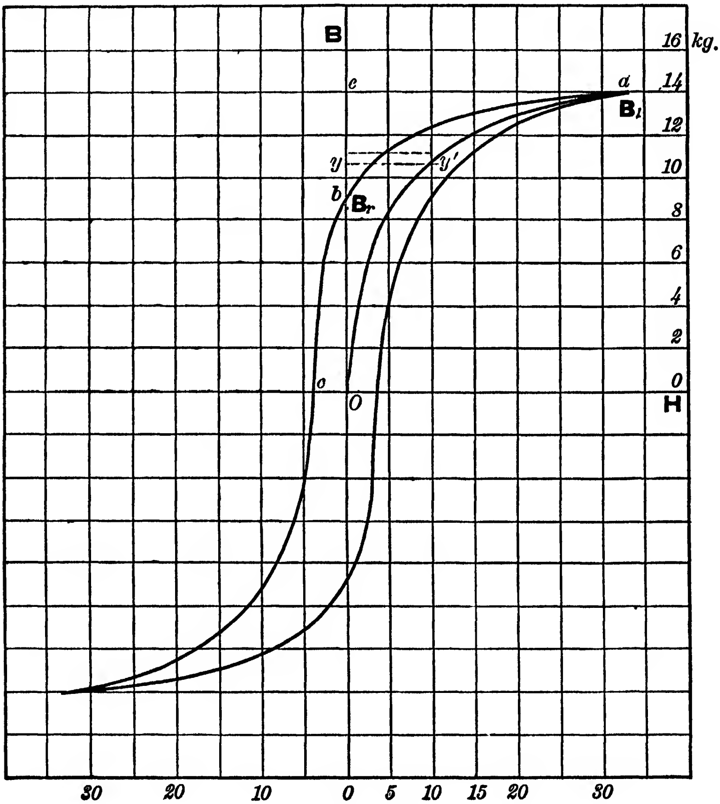

Fig. 149.

magnetic flux when once it has been established. Energy is required, however, to increase the magnetic flux, and energy is given out if the flux decreases.

A gradually increasing magnetizing field impressed on a piece of iron will cause the induction density to rise along

some such curve as *oa*, Fig. 149. If H is now gradually reduced in value, the iron shows a more or less marked tendency to persist in its state of magnetization. For a given decrement in H, the decrease in B is less than was the increase for the corresponding increment in H. Hence the curve returns along some such path as *ab*, which is quite different from that along which it rose to the point *a*. The intercept *ob* represents the ***remanence***, or the ***residual magnetism*** retained by the iron.

If the magnetizing field is reversed in direction at this point and again gradually increased in the negative direction, the curve drops along some such path as *bc*. The intercept *oc* represents the ***coercivity*** of the iron. This is conveniently measured in terms of the ***coercive*** force, which is the value of the reversed magnetizing field *oc* required to reduce the residual magnetism to zero.

The ratio of the residual magnetism to the previous maximum value of B, or *ob/oc*, is called the ***retentivity***. Some writers use retentivity as synonymous with residual magnetism. A *closed* circuit of soft iron may retain 85 % of its maximum induction, and a coercive force of less than two gausses is sufficient to reduce it to zero. On the other hand hardened steel may require a coercive force of 40 or 50 gausses to demagnetize it.

212. Hysteresis. It has been shown in the preceding article that changes in the induction density always lag behind the corresponding changes in the magnetizing field strength. This tendency is called ***hysteresis***. We shall now show that the area of the entire loop (Fig. 149), is a measure of the work done in carrying the sample through a magnetic cycle. This energy appears as heat in the sample.

In order to calculate the work expended in a magnetic cycle, assume a long bar of iron, or better a ring, of length L centi-

meters and of cross section A square centimeters, overwound throughout its entire length with n turns of wire per centimeter. If the magnetizing current i is increased by some small amount di in a time dt, a corresponding increase in the induction density $d\mathsf{B}$ will be produced. In accordance with Lenz's law, this increase will set up a counter-electromotive force of value E, in opposition to the current i. Against this counter-electromotive force the current must do work whose value is

$$dW = Ei\,dt. \tag{1}$$

But $E = dN/dt$, where N is the total number of linkings. Also we have from equation (2), § 128,

$$dN = LnA\,d\mathsf{B}, \tag{2}$$

since Ln is the number of wire turns and $Ad\mathsf{B}$ is the magnetic flux. If V, the volume of the specimen, is put in place of LA, we may write

$$dW = Vni\,d\mathsf{B}; \tag{3}$$

hence, the work per cubic centimeter is given by

$$dW = ni\,d\mathsf{B}. \tag{4}$$

Substituting the value of ni from equation (33), § 109, (4) may be written in the form

$$dW = \frac{\mathsf{H}\,d\mathsf{B}}{4\,\pi}. \tag{5}$$

The total work done as H and B vary between assigned limits is given by integrating this expression for dW

$$W = \frac{1}{4\,\pi}\int \mathsf{H}\,d\mathsf{B}. \tag{6}$$

Referring to Fig. 149, as H is increased from zero to H_1, B is increased from zero to B_1, and the amount of work expended

on the iron is given by the expression

$$\frac{\text{area } aeo}{4\,\pi},$$

that is the sum of all the small strips yy'. As H is brought back to zero, B falls to a value ob, and the changing magnetization returns energy to the circuit of value $aeb/4\,\pi$. The net amount of work expended on the iron is then given by area $abo/4\,\pi$. Extending this analysis over the entire area of the loop, we find that the work expended in the complete cycle is

$$(7)\qquad W = \frac{\text{area of loop}}{4\,\pi}.$$

This will be expressed in ergs per cubic centimeter per cycle, if the area is taken in square centimeters.

Since the values of B are large compared with H, it will not be convenient to plot them to the same scale. However, if u represents the number of H units corresponding to one scale division on the cross-sectioned paper, and if v represents the number of B units per division, then

$$(8)\qquad \begin{aligned} W &= \frac{uv}{4\,\pi}\int x\,dy \\ &= \frac{uv}{4\,\pi}\Big[\text{area of loop}\Big] \text{ ergs per cubic centimeter per cycle.} \end{aligned}$$

In addition to the hysteresis loss, there is also a loss of energy due to the eddy currents in the iron core when the iron is subjected to the magnetizing field of an alternating current. This effect is greatly diminished by building the sample of thin sheets, insulated from one another by varnish. The discussion of total core-loss will be resumed in § 237.

Instead of expressing the energy loss in ergs per cubic centimeter per cycle, it is common in practice to express the energy in watts per pound or watts per kilogram, at a given frequency. Some typical values are given in the accompanying table for

samples of steel and iron carried through various ranges of induction density.

Dynamo-Magnet Steel			Transformer Iron		
B limits	Ergs per cu. cm. per cycle	Watts per lb. freq. 100	B limits	Ergs per cu. cm. per cycle	Watts per lb. freq. 100
2000	550	0.32	2000	240	0.14
5000	2030	1.20	5000	1190	0.70
9000	5250	3.09	7000	2020	1.20
12000	8500	5.01	9000	3050	1.80
16000	13900	8.20			

Hysteresis Loss (Ewing)

The first hysteresis loop taken on a neutral sample of iron, in which all effects of previous history have been destroyed, will not close at the point a, Fig. 149. Successive loops, however, will progressively show less and less of this defect. After repeated reversals (for practical purposes, 20), the position of the cusps will be invariable, and a strictly cyclic state will be established.

The ***normal induction curve*** is the locus of the cusps of a series of hysteresis loops which are strictly cyclic. The shape of the hysteresis loop may be inferred with a fair degree of accuracy if three points are located: (1) the extreme cusp, (2) the residual magnetism intercept, (3) the coercive force intercept. It is seen from the above table that the hysteresis loss varies greatly with the conditions of the test. In order to compare results, it is important to specify standard conditions. These are usually taken as 60 cycles per second, and $B_{max} = 10000$ gausses.

Typical hysteresis loops for three samples of widely different materials are shown in Fig. 150 and Fig. 151. In I, Fig. 150, a small value of H sets up a large induction density which may be easily reversed, the hysteresis loss being small. Such

material is useful for alternating current transformer cores. A low hysteresis loss is important in this case because the loss is continuous, even during the time for which there is no current output from the transformer. Punching and shearing

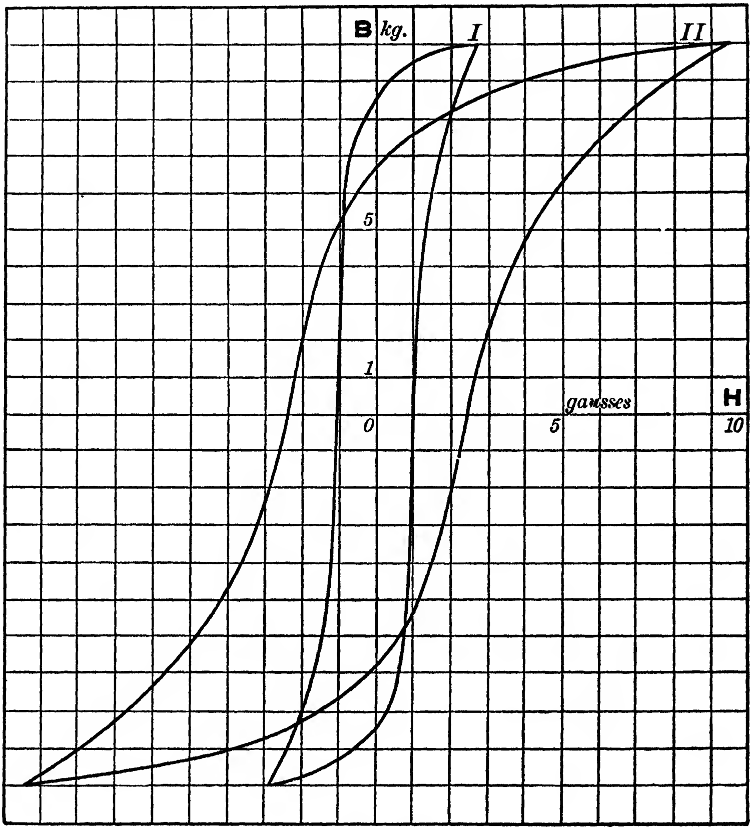

FIG 150.

of the material in the course of manufacture will increase the hysteresis loss somewhat, but this is in part overcome by subsequent annealing.

In II, Fig. 150, B increases rapidly, the residual magnetism is large, and the material is easily demagnetized. These

properties make the material useful for dynamo field magnets.

In Fig. 151, the retentivity and coercive force are both high, which are desirable qualities for steel to be used for permanent magnets.

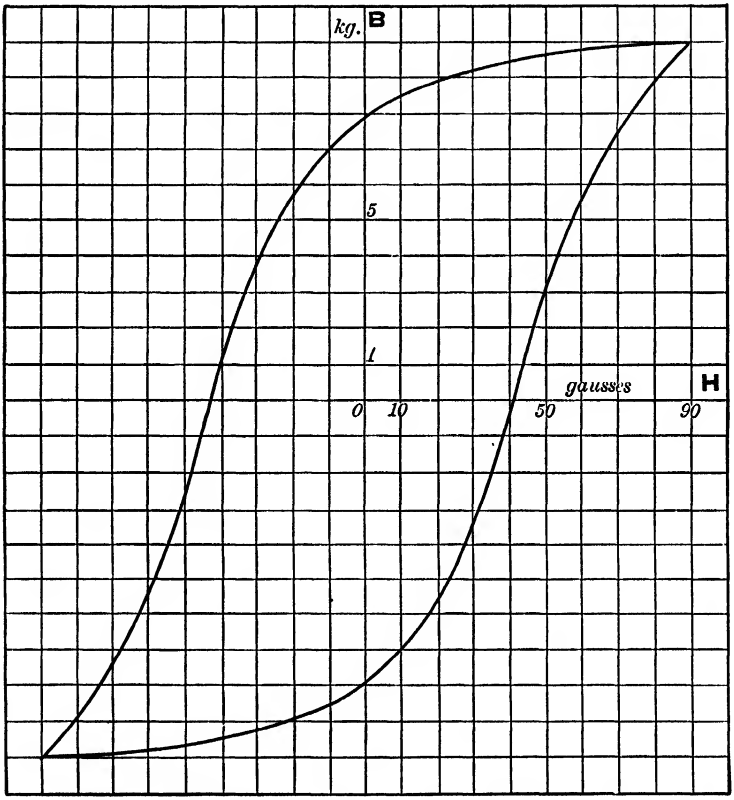

Fig. 151.

213. The Hysteretic Constant. If values of the hysteresis loss for different values of B_{max}, expressed in ergs per cubic centimeter per cycle, are plotted against the corresponding values of B_{max}, a curve of the form shown in Fig. 152 is obtained. Steinmetz has given an empirical formula which approximately expresses this relation; it is

(9) $$W = \eta \mathsf{B}^{1.6} \text{ ergs per cubic centimeter per cycle.}$$

The factor η is called the hysteretic constant, and for average samples of sheet steel its value may vary from 0.001 to 0.003. For hardened tungsten magnet steel $\eta = 0.06$, and for a high grade of silicon steel $\eta = 0.0006$.

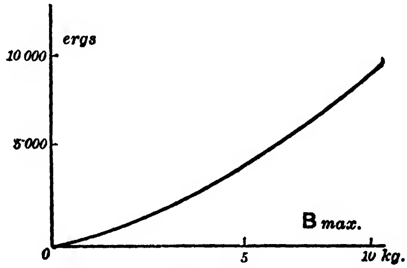

FIG. 152.

214. The Temperature Rise. The rise in temperature of a sample of iron due to hysteresis, when subjected to a cyclic magnetization, is calculated as follows. When all the heat is assumed to be retained in the iron, we may write

$$W = JH$$

where H is the heat in calories, equivalent to W ergs, and where J is the mechanical equivalent of one calorie of heat expressed in ergs. Consider a cubic centimeter of the iron of specific heat 0.115 and of specific gravity 7.6. The heat accumulated in the iron is given by the product of the mass, the specific heat, and the rise in temperature; that is

$$H = 7.6 \times 0.115 \times t.$$

The energy equivalent of this heat is

$$W = JH = (4.2 \times 10^7)(7.6 \times 0.115 \times t),$$

which is, by equation (6), equal to

$$\frac{1}{4\pi}\int \mathsf{H}\, d\mathsf{B}.$$

It follows that the expression for the temperature rise becomes

$$(10) \qquad t = \frac{\int \mathrm{H}\, d\mathrm{B}}{4\pi(4.2 \times 10^7)(7.6 \times 0.115)} \text{ degrees } C.$$

215. Alloys of Magnetic Materials. The influence of the admixture of various chemical elements on the magnetic properties of iron and steel has been intensively studied, and the adaptability of the material to its several uses has been greatly increased by this means.

The addition of 2.5 % of silicon to high grade soft iron ($\mathrm{B}_{max} = 4000$, $\mu = 2000$) increases the permeability over two-fold, while the hysteresis loss is reduced 34 % and the coercivity 27 %.

Small percentages of silicon improve steel also, but in a somewhat less degree, and at the same time render it less liable to deterioration at high temperatures. The resistivity also is increased, and the eddy current core-loss is reduced. Improved sheet steel of this character shows a core-loss as low as 0.9 watt per pound, at a frequency of 60 cycles per second, with $\mathrm{B}_{max} = 10000$ gausses. For $\mathrm{B}_{max} = 2000$, the loss falls as low as 0.05 watt per pound.

The addition of 12 % of manganese reduces the susceptibility of steel practically to zero. Certain alloys containing as high as 88 % of iron have been made which are non-magnetic. Recent experiments have shown that the permeability of electrolytic iron is very greatly increased when melted in a vacuum, while at the same time the hysteresis loss is reduced. These effects are accompanied by a low resistivity which is favorable to a large eddy current loss, but the resistivity may be increased by the addition of such elements as silicon or aluminum without materially affecting the other magnetic qualities.

Further information on the influence of the composition on

magnetic properties will be found in the various electrical handbooks.

216. Alloys of Non-magnetic Materials. Many alloys of practically non-magnetic components are themselves more or less magnetic. An alloy of 25 % manganese and 75 % copper, which is itself non-magnetic, was rendered strongly magnetic by decreasing the copper content and adding 13 % of aluminum.

Maganese and aluminum compounds are in general magnetic, and certain of them approach cast iron in magnetic quality. The effect of the copper in the alloy mentioned above appears to be solely that of keeping the alloy soft enough to be worked. Hysteresis is large in most manganese-aluminum alloys.

Very surprising reversals of properties occur for extremes of both heat and cold. A certain 25 % nickel steel, practically non-magnetic at ordinary temperatures, is strongly magnetic at −190° C., and retains the property when restored to room temperature. Heating to 600° C., and, then cooling slowly, destroys the magnetism for many of these alloys.

217. Residual Magnetism and Retentivity. With the rapid development of electro-magnets, the permanent magnet was displaced from the position of importance which it had previously occupied, and research was for a time diverted along lines of more immediately practical application. In recent years, however, permanent magnets have entered largely into the manufacture of galvanometers, quantity meters and other measuring instruments of many kinds, magneto-ignition devices, speedometers, toys, and automatic machines in great variety. Accordingly, their production and properties have been intensively studied, and they now constitute a large factor in the electrical industry.

Magnetic materials vary greatly with regard to the amount of magnetism retained after the magnetizing field has been withdrawn. Soft iron has a high residual magnetism, often

85 %, but it is loosely held, the coercive force sometimes being as low as two gausses. Hardened steel on the other hand has much less residual magnetism, but it is tenaciously held, and requires a strong reversed field of perhaps 50 gausses to remove it. A *closed* magnetic circuit of soft iron will show large remanence as long as it is not broken. After introducing an air gap, however, the iron is quickly demagnetized.

When the magnetizing force is removed from a bar it tends to demagnetize itself. This effect is much greater for short bars than for long ones, and soft iron bars for which the ratio of length to diameter is as small as ten show scarcely any residual magnetism. Bars for which the ratio of length to diameter is as great as 400 retain a large part of the induced magnetism after the magnetizing force has been withdrawn.

The total induction in a magnetic circuit of iron may be regarded as comprising three components, (*a*) the temporary magnetism, which vanishes with the removal of the magnetizing field; (*b*) the sub-permanent magnetism, which is removed by the polar field or by special treatment; (*c*) the permanent magnetism, which can only be removed by the application of a sufficiently strong reversed field, perhaps accompanied by vibration.

For permanent magnets the necessary characteristics are large retentivity and coercive force, with small tendency to deteriorate with lapse of time. Steel for such magnets is alloyed with 3 to 5 % of tungsten, with the addition of a fraction of a per cent of chromium, which improves the stability of the magnetism. The sub-permanent magnetism is removed by an artificial ageing process in which the finished magnets are alternately heated and cooled in a water or oil bath, and then subjected to a rapid mechanical vibration with the addition of a slight demagnetizing effect due to an alternating current field. This treatment tends to bring about the

same changes as long continued use, and magnets so treated deteriorate very little with the lapse of time.

The effective strength of bar magnets, measured in terms either of B or I, will depend upon the dimensions of the bars and the shape into which they are formed, as well as upon the quality of the material used. Tests for the residual magnetism of permanent magnets must be made on specimens which are virtually closed magnetic circuits to secure results which are characteristic of the material used. Any test on a short bar will yield results characteristic of that length and shape of specimen only, due to the demagnetizing effect of the poles.

If a hysteresis loop is drawn for a perfect magnetic circuit of the given material (Fig. 153), the retentivity is represented by OB_1 and the coercive force by OH_1. The magnetic circuit then has an intrinsic M. M. F. sufficient to maintain the induction B_1, which is measured by H_1 gilberts for every centimeter of length of the circuit. An air gap introduced into the circuit will reduce the remanence to some point B_2. If L_1 and L_2, respectively, represent the lengths of the steel and air portions of the circuit, then $(H_1 - H_2)L_1$ gilberts is the magnetomotive force reserved for the steel, while H_2L_2 is the magnetomotive force required to maintain the reduced induction B_2 across the air gap. From such a curve it is possible to determine in advance the dimensions of the magnet for any required induction density. The area of the cross section is generally chosen so that the value of B is from 2000 to 4000 gausses, and the corresponding value of H is read from the curve, Fig. 153.

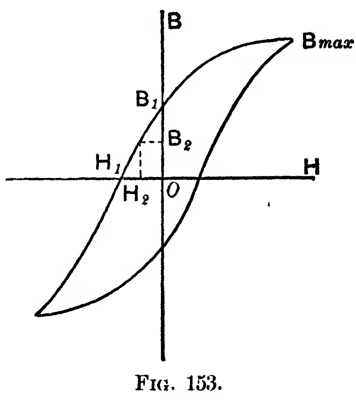

Fig. 153.

Corrections in the form of empirical constants are applied for the influence of any joints or pole pieces, as well as for leakage and polar field. A large value of the ratio $(H_1 - H_2)/H_1$ indicates good keeping quality.

218. Testing of Permanent Magnets. It is seldom that a precise measurement of the actual flux density in a bar magnet is required. The usual specifications simply require that it shall fall within certain defined limits, both of flux and of constancy.

A method for determining the flux density is given in § 236. If the magnet is a straight bar, the intensity of magnetization may be derived from magnetometer readings. If in the form of the letter U, the magnet should be provided with smooth end faces across which a soft iron armature is placed. A coil of a known number of turns is slipped over the middle point and connected to a ballistic galvanometer. The quick removal or replacing of the armature gives a throw on the galvanometer which may be compared with that taken in a similar way on a standard magnet. The total flux also may be computed by this method, using a calibrated ballistic galvanometer.

The given magnet may also be compared with a standard magnet by measuring the forces required respectively to detach the armature, or by comparing the torques due to eddy currents, when a copper disk is rotated between the poles.

219. Method of Current Reversals. In order to remove all traces of existing magnetism or to annul any previous magnetic history, it is only necessary to raise the iron to a red heat and cool it in a magnetic field of zero strength. However, this method is not practicable. A sufficiently effective demagnetization is obtained conveniently by placing the test piece in an alternating current field and gradually reducing its intensity to zero by introducing series resistance, by

reducing the voltage with a potentiometer device, or by shutting down the generator.

Another method is that of rapidly reversing a direct current, to which the magnetizing field is due, and at the same time reducing its strength by a gradually increasing series resistance. The current should be reduced so that B decreases uniformly, and the reversals should not be more rapid than one per second.

Part II. Methods of Magnetic Testing

220. Classification. The approved methods for studying magnetic properties may be arranged under four headings as follows:

(1) Magnetometer methods.
(2) Induction-ballistic methods.
  (*a*) Ring method.
  (*b*) Bar and yoke methods.
(3) Traction methods.
(4) Air gap methods.

221. The Magnetometer Method. This method is applicable only to open or imperfect magnetic circuits, that is, with samples having free poles.

The experimental work is carried out with apparatus somewhat like that used in § 203 for finding the horizontal component of the earth's field.

A sensitive magnetometer is set up at a place where H is accurately known, and the effect of the poles of the magnetic test piece, under different values of magnetizing field, is found in terms of the deflections of the magnetometer needle and H.

The method is useful only in certain lines of magnetic research and has no place in commercial testing. A serious objection to this method is that the shape of the test piece largely influences the results.

We have seen that any bar or rod exerts a demagnetizing influence upon itself. This form of the test piece can be used without corrections only when the length is 300 or 400 times the diameter.

Approximate correction factors can be computed and applied for the end effects in the case of round or square bars. With ellipsoidal specimens, absolute and reliable results can be

obtained, but such test pieces are difficult to prepare, and offer a better test of the skill of the mechanician than of magnetic quality.

Details of the method together with its limitations will be found in the larger treatises on magnetism. It will not be considered further in this book.

222. The Ring-Ballistic Method. Most of the practical magnetic testing to-day is based on the induction-ballistic method. The ring-shaped test piece is superior because there is no free magnetism, and hence there are no poles. We have seen that the presence of poles exerts a demagnetizing effect on the test specimen. The ring is uniformly overwound throughout with turns of a wire which is large enough to carry the desired magnetizing current without heating. A secondary coil of fine wire is wound over the primary, and is so arranged that any chosen number of turns may be connected to the ballistic galvanometer.

The ring may be cut or stamped from a plate of the given material, or a bar may be bent into a circular form and the ends welded together.

The radial thickness of the ring should be small, as it is found that the magnetic changes do not occur instantly, and the time required to bring about a change in the magnetic flux is greater as the thickness increases.

Since the induction method is based on the measurement of the quantity of electricity induced in the secondary coil, it follows that the galvanometer throw may occur before the magnetization has reached its final value, in which case the observed throw will not represent the total charge. This effect is not troublesome in thin rings and disappears at the higher values of flux density. The liability of error is least when a ballistic galvanometer of fairly long period is used.

The theory of the method is as follows. Let us assume

a circuit arranged as shown in Fig. 154. A storage battery of suitable voltage is connected through an ammeter A and a control rheostat R' with the middle points of a reversing switch W. This switch is connected also to a double pole double throw switch K. With the switch K on the right hand points, current flows through the primary or magnetizing coils of the ring R. With the switch K on the left hand points, current flows through the primary coils p of a known mutual inductance M. The secondary coils of R and M are connected in series with a ballistic galvanometer g, and a resistance box r'. The total resistance of the combined secondary circuits will be kept constant when once adjusted. The primary coils on the ring must be closely and uniformly wound over the entire length of the iron core. The secondary coils may be bunched at any convenient place on the ring.

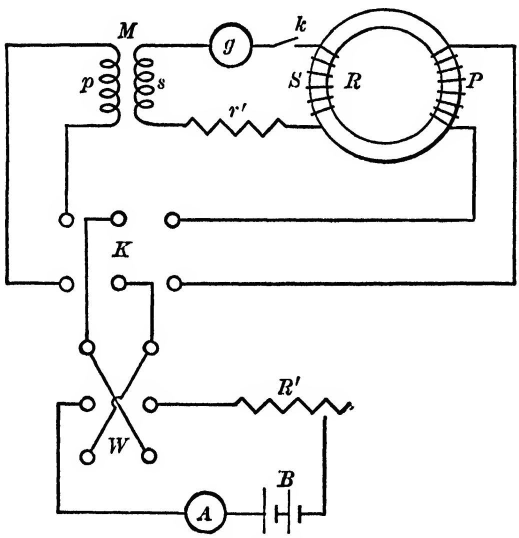

Fig. 154.

With K thrown to the ring side, a reversal of W will reverse a current I in the ring primary. This withdraws and reëstablishes the flux through the secondary coil, which, by linking with S wire turns, induces a charge Q_1 in the galvanometer circuit.

Let ΔN_1 represent the corresponding total change in the linkings of flux and wire turns. Then we have, by equations (76), § 142 and (18), § 149,

$$Q_1 = \frac{\Delta N_1}{r} = GD, \tag{11}$$

where D is the galvanometer throw which occurs when the switch is reversed, r is the total resistance of the secondary circuit, and G is the ballistic constant of the galvanometer. By equation (2), § 128, we have also

$$N_1 = \phi S, \tag{12}$$

which may be written in the form

$$N_1 = \mathsf{B}AS, \tag{13}$$

where B is the induction density in the iron and A is the area of cross section of the ring. Any change in N_1 is due to a change in B; hence the expression $\Delta N_1/r$ becomes

$$\frac{\Delta(\mathsf{B}AS)}{r},$$

and equation (11) may be written in the form

$$\frac{\Delta \mathsf{B}AS}{r} = GD. \tag{14}$$

Since there are two unknown quantities, $\Delta\mathsf{B}$ and G, in this equation, it is necessary to have another equation which gives the value of G, in order to calculate $\Delta\mathsf{B}$. If K is now thrown to the mutual inductance side, a reversal of W will reverse the current of strength i through the primary of the mutual inductance of value M, and the induced charge in the secondary will cause a deflection d on the galvanometer. The charge Q_2 induced by this reversal of the current, is given by equation (18), § 149, and is

$$Q_2 = \frac{2\,Mi}{r} = Gd. \tag{15}$$

Eliminating G between this equation and (14), and solving for ΔB, we find

$$\Delta B = \frac{2\,MiD}{ASd}. \tag{16}$$

Since A, S, and M are constants, and since i/d is a constant for the mutual inductance used, it is convenient to write

$$\Delta B = KD. \tag{17}$$

It is frequently found that the deflection D is either too small or too large for convenience, or it is not of the same order of magnitude as d. Values of D and d can be controlled by changing r'. However, if the total secondary resistance is not kept constant throughout, it becomes necessary to measure the secondary resistances for each case, and the calculations are somewhat longer.

A more convenient way to control the value of D is to have a variable number of turns in the ring secondary, each coil, however, having the same resistance, so that the total resistance remains constant as the number of turns is changed. In case it is found necessary to change the number of turns during the progress of an experiment, equation (16) may be written in the form

$$\Delta B = \left[\frac{2\,Mi}{Ad}\right]\cdot\frac{D}{S}. \tag{18}$$

When current is flowing through the primary of the ring, the reversal of this current causes the collapse of the magnetizing field H, and the establishment of a numerically equal field in the opposite direction. This change in H causes a corresponding reversal in B. By reference to Fig. 149 it will be seen that if H is changed from some positive value at a to the corresponding negative value at a', the change in B is equal to twice the value of B corresponding to the value of H calculated from the current read on the ammeter. This gives

$$\Delta B = 2\,B;$$

hence, putting this value in equation (18), we find

$$\text{(19)} \qquad \mathsf{B} = \left[\frac{Mi}{Ad}\right]\frac{D}{S}.$$

This equation gives B in C. G. S. units, or gausses, provided M and i are in C. G. S. units. If M is in henrys and i is in amperes, we shall have

$$\text{(20)} \qquad \mathsf{B} = 10^8 \frac{MiD}{AdS} \text{ gausses.}$$

The value of the magnetizing field H for a current of strength I amperes is given by the equation

$$\text{(21)} \qquad \mathsf{H} = \left[\frac{4}{10}\pi\frac{T}{L}\right] I \text{ gausses,}$$

where T is the total number of turns in the primary, and L is the mean circumference of the ring.

If the magnetizing current on the ring is brought to any desired maximum value I and is then reduced to zero, the galvanometer throw is not a measure of the maximum value of B, but is proportional to the difference between the maximum value of B and the residual induction remaining after I is zero. (See 5, § 223.) It must be remembered that in this and the following experiments, the galvanometer throw is a measure of the *change in the induction.* The above method of reversals is used in order to avoid the influence of residual magnetism.

For a neutral test piece, different values of B will result by suddenly or slowly increasing H. Moreover, the first few successive reversals of H will not yield the same value of B. After several reversals, say twenty, B becomes constant, and then its value, computed from half the throw, gives the ***normal induction density.*** The locus of several such points, for a suitable range of H values, is called the ***normal induction curve*** (§ 212). This is the curve that always should be used in specifying magnetic quality.

223. Laboratory Exercise L. *To determine the magnetic quality of a sample of iron by the ring-ballistic method, with current reversals.*

APPARATUS. Iron or steel test ring with two windings, ballistic galvanometer, standard mutual inductance, double pole double throw switch, reversing switch, tap key, ammeter, adjustable resistance, control rheostat, and a few cells of storage battery.

PROCEDURE. (1) From the primary ring constants, compute the value of I for the desired maximum value of H. With the circuit arranged as in Fig. 154, throw K to the ring side, set r' at some high trial value, say 10,000 ohms, and note the deflection when W is reversed, using 10 secondary turns. Reduce r' until the reversal of I gives a full scale deflection. Keep this value of r' unchanged throughout the experiment.

(2) Carefully demagnetize the ring. In doing this, make I slightly greater then in (1), and rock the reversing switch with the galvanometer circuit open, meanwhile bringing the current to zero by means of R'. (See § 219.)

(3) Throw K to the mutual inductance side with k open, and note the zero position of the galvanometer. Close k and set the rheostat R' so that the current i, when reversed through the primary of M, gives a full scale deflection. Reverse several times, and read and record values of i and d. Repeat these readings for values of i approximately one half and one third as large as before, and calculate the ratio of the current to the mean value of the corresponding deflections. This gives the value of i/d in equation (20).

(4) Throw K to the ring side with W open. Close W with R' set at its maximum value. Then begin reducing R' until H is only a few units, say two or three gausses, and reverse the current about 20 times, having previously opened the galvanometer circuit. Close the galvanometer circuit and reverse the current again, this time reading the throw D. Diminish

R', bringing the current to a slightly higher value, reverse about 20 times as before, and then read the throw for the next reversal. These reversals are necessary to establish a strictly cyclic state in the magnetism of the ring. During these reversals the galvanometer circuit must be open. A tap key may be used for this purpose.

Continue the above procedure for 12 or 15 steps, until the desired maximum value of H is reached. The instructor will advise concerning the maximum values of H and B for the test piece used. The first few steps should be small ones, because here the B-H curve changes its slope most rapidly for small values of B and H. Readings thus taken after several reversals may be repeated as a check. In case it is necessary to repeat a reading for a value of H which is less than the one before, the test piece must be demagnetized again.

(5) In order to study the residual magnetism in the ring, proceed as follows. After taking the reading of the throw for each reversal of the current, bring the reversing switch to its middle position, thus breaking the circuit. A throw D_r will then be observed which is proportional to the difference between the value of B and the value of the residual induction density B_r. This will be clear from a study of Fig. 149.

If the magnetic condition of the iron at any instant is represented by the point B_1, reducing I (and hence H) to zero will bring the induction density back to some point B_r. The accompanying galvanometer deflection D_r is then proportional to $B_1 - B_r$. In order to compute the value of B_r which corresponds to the original magnetizing field H_1 it is necessary to subtract D_r from the throw corresponding to B_1, that is, from half the throw for a complete reversal of I or $D/2$. Substituting $D/2 - D_r$ in equation (20), we find the value of the residual induction density B_r which corresponds to the previous excitation H_1. Immediately after throwing over the switch W and reading D as described in (4) above, quickly

open the double pole double throw switch, and read the throw D_r.

(6) After the set of readings on the ring is complete it is advisable to take another set of calibration readings, as described under (3) above. The mean of the two sets of calibration results should be used.

(7) The constants of the ring and the mutual inductance will be found marked on the apparatus. The other data should be arranged somewhat as follows.

CALIBRATION DATA

i	THROW	MEAN THROW	i/d

RING DATA

I	D	D_r	$(\frac{D}{2} - D_r)$	H	B	B_r	μ

(8) Calculate first the calibration ratio i/d, and compute once for all the values of the constants in equations (20) and (21). Using the values of I, calculate the corresponding values of H. With the values of D and D_r, calculate the values of B and B_r.

It must be remembered that half of the throw D is the measure of the induction density B. The factor 2 may be avoided by reversing the current also during the calibration. Then d is twice as great as for a single make or break. Enter all these values in the table and calculate the ratio $\mu = B/H$.

(9) Choose scales appropriate for the cross-section paper at hand and plot the B–H curve, using values of H in gausses as abscissas. Values of B, expressed either in gausses or kilogausses, should be plotted as ordinates. On the same sheet, and to the same scale, plot the B_r–H curve. On a separate sheet plot the μ–B and the μ–H curves. A B–H curve should also be plotted with H expressed in ampere-turns per inch, and B in lines per square inch.

(10) As the value of the current increases, the galvanometer throws increase, and may exceed the limits of the scale. The ring is provided with a variable number of secondary turns, and the throw can be controlled by using fewer turns. Changing the number of turns does not change the total secondary resistance, because compensating series coils are introduced.

(11) For low values of the magnetizing force the previous magnetic treatment of a specimen has considerable influence on the values of B. This is the reason why complete demagnetization is necessary. For higher values of H, the values of B taken after several reversals seem to be the same, whether or not the ring was demagnetized.

224. The Fluxmeter. Instead of a ballistic galvanometer, an instrument called the ***fluxmeter*** is frequently used for determining either induction density or total flux. This is essentially a suspended coil galvanometer, characterized by a strong and uniform magnetic field, negligible torsion in the suspension fiber, and excessive electromagnetic damping. The motion of the indicator is not impulsive, but follows the changing flux, the limit of its motion being a measure of the total charge which passes through its coil. The deflection of the suspended system is independent of the rate of change of the charge. When used with a given test coil, its scale may be graduated to read directly either in units of total flux, or of flux density. A precise zero setting is not important, since

readings can be taken by observing differences in the position of the indicator. The fluxmeter scale is calibrated by means of a given test coil and a magnetizing field of known strength.

It has been pointed out previously that changes in the magnetic flux do not occur instantaneously in iron, especially in thick samples, but require a certain time for their completion. If this time is comparable to the time of throw of a ballistic galvanometer, the throw does not measure the full change in the flux. The fluxmeter, however, accurately follows the changes in the flux, and it is under such conditions that the instrument is chiefly used. It also has the advantage of being direct reading.

225. Bar and Yoke Methods. The labor and time required to prepare and wind rings of magnetic material have led experimenters to give much thought to methods which would permit the use of short cylindrical bars, which are easily prepared and for which the magnetizing and test coils may be wound once for all.

An early method is illustrated in Fig. 155. A massive yoke *YY* of soft iron is forged, and holes are drilled at *AA′*, through which the rod *R* slips with the least possible clearance. Primary and secondary coils are wound on spools which slip over the portion of the bar within the yoke,

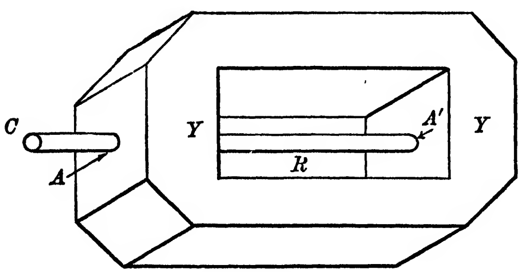

FIG. 155.

and the test is carried through as in the case of the ring. If the yoke is made from highly permeable material, the error introduced by neglecting its reluctance is not great, and

the length of the test bar between the inner yoke faces may be taken as the length of the magnetic circuit. However, no matter how closely the bar may fit in the holes, there is a slight air gap at the joints which introduces relatively great reluctance. These errors will prove least troublesome when the bar itself has a high reluctance. For very permeable material they are too large to be neglected. The effect of the reluctance of the yokes and joints is to shear the B–H curve away from the B axis, and approximate corrections may be determined.

In a further modification of the method, the bar is carefully faced on the end A' and made to abut against a faced surface inside of the yoke. When magnetized, a force will be required to separate the bar from the yoke, and this force is a measure of the induction density in the bar. From the pull at C necessary to detach the rod from the yoke the value of B may be calculated.

In the original form of the apparatus as used by Hopkinson, the procedure was as follows. The test bar was divided at its middle, and surrounding this plane of division was placed a test coil of a suitable number of turns, its terminals being connected to a ballistic galvanometer. The spool carrying the magnetizing windings was divided, with a proper space between its inner ends to permit the test coil to pass through. On pulling out the left-hand half of the rod by a force applied at C, the test coil was thrown out to one side by a spring. The magnetic flux between the two portions of the bar was then cut by the test coil, giving a throw on the ballistic galvanometer from which B was calculated.

None of the methods mentioned above are capable of giving other than approximate results, and it remained for Ewing to describe a modification which represented a great advance from the standpoint of convenience of manipulation, as well as accuracy of results.

226. Ewing's Double Bar and Yoke Method. This method requires two test bars of the given material and two yokes of soft iron. Magnetizing and test coils are wound on brass spools which can be slipped over the bars. A cross section of the arrangement is shown in Fig. 156. The yokes YY' are drilled to fit the standard sized bars, which may be firmly clamped in place by set screws in each yoke. A wooden box contains the spools with the magnetizing and test coils. A small switch playing over brass studs enables a variable num-

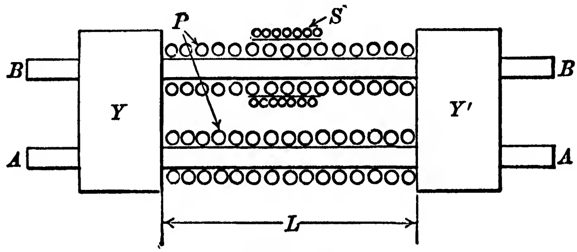

FIG. 156.

ber of turns to be included in the test coil, while the resistance is kept constant.

The condition of a perfect magnetic circuit is practically attained, and if the apparatus is used in the same manner as the ring in Experiment L, § 223, data for a B–H curve may be taken and plotted, as shown in curve 1, Fig. 157. This curve is not the true B–H curve for the material of the bars alone, as no correction has been made for the reluctance of the yokes and joints. If the free length of the bars is reduced one half, however, by substituting for the magnetizing coils others half as long and pushing the yokes together, another test carried through in the same manner as before will yield a B–H curve which falls slightly below the former, as shown in 2, Fig. 157. The true curve for the bars alone, corrected for yoke and joint reluctance, can be found in the following way. For any chosen value of B, set back the value

of H for curve 1 by an amount equal to the distance between curves 1 and 2. Repeat this process for a sufficient number of points, and the smooth curve drawn through these points, curve 3, Fig. 157, will be the true B–H curve for the bars.

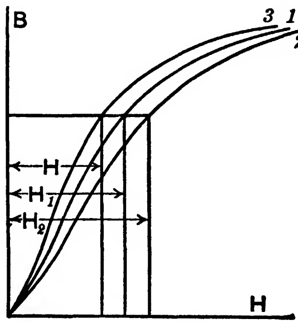

FIG. 157.

In the proof below, we shall use the following notation:

H_1 is the magnetizing force in the entire circuit for the full length of bar.

H_2 is the magnetizing force in the entire circuit for half length of bar.

H is the magnetizing force in the bars alone.

L_1 is twice the free distance between the yokes, 1st position.

L_2 is twice the free distance between the yokes, 2d position.

$L_1 = 2\,L_2$.

N_1 is the total number of magnetizing turns on L_1, 1st position.

N_2 is the total number of magnetizing turns on L_2, 2d position.

I_1 is the magnetizing current when L_1 is used.

I_2 is the magnetizing current when L_2 is used.

If H is the true magnetizing force in the material of the bars alone, then HL_1 and HL_2, respectively, will be the true magnetomotive forces in the bars in the two cases. The total magnetomotive force around the entire circuit is $4\,\pi$ times the

number of ampere turns. Some part of this, which may be represented by M, is required to maintain the magnetic flux through the yokes and joints. It is assumed that M remains constant throughout. The expressions for the magnetomotive forces for full and half length of the bars, respectively, may be written in the form

$$\tfrac{4}{10}\pi N I_1 = \mathsf{H} L_1, \tag{22}$$

$$\tfrac{4}{10}\pi N I_2 = \mathsf{H} L_2, \tag{23}$$

Each of these magnetomotive forces is equivalent to the sum of two components: one part is required to maintain the flux through the bars alone, and one part is required to maintain the same flux through the yokes and joints. Then we may write

$$\mathsf{H}_1 L_1 = \mathsf{H} L_1 + M, \tag{24}$$

$$\mathsf{H}_2 L_2 = \mathsf{H} L_2 + M. \tag{25}$$

Subtracting (25) from (24), we have

$$\mathsf{H}_1 L_1 - \mathsf{H}_2 L_2 = \mathsf{H}[L_1 - L_2]. \tag{26}$$

Introducing the condition that $L_1 = 2\,L_2$, we find

$$2\,\mathsf{H}_1 L_2 - \mathsf{H}_2 L_2 = \mathsf{H} L_2, \tag{27}$$

whence

$$2\,\mathsf{H}_1 - \mathsf{H}_2 = \mathsf{H}. \tag{28}$$

This may be written in the form

$$\mathsf{H} = \mathsf{H}_1 - \mathsf{H}_2 + \mathsf{H}_1,$$

or

$$\mathsf{H} = \mathsf{H}_1 - [\mathsf{H}_2 - \mathsf{H}_1]. \tag{29}$$

The true value for H in the bars for a chosen value of B is, therefore, given by subtracting from H_1 the difference between H_2 and H_1, as shown in curve 3, Fig. 157.

227. Laboratory Exercise LI. *To determine the* B — H *curve of a sample with the Ewing double bar and yoke method.*

APPARATUS. The same as for the ring method of Laboratory Exercise L, § 223, with the ring replaced by the double bar and yoke equipment.

PROCEDURE. (1) Arrange the circuit exactly as for the ring method, with the magnetizing coils in the battery circuit and with the test coils in the galvanometer circuit. The bars must be carefully demagnetized before beginning each test.

Clamp the bars in place, first using the long magnetizing coils. Do not force the clamp screws, but set them up snugly. If the rods do not pass freely through the holes in the yokes, rub them clean and add a trace of vaseline to the cleaning cloth.

Never pound the rods on the ends, as this will certainly ruin the fit.

(2) With the number of turns in the secondary windings suitably chosen, proceed with the observations precisely as in the ring method, first calibrating the galvanometer, then taking the first readings on the iron with small value of H, not exceeding two or three gausses. Carry the magnetizing current up to the desired maximum, rocking the reversing switch several times with the galvanometer disconnected, before taking each reading.

(3) Demagnetize the bars again, replace the long magnetizing coils by the short ones, and repeat the set of readings with the free bar length reduced one half.

(4) Calculate from each set of data the values of B and H from the formulas of § 222, and plot the curves 1 and 2, Fig. 157, on the same sheet. Set back the points of 1 according to the directions given in § 226, and draw curve 3. This will be the true B — H curve for the material of which the bars are made. The precision of the method is high if care is taken in making the observations.

228. The Compensated Permeameter. It has been shown in the foregoing articles that a short cylindrical bar is not a

satisfactory test piece because of the polar field developed by its own magnetic state. However, such a bar is a convenient form of test piece because of the ease of manufacture and the economy of material, and it is desirable to use it in some form of bar and yoke device. In the Ewing apparatus, § 226, the magnetomotive force required to overcome the reluctance of the yokes and joints was eliminated by a double set of observations and curves. The primary objection urged against this method is that the correction factor may not be constant for both settings of the yokes.

In the ***compensated permeameter***, as described by Burrows, a magnetomotive force is supplied by means of auxiliary current coils, which is just sufficient to maintain the existing flux through the yokes and joints. This insures a uniform flux through the entire circuit, and practically amounts to short-circuiting the bar by a path of zero reluctance. The general

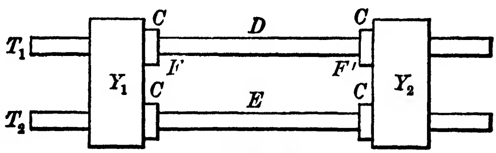

FIG. 158.

arrangement of the bars, yokes, and coils is shown in Fig. 158. The test bar T_1 and an auxiliary bar T_2, of similar but not necessarily the same material, are clamped in the yokes $Y_1 Y_2$. Both bars are inclosed by uniformly wound magnetizing coils, which fill the entire distance between the yokes. These coils are not shown in the figure.

The four compensating coils C are placed near the ends of the bars, and are connected in series. A current is sent through them of such a value that a magnetomotive force is supplied which is sufficient to overcome the reluctance of the joints and yokes. Over the middle of each bar at D and E there are wound secondary or test coils of 100 turns each. At F and F'

on bar T_1 are two coils of 50 turns each which may be connected in helping series and opposed to either D or E, or they may be connected in opposing series so that the charge induced in one of them may annul that in the other. It has been shown that when the effect of coils F and F' in helping series is exactly the same as that of coil D or E, the magnetic flux may be taken as uniform around the entire circuit, and the test bar may be treated as though devoid of poles.

The procedure for determining a point on the B–H curve for a given sample is as follows. After calibrating the ballistic galvanometer in the usual way with a known mutual inductance, the iron circuit is demagnetized. A small current is then passed through the magnetizing coil on bar T_1; from this the first value of H may be computed. The secondary coils D and E are connected in series with the ballistic galvanometer, and are opposed inductively. Current is now passed through the magnetizing coil on the bar T_2 and increased in value until, when both currents are reversed simultaneously, no galvanometer throw occurs. This shows that the magnetic flux through the two coils is the same.

Current is now passed through the four compensating coils in series, and the two coils F and F', are connected in helping series, but in opposition to D. The three magnetizing currents are now reversed simultaneously by means of a special gang switch, and the compensating current is adjusted until no galvanometer deflection can be observed. The test bar is now in a uniform state of magnetization, and the B value may be computed as in Laboratory Exercise L, § 223. By increasing the magnetizing current on the bar T_1 and repeating the procedure as above outlined, other points on the B–H curve may be obtained.[1]

[1] An extended treatment of this method is given in the following publications of U. S. Bureau of Standards: *Bulletin*, Vol. 6, p. 31; *Circular No. 17, Magnetic Testing.*

229. Traction Methods. If a magnetic circuit includes a narrow air gap, free poles will exist at the faces of this gap, and there will be an attraction between these faces which is given by the equation

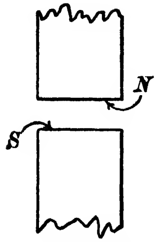
FIG. 159.

$$F = \frac{B^2 A}{8\pi}, \tag{30}$$

where A is the face area and B is the induction density in the gap. The force will be in dynes when B and A are in C. G. S. units.

This relation can be proved in the following way. Let N and S (Fig. 159) represent the two pole faces just after separation. If there is a flux density B uniformly distributed over the pole faces, we have, by equation (9), § 192,

$$BA = 4\pi m, \tag{31}$$

whence the equivalent pole strength of either face is given by

$$m = \frac{BA}{4\pi}. \tag{32}$$

The total flux across a very narrow air gap separating the attracting poles may be considered as made up of a component flux coming from and belonging to one of the poles, and another component flux in the same direction belonging to and going into the other pole. Hence we may consider one pole of strength m units, in a field of $B/2$ units due to the other pole. Hence the force that acts is given by the equation

$$P = \frac{mB}{2}. \tag{33}$$

Substituting the value of m from (32) in (33), we have

$$P = \frac{B^2 A}{8\pi}. \tag{34}$$

This gives the pull in dynes at the instant of separation. Solving (34) for B, we obtain the formula

$$B = \sqrt{\frac{8\pi P}{A}}. \tag{35}$$

Various designs of apparatus for using the above relation have been suggested,[1] the one described below being due to Fischer-Hinnen. Referring to Fig. 160, the principle of the method will be apparent. The long bar E rocks freely about a point O. The bar is graduated in centimeters and millimeters, and carries a sliding weight W of known mass. A counterweight W' is adjusted to bring the bar into equilibrium when W is at its zero position. The pull P takes place at the contact face c, when current is passed through a magnetizing coil which surrounds the test bar T. The weight W is moved

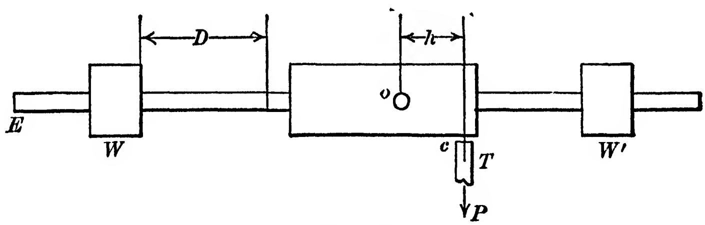

Fig. 160.

out along the graduated bar E through a distance D until the separation of the faced surfaces at c is effected. Equating the moments of the forces at this instant, we find

$$WD = Ph, \tag{36}$$

whence

$$P = \frac{WD}{h}. \tag{37}$$

[1] See Standard Handbook for Electrical Engineers (4th ed.), page 188.

Equation (35) may then be written in the form

$$\text{(38)} \qquad \mathsf{B} = \sqrt{\frac{8\,\pi WD}{hA}}.$$

In order to avoid facing the end of the test bar it is frequently provided with a cap that has a faced surface. If the area of the contact face of the cap is represented by A, and the area of cross section of the test bar by A_s, the expression for the total flux becomes

$$\text{(39)} \qquad \phi = \mathsf{B}A = \mathsf{B}_s A_s.$$

The induction density for the test bar is then given by the formula

$$\text{(40)} \qquad \mathsf{B} = \frac{A}{A_s}\sqrt{\frac{8\,\pi WD}{hA}} = K(D).$$

230. Laboratory Exercise LII. *To determine the* B–H *curve of a sample of iron with the Fischer-Hinnen traction permeameter.*

Apparatus. Permeameter with accessories, ammeter, reversing switch, rheostat, and a few storage cells.

Procedure. (1) Arrange the circuit as shown in Fig. 161, where C is the magnetizing coil about the test piece. The rocking bar should be horizontal, and the face of the soft iron

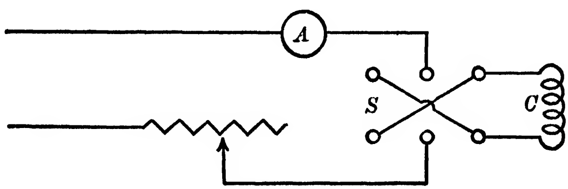

Fig. 161.

cap and the under face of the bar must be brought into intimate contact. To make this adjustment, insert the test piece and tighten the clamp screws of the cap. Let the rocking bar rest on the lower of the two adjusting screws, and adjust this screw till the contact between the faced surfaces is accurate. The operator should look across the contact surface toward a

strong light. A well lighted window is the best. Unclamp the upper screws, remove the test bar, and set W on the zero mark, adjusting the counterpoise W' for equilibrium.

(2) Having demagnetized the bar, insert the upper end in the contact piece and tighten the screws. Hold the rocking bar firmly against the lower adjusting screw, press up the test bar, clamp into contact position, and tighten the lower screws. Then unscrew the lower adjusting screw a trifle, perhaps a quarter of a turn, to insure contact between the faced surfaces.

(3) Through the magnetizing coil C pass a small current sufficient for the desired minimum value of H. Then slide out the weight W until a gentle tap on the base with the finger causes the contact faces to part, and record the position of the weight on the graduated bar. This is the value of D.

(4) Repeat the procedure in (3) for perhaps ten increasing values of the current strength up to that which gives the desired maximum value of H. Let each reading be the mean of at least three settings.

(5) Calculate B from equation (40), remembering that W, the weight of the sliding mass, is a force, and must be expressed in dynes. Corresponding values of H are computed from the equation

$$\mathsf{H} = \left[\frac{4}{10}\pi\frac{N}{L}\right]I, \tag{41}$$

where N is the number of magnetizing turns and L is the free length of the bar.

(6) Plot the B–H curve for the sample.

The utmost care is necessary to avoid injury to the faced surface of the cap. This piece is very soft, and a slight blow, or a fall to the table or floor, or careless use of the calipers in measuring its diameter may seriously impair the accuracy of the facing. Before beginning a test wipe both faces carefully with a cloth.

No traction method can be regarded as very satisfactory for investigating the magnetic quality of a specimen. Such methods are at best inexact and are not to be compared with ballistic methods for accuracy.

They afford ready means for making approximate and rapid tests, however, and they are especially useful for comparative methods and for grading, where results are desired for a single point on the curve. A joint in a magnetic circuit always increases its reluctance. Between the polished faces there will be a slight adhesion, tending to make the values of B too great, especially at low values.

The advantages of the method are that it is simple, that the test pieces are easily prepared, and that the results are not affected by stray fields. Corrections may be found and applied for the air gap and joint reluctance, and for the reluctance of the yoke.

231. Air Gap Methods. Let S be a bar for which the B–H curve is known, and let T be another bar to be tested. If both bars are clamped between two soft iron yokes, as shown in Fig. 162, the number of magnetizing turns on T may be varied by means of switches until the flux density is the same in both bars. When this condition is attained there will be no leakage between points A and B of the yokes. The magnetic potential of the points is the same, and all the flux lines that pass to the right through S, pass through T to the left. If two curved horns are erected at the points A and B as shown in Fig. 163, a magnetic needle in the air gap at G will not be deflected when the magnetizing current is reversed.

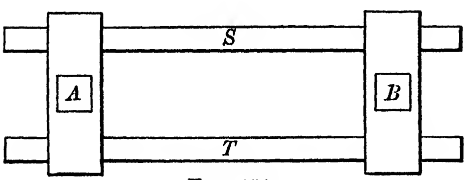

Fig. 162.

This method, which is due to Ewing, is analogous to the Wheatstone bridge method for the measurement of ohmic resistance. While it is not now in general use, it contains the essentials of several practical methods.

One of these adaptations of Ewing's method, due to Koepsel, is represented in Fig. 164. The test bar B is clamped in the pair of yokes YY_1, and magnetized by current sent through the coils mm'. At G is suspended a coil similar to that used in a d'Arsonval galvanometer, through which a weak current

of constant strength is maintained by an auxiliary battery. Any magnetic flux set up in the air gap will react with the field about the coil, and tend to rotate the suspended system. The apparatus is calibrated in terms of a standard bar, and the scale ss' may be made to read directly in gausses. An ammeter in the magnetizing coil circuit may be calibrated in C. G. S. units of H. The method has certain defects, due to the reluctance of the yoke and joints, and to the field which the coil sets up outside of the test bar.

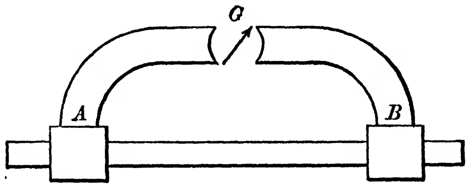

FIG. 163.

In an improved apparatus of the same general design, the pole faces at G (Fig. 164) are accurately bored out, and an armature which is independently driven by an electric motor is inserted. Therefore an electromotive force is developed if there is any flux across the gap, and that electromotive force is proportional to the flux if the speed of rotation is constant. A voltmeter connected to the armature brushes may be calibrated to read either flux density or total flux. The ammeter in the magnetizing coil circuit may be graduated to read the strength of the magnetizing field in any desired units. Full correction and compensation is effected for the yoke and joint reluctance, the independent field due to the coil, and leakage, and the results are highly satisfactory. The apparatus is calibrated by means of a standard bar, whose B–H curve has been determined by a double bar and yoke method or by a compensated permeameter.

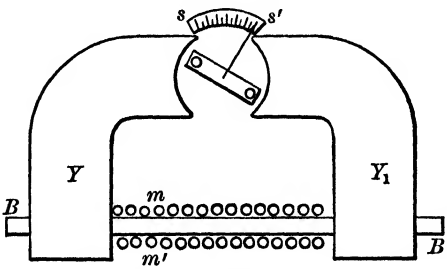

FIG. 164.

The metal ***bismuth*** possesses the characteristic of undergoing a change in its ohmic resistance when placed in a magnetic field. With values of the induction density varying from zero to 40,000 C. G. S. units, the resistance of pure bismuth wire will increase nearly threefold. Advantage is taken of this property for measuring the magnetic flux through an air gap. A fine bismuth wire is wound non-inductively in a flat spiral coil and incased in a mica capsule, the entire thickness being less than one millimeter. The normal resistance, R, of the wire having been measured, it is placed successively in fields of increasing strength, the resistance being measured for each value of H. A curve showing the relation between values of H and R enables us to interpolate any value of H for any given value of R. This apparatus is sometimes made up in the form of a Wheatstone bridge, the scale being graduated to read H units directly.

232. Hysteresis. Step-by-step Method. In order to determine the contour of the hysteresis loop, simultaneous values of H and B must be determined throughout a complete magnetic cycle. A convenient method for doing this makes use of the circuit and apparatus described in § 223, but the rheostat must be designed so that definite changes may be made in the strength of the current without breaking the circuit. The magnetizing field H is increased by suitable increments, and the corresponding deflections on the galvanometer are observed. Each deflection is proportional to the change in B which accompanies the change in H, and the value of B for any point is found by adding the increments in B from the beginning. The separate values of ΔB may be calculated from equation (18), and added together; or the deflection D for any point may be regarded as the sum of all the deflections for the preceding steps. Any value of B is a ΔB with reference to the starting point.

233. Laboratory Exercise LIII. *To determine the hysteresis loss in a sample of iron. Step-by-step method.*

APPARATUS. As in Laboratory Exercise L, § 223, with a special rheostat.

PROCEDURE. (1) Arrange the circuit as for the ring method (Fig. 154). With the double pole double throw switch toward the *M* side, calibrate the ballistic galvanometer as usual.

(2) After demagnetizing the ring, close the battery circuit with R' set for a small current value. This gives a magnetizing force of H_1, and a throw d_1 (Fig. 165). Diminish R' by a suitable step without breaking the circuit. This gives a magnetizing force H_2 and a throw d_2. Continue in this way until the point *a* is reached in about six steps. Rock the reversing switch about twenty times in order to establish this point, meanwhile having the key *k* open.

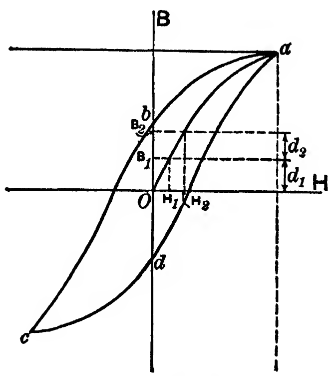

FIG. 165.

Decrease the current by increasing R', and return in a few steps to zero current at point *b*. Throw over the reversing switch and increase the current by steps as before until the lower maximum is reached at *c*. The current value, and hence H, should be the same as at point *a*. Establish this point as before by several reversals.

Return along *cd* by decreasing the current until *d* is reached, at which point the current and H are both zero. Reverse the current and increase by steps to point *a*. The entire cycle should have about twenty-five steps. The number of secondary turns may be changed at any time if necessary, if proper account is taken of the change in computing B. Great care must be taken in observing the throws, because

any error in one value of d will carry through to the end when the deflections are summed.

(3) Calculate values of H from equation (21). For each value of H the corresponding value of B is calculated from equation (18), D of the formula being the sum of all the deflections from zero up to that point. For example, the first value of B_1 corresponding to H_1 is calculated from d_1. The next value B_2 is calculated from $d_1 + d_2$, and so on. The data and results may be tabulated as shown.

I	H	d	Σd	B

(4) When the cusps at a and c are reached the deflection will change sign, because the induction changes from an increasing to a decreasing value. In view of this change it is advisable to interchange the galvanometer terminal connections at these points, in order that all the throws may be toward the same side of the scale. This obviates any error due to an inequality of fiber torsion. The values of d must be added algebraically, each with its proper sign.

(5) If at any time a resistance step is taken so great that the galvanometer throw is off the scale, the previous work must be discarded and the entire cycle repeated. In order to guard against this, a preliminary cycle should be carried through, choosing the steps so that the throw is always readable. The numbers of the steps chosen on the rheostat should be recorded and carefully followed in the subsequent observing program.

(6) It will frequently be found that the curve as drawn

does not close at *a*. It is probable that this is due to faulty readings or to errors in summation of *d*.

This method is subject to the criticism that the throw observed is not a true measure of the change in B, since there is a slow creeping up of the induction which may persist for a few seconds after the throw has taken place. This is due to the so-called *magnetic viscosity*. A magnetometer method is free from this error.

(7) The antecedent magnetic state is of great significance in all magnetic testing. If the piece has residual magnetism there is no definite relation between the impressed H and the induction density B. If the sample is not effectively demagnetized, the rising branch of the loop, *da* (Fig. 165) may cross the normal induction curve. Moreover, the loop will not be symmetrical above and below the H axis. One half of the difference between the extreme values of B, however, gives the value of B_{max}.

Sometimes the branch *oa* is omitted. The current is brought to the desired maximum value, reversed several times and decreased by steps, the starting point being taken at *a*. The values of H and B will then be plotted with reference to a temporary pair of axes as shown in the dotted lines in Fig. 165. The axes through *o* may be drawn subsequently, if desired.

(8) Determine the area of the loop with the planimeter, calculate the energy loss in ergs per cubic centimeter per cycle, and express the result also in watts per pound at a frequency of 60. The value of B_{max} should always be stated in connection with this result.

The calculation of the B and H values is somewhat laborious. It is advisable to make some trial calculations which will indicate whether the data may be expected to yield satisfactory results. Two methods for checking the correctness of the data are given in the following paragraphs.

Beginning at *o* (Fig. 165), the sum of the deflections from *o* to *a* should be one half of the value found by summing the deflections from *a* to *c*. Moreover, the summation of deflections over *cda* should be equal

to that over *abc*. If these relations are not found, we may infer that the loop will not close at *a*.

Another method is that of plotting roughly a trial curve between the sums of the deflections and the corresponding current values. If this curve closes and has a proper contour, success may be anticipated with the B–H curve, since B values are proportional to Σd, and H values are proportional to the current.

234. Hysteresis. Fixed Point Method. It has been seen that the area of the hysteresis loop is a measure of the energy loss per cycle, when iron or steel is subjected to a cyclic magnetization. The determination of the B values, as outlined in the step-by-step method of § 232, is open to certain objections. Errors in the readings are cumulative, and the readings for individual points cannot be repeated. The

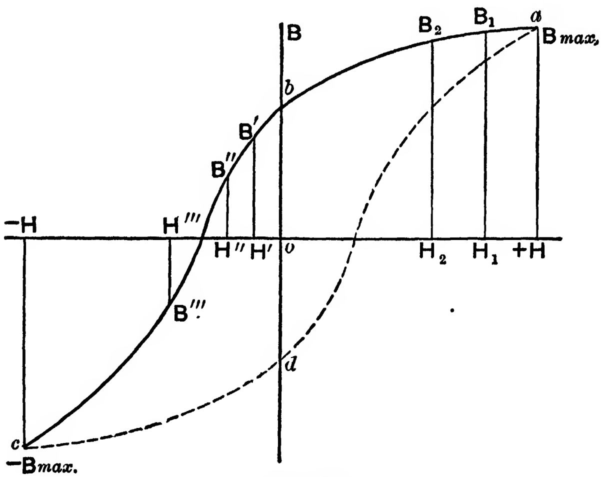

FIG. 166.

method described in the present article is free from these faults, inasmuch as each reading is referred to a fixed point.

Assume any chosen magnetizing force H applied to the test piece, which brings the iron to some point B_{max} (Fig. 166). If H is now reduced suddenly to a value H_1, the induction density drops to B_1, and the change in B which causes the

galvanometer throw D may be called $\Delta_1 B$. The relation between D and the value of ΔB is seen in equation (18), § 222. The iron is now brought back to its original state at a and again H is reduced, this time to H_2, which brings the iron to B_2, and the corresponding change in induction is $\Delta_2 B$. Continuing in this way for several steps, we reach the point b on the curve.

Points from b to c may be found by so arranging the apparatus that the current corresponding to the value of H at a is, by a single throw of the switch, (1) reduced to zero, (2) reversed, and (3) brought to some small initial value in the opposite direction. This impresses on the iron a magnetizing force H', which corresponds to an induction density B'. The change in induction density from its previous value may be represented by $\Delta' B$. From the original point a, proceeding in the same manner, we reach the point H'', with the corresponding induction density B''. In this way successive values of H are established, until finally the point c is reached, which corresponds to a reversal of the original value of the magnetizing current.

The galvanometer throw observed when the iron is carried from the point a to the point c is due to a change in induction density given by the equation

$$\Delta B = 2\, B_{max}.$$

The value of B_{max} is taken as one half of the total change in B, corresponding to a complete reversal of the original magnetizing current, and other values of B may be found by subtracting in succession the various values of ΔB from B_{max}. The values of H may be calculated from equation (21). With these corresponding values of H and B the curve from a through b to c may be plotted. The other half of the loop may be drawn in from symmetry. Further details of the method will be given in connection with the Laboratory Exercise LIV, § 235.

235. Laboratory Exercise LIV: *To determine the hysteresis loss in a sample of iron by the fixed point method.*

Apparatus. The same as in Laboratory Exercise L, § 223, with an additional rheostat and a knife switch.

Procedure. (1) Connect the circuit as in Fig. 167. The reversing switch W is so arranged that the points c and d are connected in the usual manner, while the points a and f are connected through an adjustable resistance R_2. The knife

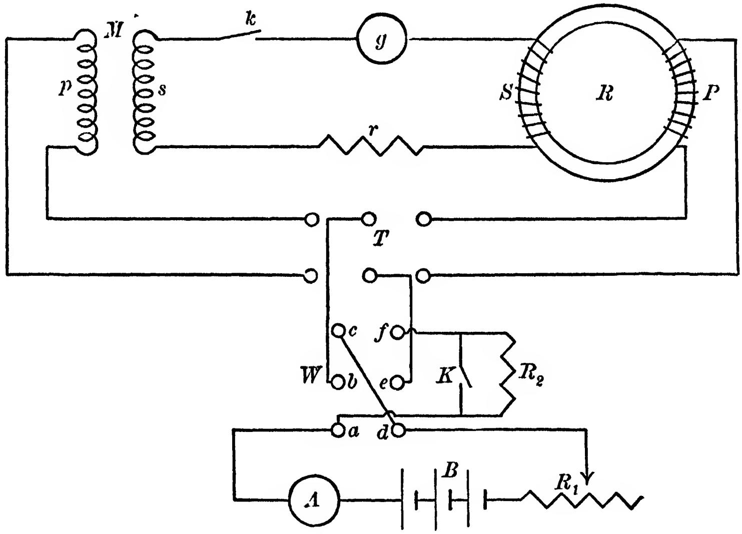

Fig. 167.

switch or key at K is used to short-circuit this resistance if desired. An examination of the figure will show that the switch W is an ordinary reversing switch when K is closed. When K is open, and W is on the points ad, the current has a maximum value determined by R_1. When W is on the points cf, the current will be diminished by the introduction of R_2 if K is open.

(2) With the double pole double throw switch T thrown to the mutual inductance side of the circuit, and with the key K closed, calibrate the galvanometer in the usual way.

(3) With the switch T on the ring side of the circuit and with W on the points cf, in which case the cross connections are in the magnetizing current circuit, adjust R_1 until a current value is reached such that the B limits will be about 6000 or 8000 gausses. It is left as an exercise for the student to show how the current is determined which corresponds to this value of B_{max}. With the key k open, and K closed, rock the switch back and forth several times in order to bring the iron into a cyclic state, finally resting on the points cf.

(4) To obtain points on the curve ab (Fig. 166), between the maximum positive value of H and zero, proceed as follows. With the switch W on the points cf, and with a small resistance in R_2, quickly open K and read the throw due to the sudden decrease in the induction density $\Delta_1 B$. Close K and rock W several times in order to establish again the original point a, with k open to avoid damaging the galvanometer. Increase R_2, open K again, and read the throw as before; this reading corresponds to a change $\Delta_2 B$. Continue in this way for about six steps until the magnetizing current has been reduced to zero.

(5) Points on the curve from b to c now remain to be determined. With the original value of the current, rock the reversing switch W several times as before with K closed, resting finally on points ad, in which case the cross connections are not in the current circuit. The iron is now at the point c (Fig. 166); to regain the value of B_{max} at a, the current through the magnetizing turns of the ring must be reversed. This may be done conveniently by interchanging the current wires at the primary terminals of the ring.

Reversing the switch W now will bring the original value of the magnetizing current to zero and reëstablish it in the opposite direction. If at the same time the resistance R_2 is introduced into the circuit, the limit of the magnetizing field in the negative direction is fixed at some value less in magnitude

than the preceding positive value. With K open, set R_2 at some high value which ensures a suitable small negative increment H', and read the throw when W is thrown over. This corresponds to some change in the induction density $\Delta' B$, and the iron is now at the point B'.

Again establish the point a by repeated reversals with K closed. With R_2 slightly decreased and with K open, reverse the switch and read the throw corresponding to a change $\Delta'' B$, from B_{max} to B''. Proceed in this way until the point c is reached.

The last step is taken with K closed, which corresponds to a complete reversal of the magnetizing field H, and gives the iron an induction density of $-B_{max}$.

Throughout the foregoing procedure the rheostat R_1, has not been changed, and the current should return to its original value.

(6) In order to prevent dangerous throws of the galvanometer the key k should be kept open except when a reading is to be made. Care should be taken to rock the reversing switch W several times before each reading in order to bring the induction density to a point which corresponds to the maximum value of H.

(7) This method possesses a distinct advantage over the step-by-step method, in that each point on the curve is determined independently, and is reached by a single step from the end of the cycle. This prevents the carrying forward and accumulation of errors. Each reading may be repeated as many times as desired, and its correctness checked. The cusps of the loop fall on the normal induction curve.

(8) Calculate values of ΔB and H from equations (18) and (21). Beginning at point a, plot the portion of the curve abc from the values of ΔB and H as computed above. Draw in the other side of the loop cda by symmetry and locate the permanent axes through o. Enter on these axes the appropriate scales of B and H values. On the curve the value of B_{max} is

represented by one half of the extreme vertical distance between the cusps.

(9) Determine the area of the completed loop with a planimeter, and calculate the energy loss in ergs per cubic centimeter per cycle. Also convert this into the equivalent watts per pound at a frequency of 60 cycles per second.

(10) Repeat the experiment, using a magnetizing current sufficient to produce B limits of 10,000 gausses.

236. Laboratory Exercise LV: *To determine the flux density in a permanent magnet.*

Apparatus. Standard mutual inductance, ballistic galvanometer, slip coil and magnet to be tested, battery, reversing switch, and ammeter.

Procedure. (1) Arrange the circuit as in Fig. 168. Connect the slip coil in series with the ballistic galvanometer and the secondary of the mutual inductance. Place the slip coil over the middle of the bar magnet so that the maximum number of lines of force link with it. Withdraw the coil quickly and read the throw d_1. The quantity induced in the galvanometer circuit is given by the formula

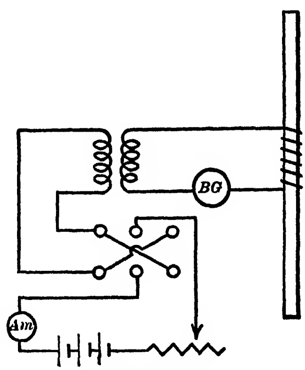

Fig. 168.

$$(42)\quad Q_1 = \frac{\Delta N}{R} = \frac{\Delta \mathsf{B} A S}{R} = G d_1$$

where $\Delta \mathsf{B} A$ is the total number of flux lines linked with the coil. In this equation, ΔN is the change in the number of linkings, $\Delta \mathsf{B}$ is the change in the induction density (from B to zero), S is the number of wire turns in the coil, A is the cross section of the bar, and R is the total secondary circuit resistance.

(2) Adjust R' so that a suitable current of strength i passes through the primary coil of the mutual inductance M, and reverse the switch, reading the throw d_2. The quantity induced in the galvanometer circuit is

$$Q_2 = \frac{2\,Mi}{R} = Gd_2. \tag{43}$$

Dividing (42) by (43), and solving for ΔB we have

$$\Delta B = \frac{2\,M}{AS} \cdot \frac{id_1}{d_2}. \tag{44}$$

It is evident in this equation that ΔB is the value of B in the bar.

If M, i, and A are in absolute C. G. S. units, then B will be given in gausses. If M is in henrys and i is in amperes, the equation for B becomes

$$B = \frac{2\,Mi}{AS} \cdot \frac{d_1}{d_2} 10^8. \tag{45}$$

(3) Repeat the readings for d_1 and d_2 several times and calculate the value of B.

(4) State in connection with the value of B the dimensions and shape of the test piece, and calculate the value of the intensity of magnetization.

It must be remembered that this test gives information for this particular form of the test piece, and yields results characteristic of the material used only when the bar is very long.

237. The Measurement of Core-Loss. Hysteresis has been defined as the lag of the change in induction density behind the change in the magnetizing force. In order to carry a piece of iron through a complete magnetic cycle a certain amount of energy is required. This ultimately takes the form of heat, and is known as the hysteresis loss. Moreover, on account of the cyclic magnetization, eddy currents are set up in the iron, and these also represent a definite energy equivalent.

In iron cores used with alternating current circuits these effects are closely associated, and can only be determined separately by careful measurements. These combined effects are known as ***core-loss***, and this may be expressed in terms of energy units per unit mass of material per cycle, or better, it may be expressed by the power consumption in watts per pound or per kilogram of material, for a given frequency and a given maximum induction density.

A method due to Epstein is widely used for making core-loss tests. The following paragraphs which describe this method are adapted from the standard specifications for magnetic tests as adopted by the American Society for Testing Materials in 1911[1].

The standard core-loss is the power in watts consumed in each kilogram of material at a temperature of 25° C., when submitted to a harmonically varying induction, having a maximum of 10,000 gausses and a frequency of 60 cycles per second, when measured as specified.

The magnetic circuit consists of 10 kilograms (22 pounds) of the test material, cut into strips 50 centimeters long and 3 centimeters wide, half parallel and half at right angles to the direction of rolling. These are assembled in four equal bundles and arranged in the four sides of a square with butt joints, the opposite sides consisting of strips cut from the sheets in the same way with reference to the direction of rolling.

No insulation other than the natural scale of the material (except in the case of scale free material) is used between laminations, and the corner joints are separated by paper strips, 0.01 inch in thickness.

The magnetizing winding consists of four solenoids surrounding the four sides of the magnetic circuit, and joined in series. This is represented in Fig. 169 as a single coil M. A secondary

[1] *Transactions of the American Society for Testing Materials*, 1911; Vol. XI, p. 110.

coil *P* is used for energizing the voltmeter and the potential coil of the wattmeter. These coils are wound on forms of non-magnetic, non-conducting material, of the following dimensions:

Inside cross section:	4 by 4 centimeters,
Thickness of wall:	not over 0.3 centimeter.
Winding length:	42 centimeters.

The winding on each solenoid should consist of 150 turns of copper wire uniformly wound over the 42 centimeters of length. The total resistance of the magnetizing coils is between 0.3 and 0.5 of an ohm. The secondary winding of 150 turns of copper wire on each solenoid is similarly wound beneath the primary coils. Its resistance should not exceed one ohm.

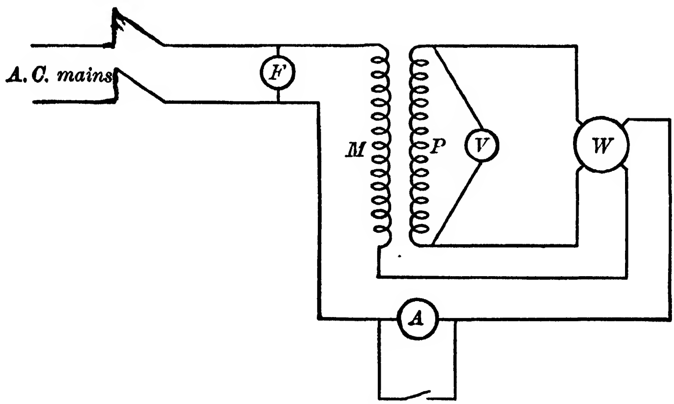

Fig. 169.

The arrangement of the circuit is shown in Fig. 169. A voltmeter *V*, and the potential coil of a wattmeter *W*, are connected in parallel to the terminals of the secondary winding of the apparatus. The current coil of the wattmeter is connected in series with the primary winding. A frequency meter is placed at *F*. An alternating E. M. F. of the pure sine curve type is impressed on the primary windings, and is adjusted

until the voltage of the secondary circuit is given by the equation

$$E = \frac{4 fNn\mathsf{B}M}{4\,LD\,10^8}, \tag{46}$$

where the letters denote the following quantities:

f = Form factor[1] of the primary E. M. F. wave = 1.11 for a sine wave.

N = Number of secondary turns = 600.

n = Number of cycles per second = 60.

B = Maximum induction density = 10,000.

M = Total mass in grams = 10,000.

L = Length of strips in centimeters = 50.

D = Specific gravity.

E = 106.6 volts for a sine wave with high resistance steel.

E = 103.8 volts for a sine wave with low resistance steel.

A specific gravity of 7.5 is assumed for all steel samples having a resistivity of more than two ohms per meter-gram, and 7.7 for all steel samples having a resistivity of less than two ohms per meter-gram. These are designated as high resistance and low resistance steels respectively. The formula is derived as follows.

The maximum magnetic flux through the iron or steel may be represented by ϕ. This will rise and fall and reverse its direction with the magnetizing field of the alternating current, and it will cut the wire turns of the voltage coil four times per

[1] If the average is taken of a large number of equidistant ordinates for a sine curve, it is found to be 0.6369 of the maximum ordinate. That is, the average value of the E.M.F. is 0.6369 of the maximum value. The reading on the voltmeter, however, is not the maximum value, nor is it the average value, but it is that value of the alternating voltage which is equivalent to the square root of the mean of the squares of the several ordinates. This is called the effective value, and may be shown to be 0.707 of the maximum ordinate. The ratio of the effective to the average value is 1.11 for a sine curve, and this ratio is called the form factor of the wave. Proofs of these relations may be found in FLEMING, *The Alternating Current Transformer*, Vol. 1, pp. 98–103.

cycle. For a frequency n the flux will cut the wire turns $4n$ times per second. If there are N wire turns on the voltage coil there will be $4nN$ flux turns or linkings per second. From the Faraday equation, § 129, the average voltage induced is equal to the time rate of change of the number of linkings; whence we have

$$E_{av} = \frac{4\,nN\phi_{max}}{10^8}\text{ volts.} \tag{47}$$

The value of ϕ_{max} may be replaced by $A\mathsf{B}_{max}$, where A is the area of cross section of the core. Moreover, since we know that

$$\text{volume} = \frac{\text{mass}}{\text{density}},$$

we may write

$$4\,LA = \frac{M}{D},$$

or

$$A = \frac{M}{4\,LD}, \tag{48}$$

It follows that

$$\phi_{max} = \frac{\mathsf{B}_{max}M}{4\,LD}. \tag{49}$$

Combining equations (47) and (49), we obtain the equation

$$E_{av} = \frac{4\,nN\mathsf{B}_{max}M}{4\,LD\,10^8}. \tag{50}$$

Since form factor is defined as the ratio of the effective to the average value of the voltage during a half cycle, equation (50) may be written in the form

$$E_{eff} = E_{av}f = \frac{4\,fnNM\mathsf{B}_{max}}{4\,LD\,10^8}, \tag{51}$$

whence

$$\mathsf{B}_{max} = \frac{4\,LDE_{eff}\,10^8}{4\,fnNM}. \tag{52}$$

The wattmeter gives the power consumed in the iron and the secondary circuit. Subtracting the correction terms from the total, and dividing by the mass in kilograms, gives the loss in watts per kilogram under standard conditions.

It is often desirable to separate the losses due respectively to hysteresis and to eddy currents. This may be done by taking advantage of the fact that for a given value of the maximum induction density, hysteresis varies directly with the frequency, while the eddy current loss varies with the square of the frequency. If the total core-loss is determined for two known and different frequencies for the same value of B_{max}, two simultaneous equations may be written with two unknown quantities, from which either one of the two factors may be found.

The method described above requires expensive equipment and massive samples, and it may be replaced oftentimes by the following method, which yields results of sufficient accuracy for many purposes. A small bundle of strips of the test material is placed in a solenoid energized by alternating current, with the current coils of the wattmeter in series with the magnetizing circuit. Two separate secondary windings are provided for the potential coils of the wattmeter and for the voltmeter. A low frequency is used in order to minimize the eddy current effects. Empirical corrections are applied for the irregular flux distribution in the sample. Measurements thus made are accurate within 5 %.

A still further simplification gives results sufficient for comparative purposes, in which a bundle of strips of the test material, of specified dimensions, is placed in a magnetizing solenoid, and wattmeter readings are compared with others taken for a standard sample under the same conditions.

The various electrical handbooks give valuable information on these methods of testing, and also on the values of core-loss in representative materials.

238. Laboratory Exercise LVI: *To determine core-loss with the Epstein equipment.*

APPARATUS. Epstein permeameter and accessories, source of suitable alternating current, voltmeter, ammeter, wattmeter, frequency meter, and samples to be tested.

PROCEDURE. (1) Arrange the circuit as shown in Fig. 169. Place the bundles of steel in the solenoids, with strips cut the same way in opposite sides, and with one thickness of paper separating the ends. Clamp the bundles in place before closing the line switch. Rap the joints with a wooden or raw-hide mallet until the ammeter shows a minimum magnetizing current. Short-circuit the ammeter with a knife-switch. If standard conditions are to be used, adjust the voltage and frequency to normal. Never adjust the voltage by inserting inductive resistance, since this will change the wave form.

(2) Read the wattmeter and voltmeter simultaneously and note the frequency. Subtract from the wattmeter reading the E^2/R loss in the potential coils of the two instruments. The losses in coils of the permeameter are negligible.

(3) Divide the corrected loss by the mass. This gives the core-loss in watts per kilogram. Express this result also in watts per pound.

(4) Calculate B from equation (52).

(5) Repeat for three other samples.

(6) In order to separate the hysteresis effects from the eddy current loss, the test may be repeated at a different frequency.

EXERCISES

1. Reduce a loss of 0.9 watt per pound, $\mathsf{B}_{max} = 10{,}000$, at 100 cycles per second, to ergs per cubic centimeter per cycle.

2. Given an iron ring 40 centimeters in diameter, of cross section 5 square centimeters, overwound with 200 primary turns, and 100 secondary turns. A current is passed through the primary of such value that the iron has a permeability of 1000. Calculate the mutual inductance of the coils in millihenrys.

3. Show that the number of ampere turns necessary to establish a flux density B through an air gap of length L cm. is 0.8 $\mathsf{B}L$.

APPENDIX

PART I. ABSOLUTE MEASUREMENTS

239. Absolute Measurements. An important class of electric measurements, which can only be undertaken in a well equipped precision laboratory, includes the determination of the fundamental electric quantities, current, resistance and potential difference, directly in terms of the centimeter, the gram, and the second.[1]

Bodies which are electrically charged are known to attract or repel one another with forces which are proportional to the magnitudes of the charges. An instrument called the ***electrometer*** is used to measure directly the force action between parallel plates, when they are charged with a given potential difference. The theory and description of this instrument in its various forms will be found in the larger textbooks on electricity. In brief, the determination of a potential difference is made to depend upon force measurements between parallel charged plates. On account of the difficulty of making plates perfectly plane, and keeping them exactly parallel, the precision attained is not high, and the direct determination of a potential difference in absolute measure is not attempted.

The International Conference, London, 1908, selected the ampere and the ohm as the two units which could be most easily evaluated in absolute measure, and from these two, the volt is readily fixed by the relation given in Ohm's Law.

240. The Absolute Measure of Current Strength. The forces due to the magnetic fields about current-carrying con-

[1] See GRAY, *Absolute Measurements in Electricity and Magnetism*, vol. II.

ductors may be measured directly in dynamic units in any one of the three following ways.

I. If a standard magnetic field is available, equations similar to those for the tangent galvanometer will give the value of a current strength in terms of the field strength, together with certain geometric constants of the circuit.

II. The torque due to the reaction of magnetic fields about fixed and movable coils may be expressed in terms of the elastic constants of a suspension fiber, when the suspended system moves about a vertical axis.

III. The force or torque due to the magnetic field reactions about adjacent coils may be balanced against the gravitational pull of the earth on known masses, if the axis of rotation of the system is horizontal.

Absolute methods of measurement of current strength have been based upon all of these principles, but it is by means of

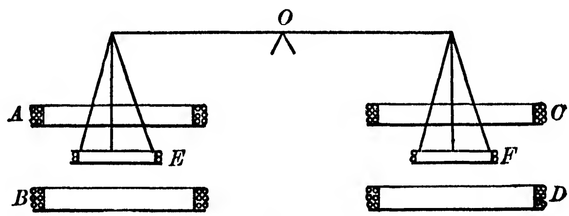

FIG. 170.

the third that the most recent and the most accurate determinations have been made. The principle is illustrated by Fig. 170.

The coils *A*, *B*, *C*, and *D* are fixed in position, and coils *E* and *F* are hung symmetrically between them, from the arms of a balance. If the same current passes in series through all six of the coils, in such direction that the acting torques are all in the same sense with respect to the axis of rotation *O*, the tendency of the beam to turn may be compensated by the pull of the earth on standard masses placed on the beam or on pans

attached to the coils E and F. The total torque is proportional to the square of the current strength, and the value of the current may be calculated from the forces acting, the dimensions of the coils, and the number of wire turns. In recent work the result is made to depend upon the ratio of the radii of the fixed and movable coils. This ratio can be obtained by an electric method with greater precision than by direct measurement.

The current strength determined in this way is expressed in absolute C. G. S. units. A silver voltameter in series with the coils enables the experimenter to express the result in terms of the mass of silver deposited in one second. Moreover, if a standard resistance is in series with the circuit, the potential difference across its terminals when the measured current is flowing, may be compared with a Weston normal cell. In this way the value of the International ampere may be expressed in terms of absolute units.

The precision attainable in these measurements is of the order of two or three parts in 100,000.[1]

241. The Absolute Measurement of Resistance. The necessity of finding the resistance of some particular conductor in absolute measure was recognized early. In 1863 such determinations were undertaken by a committee of the British Association. Many methods have since been suggested and developed. That due to Lorenz is probably the simplest in theory and in practice.

In § 109 it was shown that a standard magnetic field of known strength is realized at the center of a long solenoid which carries a steady current. Let the outer circle AA, Fig. 171, represent one turn at the middle of such a solenoid. The magnetic field is here perpendicular to the plane of the paper.

[1] For a detailed discussion of these methods, together with references to standard papers on the subject, see U.S. Bureau of Standards, *Bulletin*, Vol. 8, p. 269, 1912.

With the battery polarity as indicated, the direction of the field is away from the reader. A circular disk D of radius r is mounted with its plane at right angles to the axis of the solenoid. This disk can be maintained in uniform rotation by an auxiliary motor of any kind, and it is equipped with a device for accurately counting the number of revolutions per second. Brush contacts are applied at the axis and rim of this disk. The radial filament of the disk connecting the points of contact of the brushes may be regarded as a conductor which is cutting lines of force. A constant potential difference exists between the brushes when a uniform rotation is maintained. Let H represent the flux density through the disk and A its effective area. The total flux through the disk is then given by

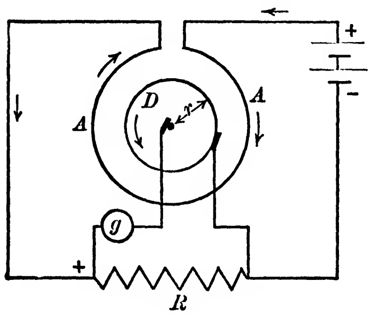

FIG. 171.

$$\phi = HA,$$

and this flux is cut once by the line connecting the brushes, for each complete revolution. If the time of one revolution is t seconds, then the number of lines of magnetic flux cut in one second is given by ϕ/t. This may be regarded as the rate of change of linking of magnetic lines with the radius of the disk which connects the brushes, and the potential difference between the contact points may then be found by means of Faraday's equation (7), § 129.

We may then write

$$E = \frac{dN}{dt} = \frac{\phi}{t} = \frac{HA}{t}.$$

By equation (33), § 109, we have

$$H = 4\,\pi n i,$$

whence

$$E = \frac{4\pi niA}{t}.$$

From the equations of § 131, it is seen that the mutual inductance between the disk and the solenoid, or the number of flux lines passing through the disk when unit current flows in the coil, is given by the equation

$$M = 4\pi nA,$$

whence

$$E = \frac{Mi}{t}.$$

The brushes are connected to the terminals of the resistance R which is to be determined, and a galvanometer g is included in the circuit. The connections are made so that the potential difference due to the cutting of magnetic lines is opposed to the potential drop through R. If the value of i and the speed of rotation are adjusted so that no deflection occurs on the galvanometer, then we may write

$$\frac{Mi}{t} = iR,$$

or

$$R = \frac{M}{t}.$$

Since the time of one revolution is always the reciprocal of the number of revolutions per second z, we have

$$R = Mz.$$

The value of M can be calculated directly from the geometric constants of the apparatus, and z can be determined accurately. In this way the resistance of a wire coil or of a mercury column can be directly evaluated in absolute measure.[1]

[1] For a detailed account of the construction of primary mercurial resistance standards, see U. S. Bureau of Standards, *Bulletin*, Vol. 12, p. 375.

Part II. Table

Conductivity of Normal Solution of KCl

[74.60 grams of KCl in one liter of solution.]

Temp.	Conductivity	Temp.	Conductivity
15° C.	0.09264 mhos	22° C.	0.10595 mhos
16	0.09443	23	0.10788
17	0.09663	24	0.10981
18	0.09824	25	0.11174
19	0.10016	26	0.11392
20	0.10209	27	0.11614
21	0.10402	28	0.11840

Mathematical and Physical Constants in Frequent Use

Base of Napierian logarithms	= 2.71828
To convert common logarithms into Napierian, multiply by	= 2.30258
To convert Napierian logarithms into common logarithms, multiply by	= 0.43429
Ratio of circumference to diameter of a circle	= 3.1416
π^2	= 9.8696
4π	= 12.566
$\frac{1}{4\pi}$	= 0.07957
$\log 4\pi$	= 1.0992
Density of mercury	= 13.5956 gr. per cubic centimeter
Electrochemical equivalent of silver	= 0.00111804 gr. per coulomb
Acceleration of gravity, Potsdam	=981.274 cm. sec.
Acceleration of gravity, Washington, D.C.	=980.094 cm. sec.

PART III. STANDARD REFERENCE BOOKS

Fleming — Handbook for the Electrical Laboratory, 2 vols.
Fleming — Alternate Current Transformer, Vol. I.
Karapetoff — Experimental Electrical Engineering, 2 vols.
Kempe — Handbook of Electrical Testing.
Kohlrausch — Lehrbuch der Praktischen Physik.
Roller — Electric and Magnetic Measurements.
Edgcumbe — Industrial Electrical Measuring Instruments.
Murdock and Ochswold — Electrical Instruments.
Ewing — Magnetic Induction in Iron and other Metals.
Du Bois — The Magnetic Circuit in Theory and Practice.
Northrup — Methods of Measuring Resistance.
Foster and Porter — Electricity and Magnetism.
Starling — Electricity and Magnetism.
Brooks and Poyser — Magnetism and Electricity.
Thompson, S. P. — Elementary Lessons in Electricity and Magnetism.
Gray — Absolute Measurements in Electricity and Magnetism.
Palmer — Theory of Measurements.

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Printed in the United States of America.